7.14. Using the bilinear transform frequency mapping equation,

$$\begin{aligned}\Omega_c &= \frac{2}{T}\tan\left(\frac{\omega_c + 2\pi k}{2}\right), \quad \text{k an integer} \\ &= \frac{2}{T}\tan\left(\frac{\omega_c}{2}\right)\end{aligned}$$

$$T = \frac{2}{2\pi(300)}\tan\left(\frac{3\pi/5}{2}\right) = 1.46 \text{ ms}$$

The only ambiguity in the above is the periodicity in ω. However, the periodicity of the tangent function "cancels" the ambiguity and so T is unique.

1. **7.15.** This filter requires a maximal passband error of $\delta_p = 0.05$, and a maximal stopband error of $\delta_s = 0.1$. Converting these values to dB gives

$$\begin{aligned}\delta p &= -26 \text{ dB} \\ \delta s &= -20 \text{ dB}\end{aligned}$$

This requires a window with a peak approximation error less than -26 dB. Looking in Table 7.1, the Hanning, Hamming, and Blackman windows meet this criterion.

Next, the minimum length L required for each of these filters can be found using the "approximate width of mainlobe" column in the table since the mainlobe width is about equal to the transition width. Note that the actual length of the filter is $L = M + 1$.

Hanning:

$$\begin{aligned}0.1\pi &= \frac{8\pi}{M} \\ M &= 80\end{aligned}$$

Hamming:

$$\begin{aligned}0.1\pi &= \frac{8\pi}{M} \\ M &= 80\end{aligned}$$

Blackman:

$$\begin{aligned}0.1\pi &= \frac{12\pi}{M} \\ M &= 120\end{aligned}$$

7.16. Since filters designed by the window method inherently have $\delta_1 = \delta_2$ we must use the smaller value for δ.

$$\begin{aligned}\delta &= 0.02 \\ A &= -20\log_{10}(0.02) = 33.9794\end{aligned}$$

$$\beta = 0.5842(33.9794 - 21)^{0.4} + 0.07886(33.9794 - 21) = 2.65$$

$$M = \frac{A - 8}{2.285\Delta\omega} = \frac{33.9794 - 8}{2.285(0.65\pi - 0.63\pi)} = 180.95 \rightarrow 181$$

7.17. Using the relation $\omega = \Omega T$, the specifications which should be used to design the prototype continuous-time filter are

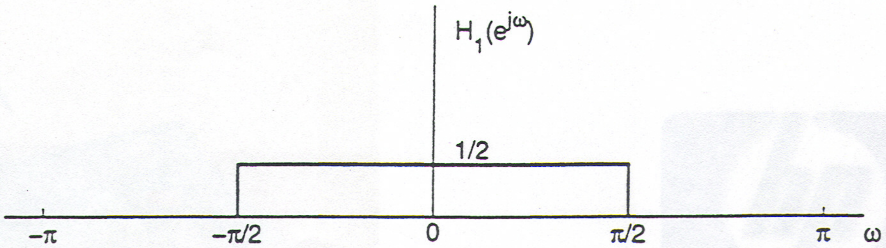
$$\begin{aligned}-0.02 < H(j\Omega) < 0.02, &\quad 0 \le |\Omega| \le 2\pi(20) \\ 0.95 < H(j\Omega) < 1.05, &\quad 2\pi(30) \le |\Omega| \le 2\pi(70) \\ -0.001 < H(j\Omega) < 0.001, &\quad 2\pi(75) \le |\Omega| \le 2\pi(100)\end{aligned}$$

2 ~~7.27.~~

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| \le \pi \end{cases}$$

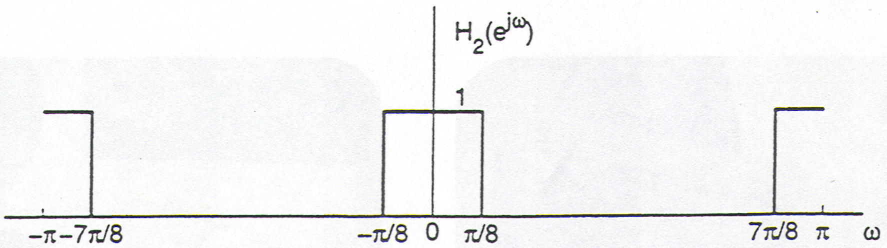
(a)

$$\begin{aligned} h_1[n] &= h[2n] \\ H_1(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} h[2n]e^{j\omega n} \\ &= \sum_{n \text{ even}} h[n]e^{\frac{j\omega n}{2}} \\ &= \sum_{n=-\infty}^{\infty} \frac{1}{2}\left[h[n] + (-1)^n h[n]\right] e^{j\frac{\omega n}{2}} \\ &= \frac{1}{2}H(e^{j\frac{\omega}{2}}) + \frac{1}{2}H\left(e^{j\frac{\omega+2\pi}{2}}\right) \end{aligned}$$



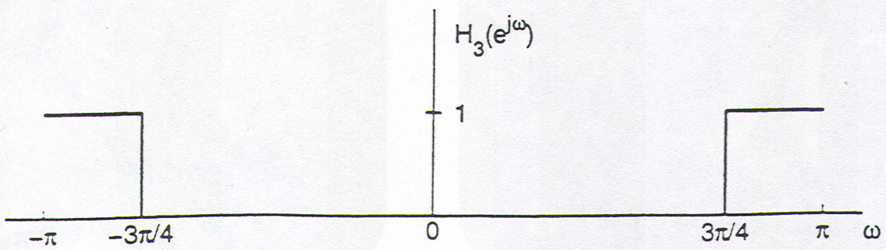
(b)

$$\begin{aligned} H_2(e^{j\omega}) &= \sum_{n \text{ even}} h[n/2]e^{-j\omega n} \\ &= \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega 2n} \\ &= H\left(e^{j2\omega}\right) \end{aligned}$$



(c)

$$H_3(e^{j\omega}) = H\left(e^{j(\omega+\pi)}\right)$$



10th order pole

(3) ~~7.37.~~ (a) The most straightforward way to find $h_d[n]$ is to recognize that $H_d(e^{j\omega})$ is simply the (periodic) convolution of two ideal lowpass filters with cutoff frequency $\omega_c = \pi/4$. That is,

$$H_d(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{lpf}(e^{j\theta})H_{lpf}(e^{j(\omega-\theta)})d\theta$$

where

$$H_{lpf}(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{4} \\ 0, & \text{otherwise} \end{cases}$$

Therefore, in the time domain, $h_d[n]$ is $(h_{lpf}[n])^2$, or

$$\begin{aligned} h_d[n] &= \left(\frac{\sin(\pi n/4)}{\pi n}\right)^2 \\ &= \frac{\sin^2(\pi n/4)}{\pi^2 n^2} \end{aligned}$$

(b) $h[n]$ must have even symmetry around $(N-1)/2$. $h[n]$ is a type-I FIR generalized linear phase system, since N is an odd integer, and $H(e^{j\omega}) \neq 0$ for $\omega = 0$. Type-I FIR generalized linear phase systems have even symmetry around $(N-1)/2$.

(c) Shifting the filter $h_d[n]$ by $(N-1)/2$ and applying a rectangular window will result in a causal $h[n]$ that minimizes the integral squared error ϵ. Consequently,

$$h[n] = \frac{\sin^2\left[\frac{\pi}{4}(n - \frac{N-1}{2})\right]}{\pi^2(n - \frac{N-1}{2})^2} w[n]$$

where

$$w[n] = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

(d) The integral squared error ϵ

$$\epsilon = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|A(e^{j\omega}) - H_d(e^{j\omega})\right|^2 d\omega$$

can be reformulated, using Parseval's theorem, to

$$\epsilon = \sum_{-\infty}^{\infty} |a[n] - h_d[n]|^2$$

Since

$$a[n] = \begin{cases} h_d[n], & -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned} \epsilon &= \sum_{-\infty}^{-(N-1)/2-1} |a[n] - h_d[n]|^2 + \sum_{-(N-1)/2}^{(N-1)/2} |a[n] - h_d[n]|^2 + \sum_{(N-1)/2+1}^{\infty} |a[n] - h_d[n]|^2 \\ &= \sum_{-\infty}^{-(N-1)/2-1} |h_d[n]|^2 + 0 + \sum_{(N-1)/2+1}^{\infty} |h_d[n]|^2 \end{aligned}$$

By symmetry,

$$\epsilon = 2\sum_{(N-1)/2+1}^{\infty} |h_d[n]|^2$$

7.38. (a) A Type-I lowpass filter that is optimal in the Parks-McClellan can have either $L+2$ or $L+3$ alternations. The second case is true only when an alternation occurs at all band edges. Since this filter does not have an alternation at $\omega = 0$ it only has $L+2$ alternations. From the figure we see there are 9 alternations so $L = 7$. Thus, $M = 2L = 2(7) = 14$.