ECE 431

Problem assignment on discrete time system realizations (Solutions)

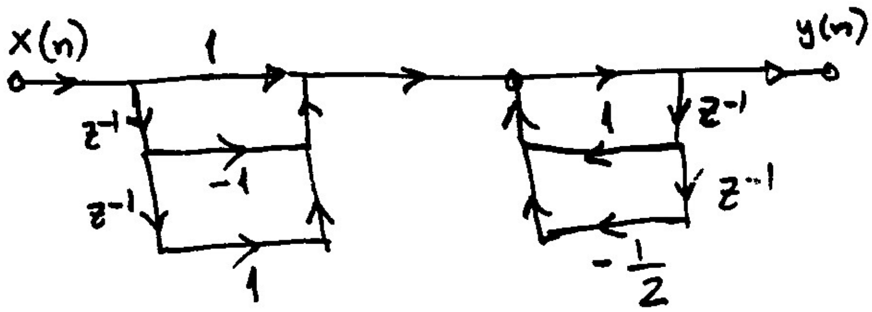
Problem 1

$$y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) - x(n-1) + x(n-2) \Rightarrow$$

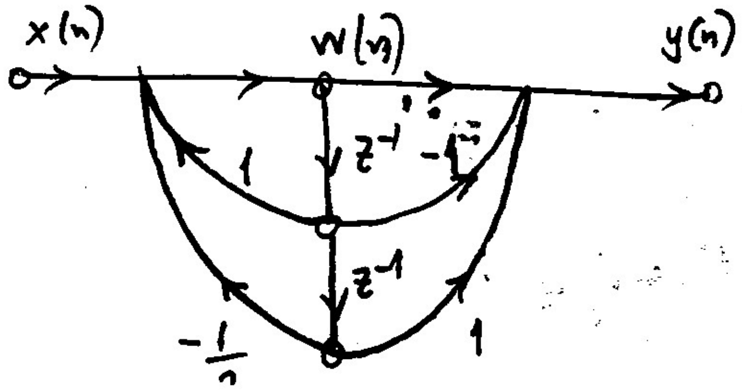
$$\Rightarrow Y(z)\left(1 - z^{-1} + \frac{1}{2}z^{-2}\right) = X(z)\cdot\left(1 - z^{-1} + z^{-2}\right) \Rightarrow$$

$$\Rightarrow \boxed{H(z) = \frac{1 - z^{-1} + z^{-2}}{1 - z^{-1} + \frac{1}{2}z^{-2}}}$$

DIRECT I



DIRECT II, Cascade, Parallel



Pr.: 2

Determine the cascade and parallel realizations for the system described by the system function

$$H(z) = \frac{10(1-\frac{1}{2}z^{-1})(1-\frac{2}{3}z^{-1})(1+2z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{1}{8}z^{-1})[1-(\frac{1}{2}+j\frac{1}{2})z^{-1}][1-(\frac{1}{2}-j\frac{1}{2})z^{-1}]}$$

Solution: The cascade realization is easily obtained from this form. One possible pairing of poles and zeros is

$$H_1(z) = \frac{1-\frac{2}{3}z^{-1}}{1-\frac{7}{8}z^{-1}+\frac{3}{32}z^{-2}}$$

$$H_2(z) = \frac{1+\frac{3}{2}z^{-1}-z^{-2}}{1-z^{-1}+\frac{1}{2}z^{-2}}$$

and hence

$$H(z) = 10H_1(z)H_2(z)$$

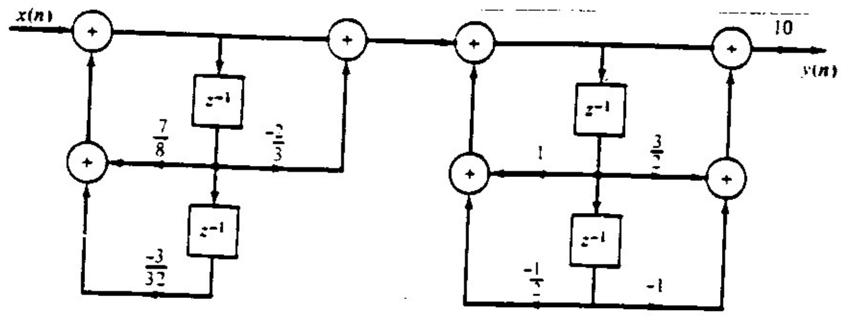
The cascade realization is depicted in Fig. 7.23a.

To obtain the parallel-form realization, $H(z)$ must be expanded in partial fractions. Thus we have

$$H(z) = \frac{A_1}{1-\frac{3}{4}z^{-1}} + \frac{A_2}{1-\frac{1}{8}z^{-1}} + \frac{A_3}{1-(\frac{1}{2}+j\frac{1}{2})z^{-1}} + \frac{A_3^*}{1-(\frac{1}{2}-j\frac{1}{2})z^{-1}}$$

where A_1, A_2, A_3, and A_3^* are to be determined. After some arithmetic we find that

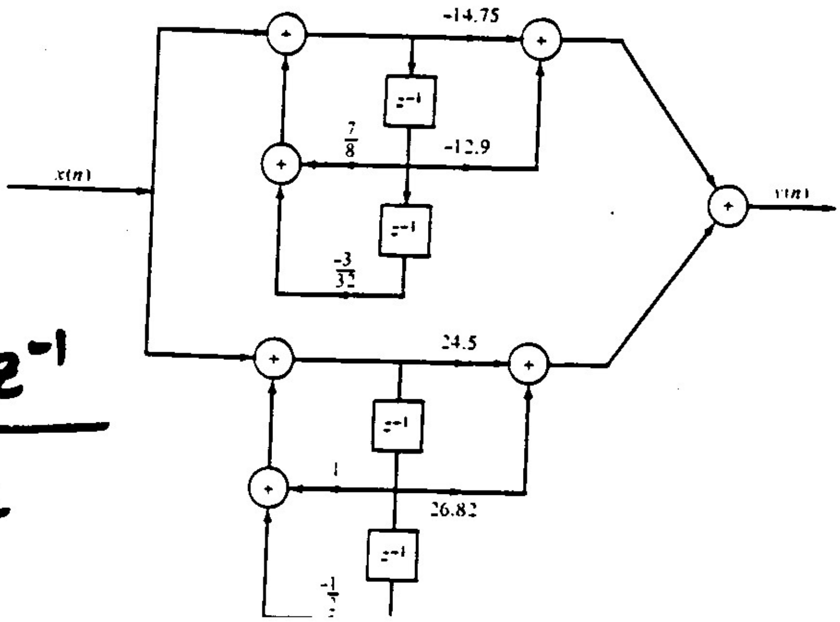
$$A_1 = 2.93, \quad A_2 = -17.68, \quad A_3 = 12.25 - j14.57, \quad A_3^* = 12.25 + j14.57$$



(a)

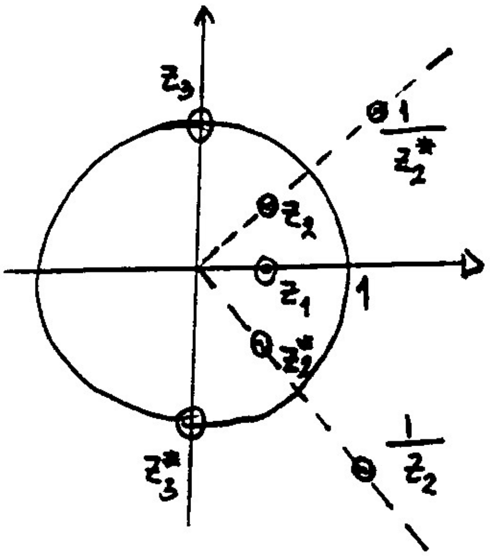
$$\frac{-14.75 - 12.9z^{-1} \text{ [redacted]}}{1-\frac{7}{8}z^{-1}+\frac{3}{32}z^{-2}}$$

$$\frac{24.5 + 26.82z^{-1}}{1-z^{-1}+\frac{1}{2}z^{-2}}$$



Problem 3

a)



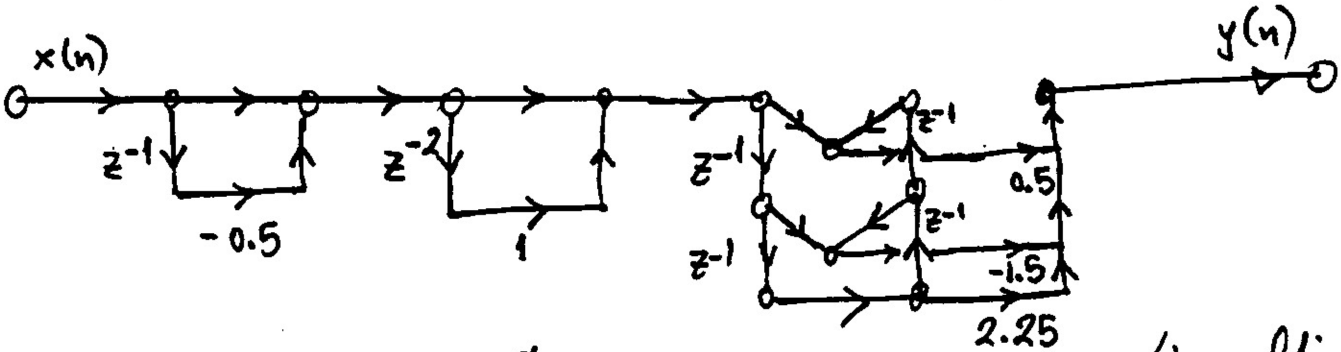
* If a real linear phase FIR Filter has a zero at $z = z_2$ then it also has zeros at $z = \frac{1}{z_2}, z_2^*, \frac{1}{z_2^*}$

* If a complex linear phase FIR filter has a zero at $z = z_2$ it also has a zero at $z = \frac{1}{z_2}$

b) $H(z) = z^{-5} \cdot (z-0.5) \cdot \underbrace{(z-j)(z+j)} \cdot \underbrace{[1-(0.5-j0.5)z^{-1}] \cdot [1-(0.5+j0.5)z^{-1}] \cdot [1-(0.5-j0.5)z] \cdot [1-(0.5+j0.5)z]} =$

$= z^{-5} \cdot (z-0.5)(z^2+1)\,(0.5z^2 - 1.5z + 2.25 - 1.5z^{-1} + 0.5z^{-2}) =$

$= \underbrace{(1-0.5 \cdot z^{-1})}_{\text{First-order non-linear phase FIR}} \underbrace{(1+z^{-2})}_{\text{Second-order linear phase FIR}} \underbrace{(0.5 - 1.5z^{-1} + 2.25z^{-2} - 1.5z^{-3} + 0.5z^{-4})}_{\text{Fourth-order linear-phase FIR.}}$



1 + 0 + 3 = 4 multiplie.