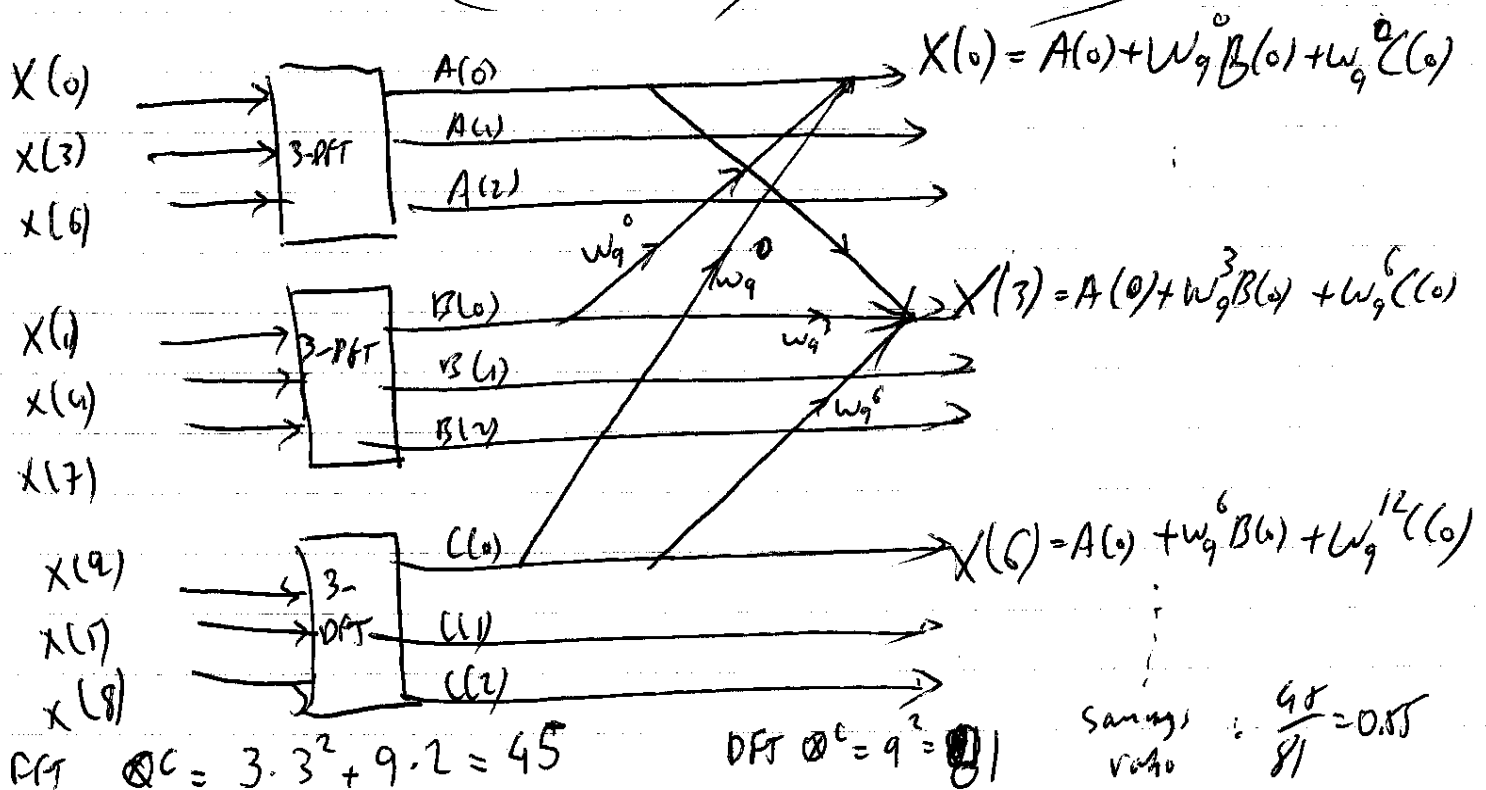


LECTURE 20

More examples on Decimation in time FFT algorithms

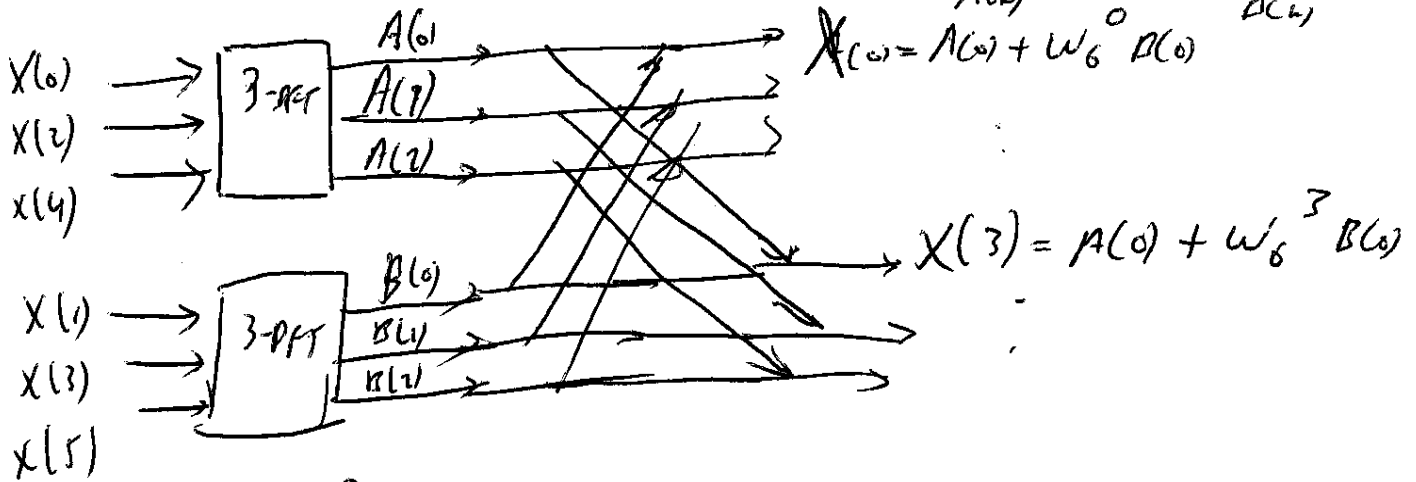
Let $N=9=3^2$. This will be a radix 3 FFT implementation.
In this case there is only one step of decimation where we implement 3 3-FFT algorithms. Accordingly:

$$\begin{aligned}
 X(k) &= \sum_{n=0}^8 x(n) W_9^{nk} = \sum_{n=3m} x(n) W_9^{nk} + \sum_{n=3m+1} x(n) W_9^{nk} + \sum_{n=3m+2} x(n) W_9^{nk} \\
 &= \sum_{m=0}^2 x(3m) W_9^{3mk} + \sum_{m=0}^2 x(3m+1) W_9^{(3m+1)k} + \sum_{m=0}^2 x(3m+2) W_9^{(3m+2)k} \\
 &= \underbrace{\sum_{m=0}^2 x(3m) W_3^{mk}}_{A(k), k=0,1,2} + W_9^k \underbrace{\sum_{m=0}^2 x(3m+1) W_3^{mk}}_{B(k), k=0,1,2} + W_9^{2k} \underbrace{\sum_{m=0}^2 x(3m+2) W_3^{mk}}_{C(k), k=0,1,2}
 \end{aligned}$$



Let $N=6=2 \times 3$. We can implement FFT as follows.
 Either combining 2 3-DFTs or 3 2-DFTs...

$$X(k) = \sum_{n=0}^{5} x(n) \omega_6^{nk} = \sum_{n=2m}^5 x(n) \omega_6^{nk} + \sum_{n=2m+1}^5 x(n) \omega_6^{nk} = \sum_{m=0}^2 x(2m) \omega_3^{mk} + \omega_6^k \sum_{m=0}^2 x(2m+1) \omega_3^{mk}$$

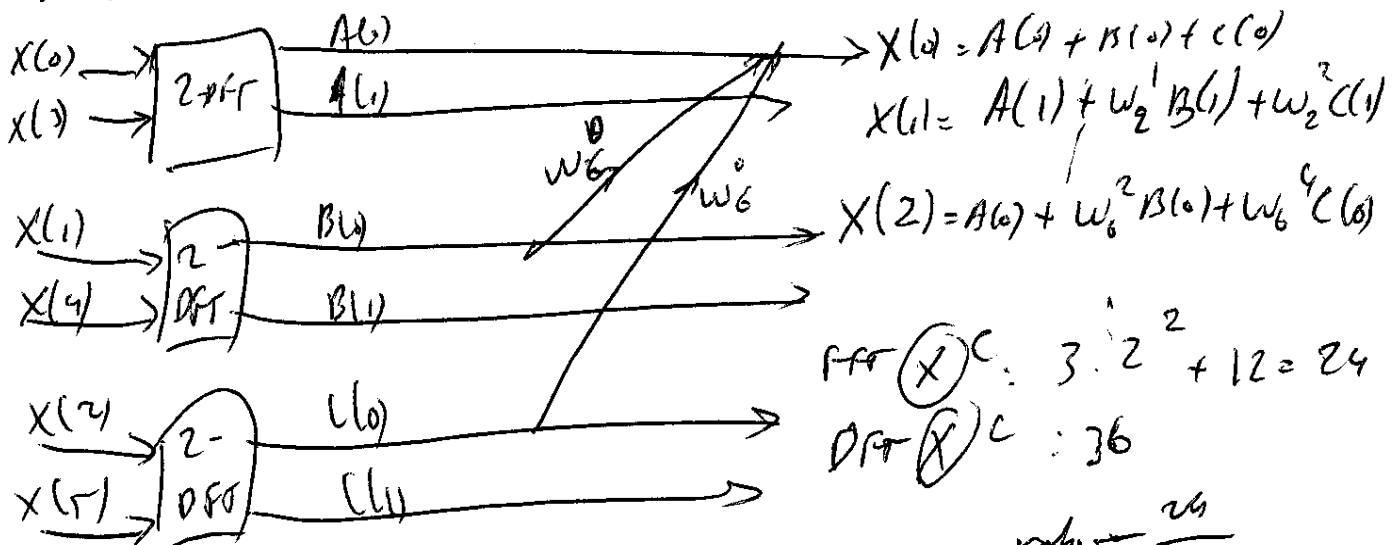


FFT $\times^C$: $2 \cdot 3^2 + 6 = 18 + 6 = 24$
 6-DFT $\times^C$: $6^2 = 36$

Ratio: $\frac{24}{36}$

Otherwise.

$$X(k) = \sum_{m=0}^1 x(3m) \omega_2^{mk} + \omega_6^k \sum_{m=0}^1 x(3m+1) \omega_2^{mk} + \omega_6^{2k} \sum_{m=0}^1 x(3m+2) \omega_2^{mk}$$



FFT $\times^C$: $3 \cdot 2^2 + 12 = 24$
 DFT $\times^C$: 36

Ratio: $\frac{24}{36}$

How can we derive an IFFT algorithm (vortex 2, decimation in time)
 Formula of N-IFFT : $x(n) = \sum_{k=0}^{N-1} \bar{X}(k) W_N^{kn} = \frac{1}{2^9} \sum_{k=0}^{N-1} \bar{X}(k) W_N^{-kn}$

So:

1. Use the same diagram as in forward DFT (FFT)
 2. Use W_N^{-r} instead of W_N^r
 3. Multiply outputs of each butterfly by $\frac{1}{2}$
 (9 stages $\cdot \left(\frac{1}{2}\right)^9 = \frac{1}{N}$).
 4. Input $X(k)$ is in bit reversed order,
 output $x(n)$ is in normal order.
-

* Note: If N is a prime number there is no FFT implementation. That is

$$N=7 \quad 7\text{-PFT} \equiv 7\text{-FFT}$$

$$N=13 \quad 13\text{-PFT} \equiv 13\text{-FFT}$$

$$N=12 = 2 \cdot 6 = 3 \cdot 4 \quad (\text{There are 4 possible FFT implementations})$$