Student Name: Student Number:

University of Toronto
Faculty of Applied Science and Engineering

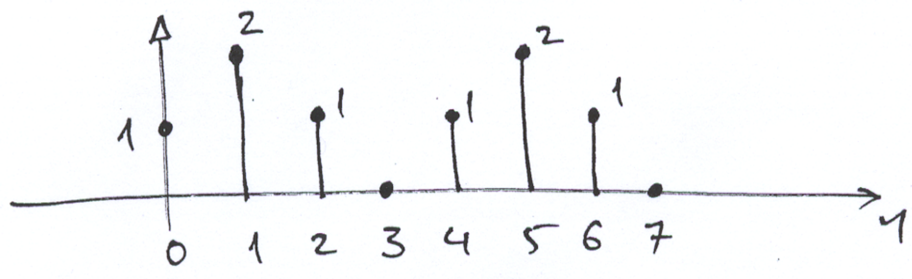
MIDTERM EXAMINATION
ECE431, Digital Signal Processing
March 5, 2002, 6:15-7:45 pm
Examiner: D. Hatzinakos

Exam type A
Non-programmable Calculators are allowed

A continuous time signal xa(t)=sa(t)+da(t) is observed over the period $t = 0$ to 1.99 sec. The sa(t) = $\sin(2\pi t)$ and the da(t)=1. To estimate the frequency of the sinusoid we decide to use discrete time techniques. Thus , the signal is ideally and uniformly sampled with a sampling period T=0.25 sec and then the N-point DFT of the obtained samples is calculated.

a) Obtain and draw and discrete signal x(n)=xa(nT) n=0,..., 7. (3 points)

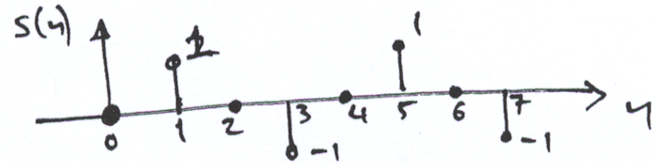
$$x(n) = x_a(nT) = 1 + \sin\left(2\pi n \frac{1}{4}\right) = 1 + \sin\left(\frac{\pi n}{2}\right) \qquad n = 0, 1, \ldots, 7$$



b) Often in practice the dc component (i.e. the mean) of the signal is removed before the application of the DFT in order to obtain unbiased estimates of the sinusoid parameters. Describe a simple process to estimate and remove the mean of x(n). Let s(n), n=0,1,...7 be the resulting zero mean signal. (2 points)

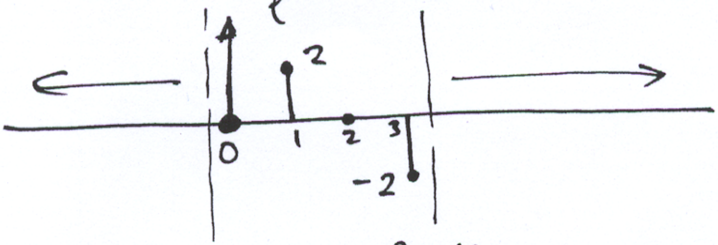
$$\text{mean} = \frac{1}{8}\sum_{n=0}^{7} x(n) = \frac{1}{8}(1+2+1+0+1+2+1+0) = \frac{8}{8} = 1$$

So $s(n) = x(n) - 1$



c) Calculate explicitly the 4-DFT S(k), k=0,1,2,3 of the discrete signal s(n) obtained in part (b). Draw the magnitude |S(k)|, k=0,1,2,3. (8 points)

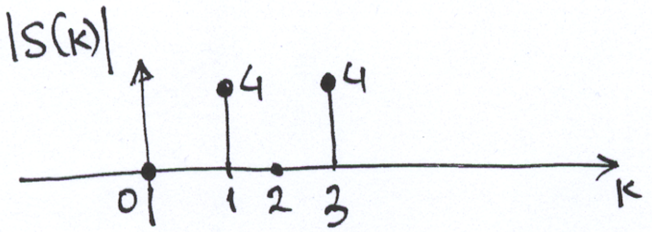
Since s(n) is of length 8 we first form the time-aliased sequence $\tilde{s}(n) = \sum_{\ell} s(n+4\ell)$ and retain one period $(n=0,1,2,3)$



Then, $S(k) = \sum_{n=0}^{3} \tilde{s}(n) e^{-j\frac{2\pi k n}{4}} = 2 \cdot e^{-j\frac{\pi k}{2}} + (-2)e^{-j\frac{3\pi k}{2}}$

$= 2e^{-j\pi k}\left[e^{j\frac{\pi k}{2}} - e^{-j\frac{\pi k}{2}}\right] = 4je^{-j\pi k}\sin\left(\frac{\pi k}{2}\right)$

$k=0,1,2,3$



$|S(k)| = 4\left|\sin\left(\frac{\pi k}{2}\right)\right|$

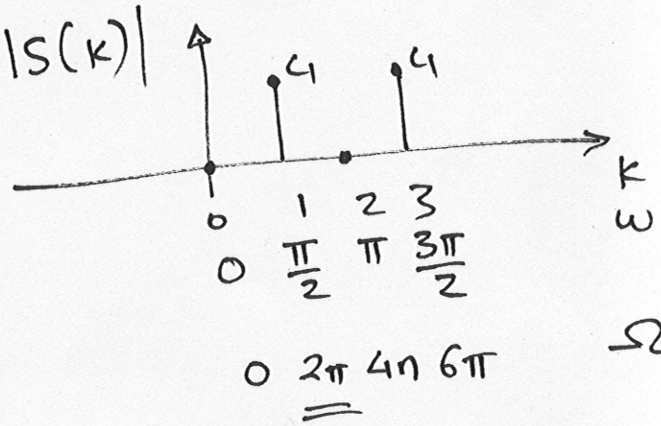
d) What should be the relation between the Fourier Transform $Sa(\Omega)$ of sa(t) and the 4-DFT S(k) of s(n)? Do you think that in this case the 4-DFT of s(n) provides a good estimate of the frequency and magnitude of sa(t)? (6 points)

$$S(\omega) = \frac{1}{T}\sum_{l=-\infty}^{\infty} S_a\left(\overbrace{\frac{\omega - 2\pi l}{T}}^{\Omega}\right) = 4\sum_{l=-\infty}^{\infty} S_a\left(\frac{\omega - 2\pi l}{0.25}\right)$$

$$S(k) = S\left(\omega = \frac{2\pi k}{4}\right) = 4\sum_{l=-\infty}^{\infty} S_a\left(\frac{\frac{\pi k}{2} - 2\pi l}{0.25}\right)$$

So $\boxed{S(k) = 4\sum_{l=-\infty}^{\infty} S_a(2\pi k - 8\pi l)}$

From the previous



→ So the estimated frequency is at $\Omega = 2\pi$ which is the true frequency of the sinusoid

→ The estimated however magnitude will be $2(4\cdot T) = 2$ (Think of the Eulers formula) while the true magnitude of the sinusoid is $\underline{1}$

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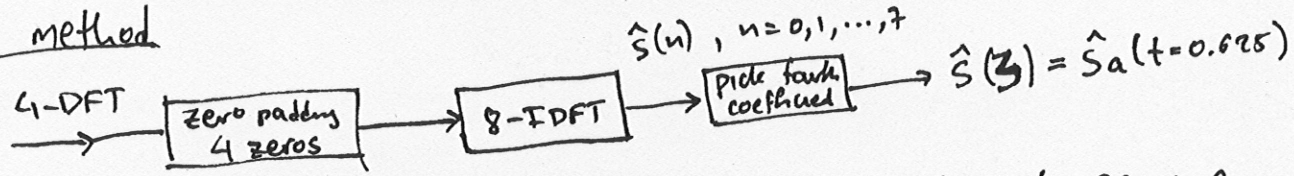
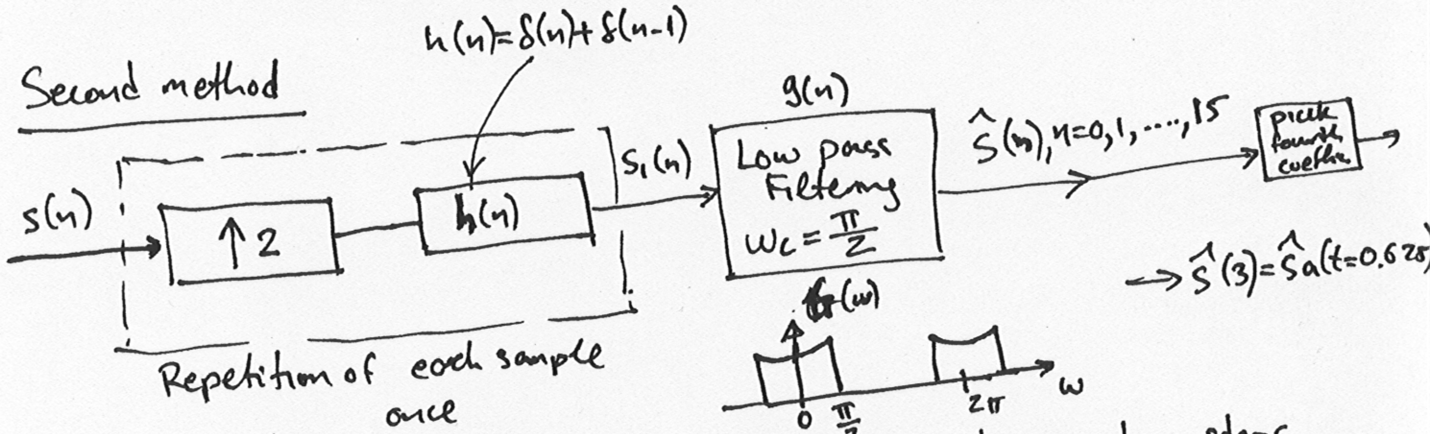
e) To estimate the value of sa(t=0.625 sec) we apply two different methods: One based on zero padding the 4-DFT of s(n) and then calculating the N-IDFT. The other by repeating once each of the samples of s(n) and then applying low pass filtering. Provide a block diagram of how would you implement each method describing and defining all important parameters. Which one of the two methods do you think will provide a better estimate? (6 points)

First method



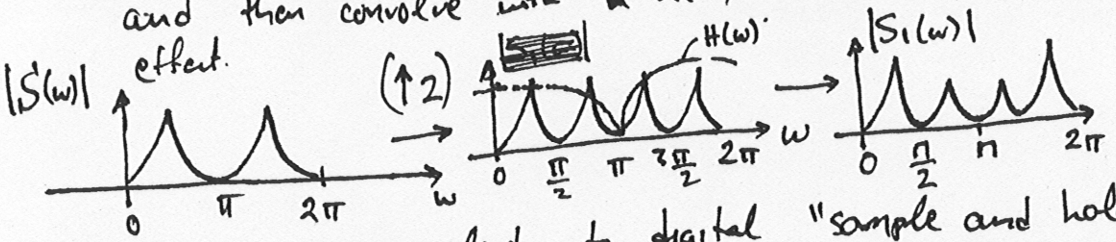
However, this method ~~that~~ interpolates the time aliased sequence $\tilde{s}(n)$ of part (c) rather than s(n). So the interpolation will not be accurate.

Second method



The process of repeating each sample can be done in two steps.

① upsampling s(n) by 2, i.e., by inserting one zero after each sample and then convolve with h(n) = δ(n)+δ(n-1). This has the following effect.



This process is equivalent to digital "sample and hold"!

② The low pass filter g(n) will remove the images and correct "~~aperture effects~~" "aperture effects"

So second method works better.