ECE 431 S

Hw Assignement # 8

1. We wish to design an FIR lowpass filter satisfying the specifications

$$0.95 < H(e^{j\omega}) < 1.05, \qquad 0 \le |\omega| \le 0.25\pi,$$

$$-0.1 < H(e^{j\omega}) < 0.1, \qquad 0.35\pi \le |\omega| \le \pi.$$

by applying a window $w[n]$ to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff $\omega_c = 0.3\pi$. Which of the filters listed in Section 7.2.1 can be used to meet this specification? For each window that you claim will satisfy this specification, give the minimum length $M + 1$ required for the filter.

2. Suppose that we are given an ideal lowpass discrete-time filter with frequency response

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi/4, \\ 0, & \pi/4 < |\omega| \le \pi. \end{cases}$$

We wish to derive new filters from this prototype by manipulations of the impulse response $h[n]$.

(a) Plot the frequency response $H_1(e^{j\omega})$ for the system whose impulse response is $h_1[n] = h[2n]$.

(b) Plot the frequency response $H_2(e^{j\omega})$ for the system whose impulse response is

$$h_2[n] = \begin{cases} h[n/2], & n = 0, \pm 2, \pm 4, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

(c) Plot the frequency response $H_3(e^{j\omega})$ for the system whose impulse response is $h_3[n] = e^{j\pi n}h[n] = (-1)^n h[n]$.

3. The frequency response of a desired filter $h_d[n]$ is shown in Figure P7.37-1. In this problem, we wish to design an N-point causal linear-phase FIR filter $h[n]$ that minimizes the integral-squared error

$$\epsilon = \frac{1}{2\pi}\int_{-\pi}^{\pi} |A(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega.$$

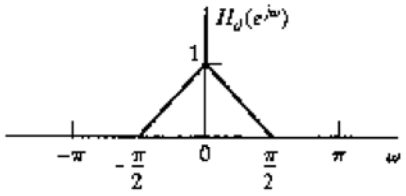

Figure P7.37-1

where the frequency response of the filter $h[n]$ is

$$H_d(e^{j\omega}) = A(e^{j\omega})e^{-j\omega(N-1)/2}$$

and N is an odd integer.

(a) Determine $h_d[n]$.

(b) What symmetry should $h_d[n]$ have in the range $0 \le n \le N-1$? Briefly explain your reasoning.

(c) Determine $h_d[n]$ in the range $0 \le n \le N-1$.

(d) Determine an expression for the minimum integral-squared error ϵ as a function of $h_d[n]$ and N.