Problem Assignment on DFT / FFT

1) A discrete filter has an impulse response h(0)=0.5, h(1)=1, h(n)=0 for all other n. (Note: This is called a Finite Impulse Response (FIR) filter or system.)

(a) Find the 2-point DFT for h(n), H(k).

(b) The following operation is performed on a 256-point data block, x(n):

x(n) {256-point FFT} → X(k)
h(n) {256-point FFT} → H(k)
Y(k) = H(k)X(k)
Y(k) {256-point IFFT} → y(n)

If $\tilde{y}(n)$ is the linear convolution of x(n) and h(n), for what values of n (on the range 0,1,...255) are y and $\tilde{y}$ equal? Write an expression for y(n) in terms of $\tilde{y}(n)$.

(c) If a decimation in time FFT algorithm is used in (b), how many stages of computation will be required? Same question for a decimation in frequency algorithm.

2) Let $x(n)$ be a real-valued N-point ($N = 2^v$) sequence. Develop a method to compute an N-point DFT $X'(k)$, which contains only the odd harmonics [i.e., $X'(k) = 0$ if k is even] by using only a real $N/2$-point DFT.

3) A designer has available a number of eight-point FFT chips. Show explicitly how he should interconnect three such chips in order to compute a 24-point DFT.

4) Compute the eight-point DFT of the sequence

$$x(n) = \{\tfrac{1}{4}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, 0, 0\}$$

using the in-place radix-2 decimation-in-time and radix-2 decimation-in-frequency algorithms. Follow exactly the corresponding signal flow graphs and keep track of all the intermediate quantities by putting them on the diagrams.