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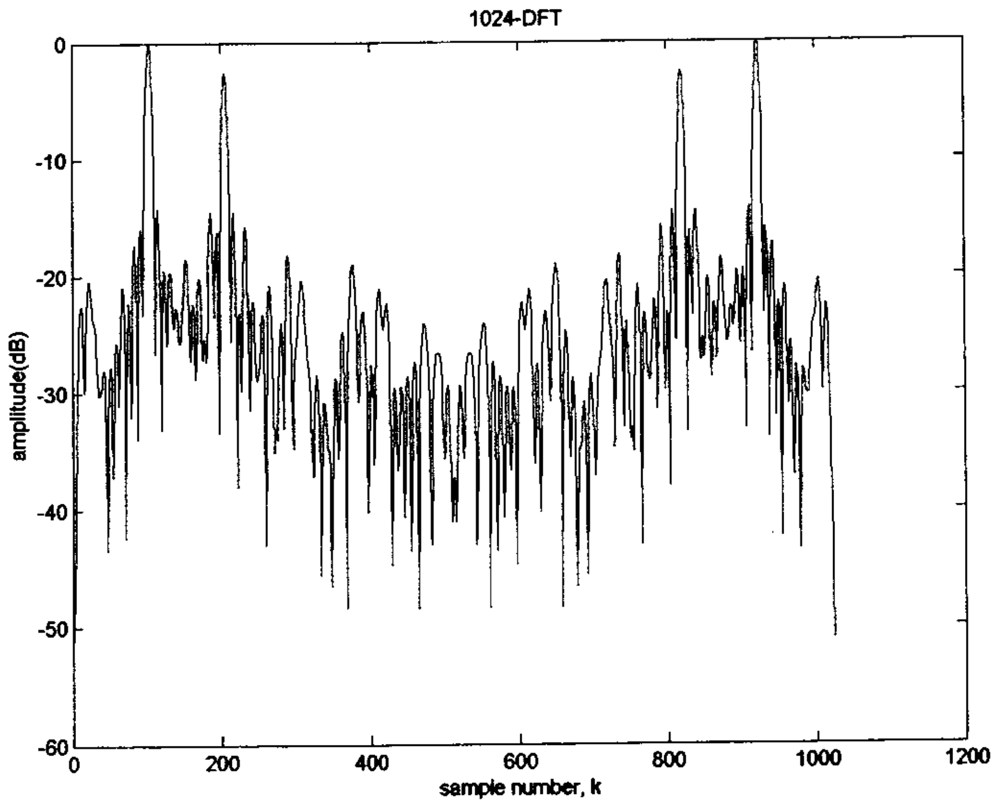
MIDTERM EXAMINATION
ECE431H1S, Digital Signal Processing
Time allowed: 90 minutes
February 23, 2004
Examiner: D. Hatzinakos

Exam type A
Non-programmable Calculators are allowed

The plot below is the 1024-DFT of a set of data made up of 1024-samples taken with sampling period T=0.001 sec. The horizontal axis is labeled with the index *k*, the sample number of the DFT entries.



SOLUTIONS + MARKING SCHEME

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a) What length is the time interval from which these samples are taken? What is the Nyquist frequency for this system? At what frequency in (Hz) are the two dominant peaks located? (3 pts)

1 1024 SAMPLES BETWEEN 0 AND $1024T = 1.024$ s.

1 $F_s = 1$ KHz. NYQUIST FREQUENCY : 500 Hz.

1 FROM DFT, SEE PEAKS AT $K_1 = 100$ AND $K_2 = 200$ (ASSUMING THE SIGNAL IS REAL THE OTHER TWO PEAKS ARE JUST THE MIRROR IMAGE),

SO $F_1 = \left(\frac{K_1}{1024}\right) F_s \approx 100$ Hz, $F_2 = \left(\frac{K_2}{1024}\right) F_s \approx 200$ Hz.

b) Assuming that a rectangular window of length less than 1024 T sec was applied to the signal prior to sampling, what would be the shortest such window you could use which would provide sufficient spectral resolution to distinguish the two dominant spectral peaks? (3pts)

1 A WINDOW OF WIDTH T' CAN RESOLVE TWO SPECTRAL PEAKS SEPARATED IN FREQUENCY BY $1/T'$.

1 IN THIS CASE $\Delta F = F_2 - F_1 = 100$ Hz.

1 THEREFORE $\frac{1}{T'} = 100 \Rightarrow \boxed{T' = 0.01 \text{ s}}$

SO WINDOW LENGTH ≥ 0.01 s CAN DISTINGUISH THE TWO PEAKS.

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c)Using the given 1024-DFT describe how to obtain an estimate of the corresponding signal at time 10.5 msec. (3pt)

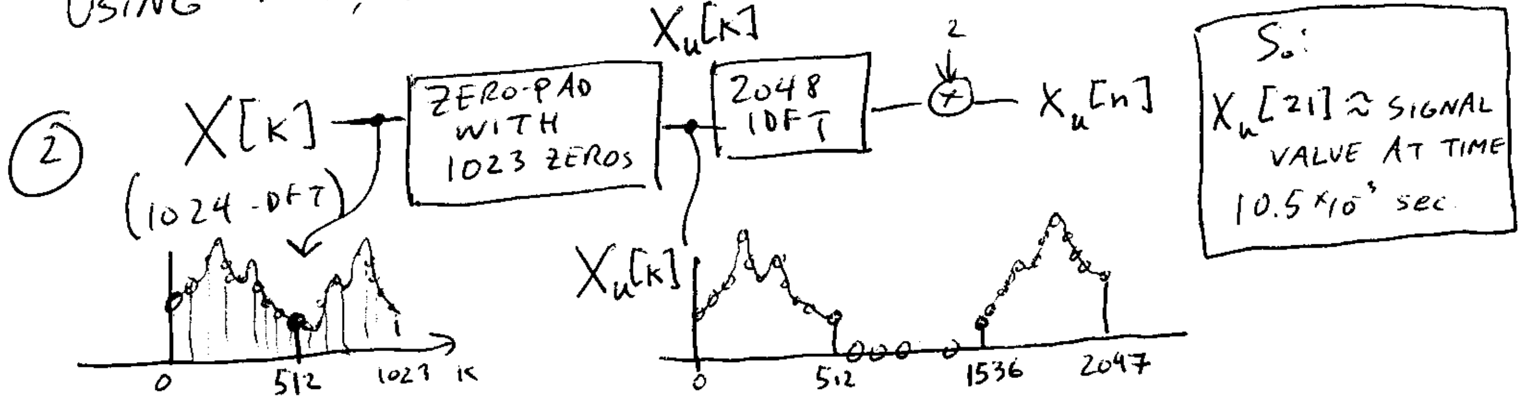
- SAMPLES ARE AT EVERY MSEC: 0, 1, 2, ..., 10, 11, ... msec.

WANT TO INTERPOLATE SAMPLE HERE.

(1) · NEED TO INTERPOLATE BY A FACTOR OF 2

USING DFT, THIS IS ACHIEVED AS FOLLOWS

(2)



d) Assuming that the signal is well matched to the dynamic range of the quantizer, what is the minimum number of bits per second required for the transmission of this signal with signal-to-Quantization noise ratio at least 30 dB ? (3pts)

FOR A (B+1) BIT QUANTIZER, USING EQ. (4.124) OF TEXT,

$$SNR = 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right) \ dB$$

ASSUMING SIGNAL AND QUANTIZER ARE MATCHED

$$20\log_{10}\left(\frac{X_m}{\sigma_x}\right) = 0, \text{ so}$$

(1) $SNR = 6.02B + 10.8 \ dB$

So, IF $B=4$, $SNR > 30\ dB$, THEREFORE NEED

(1) $\boxed{5 \text{ bits}}$

$\therefore$ MINIMUM DATARATE: $1\times10^3 \frac{\text{SAMPLES}}{\text{SEC}} \cdot 5 \frac{\text{bits}}{\text{SAMPLE}}$

(1) $= \boxed{5 \text{ Kbps}}$

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e) By observing the given DFT spectrum, we realize that we are wasting bandwidth with the applied discritization process (why is that?). Therefore, in addition to the given signal $x_1[n]$, we have been asked to transmit or store a second digital signal $x_2[n]$ with similar spectral properties to those of $x_1[n]$. To achieve this, the following two candidate procedures have been proposed:

1. Form and store or transmit the signal $y[n] = x_1[n] + (-1)^n x_2[n]$.

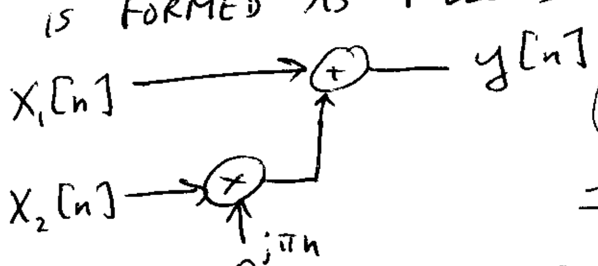
2. Form and store or transmit the signal $z[n]$ where $z[2n] = x_1[2n]$ and $z[2n+1] = x_2[2n]$.

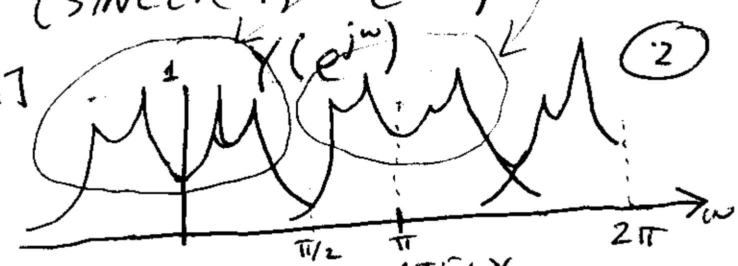
Are the described procedures reversible? Draw approximately the corresponding frequency content of the y[n] and z[n] and comment appropriately. Describe the process to reconstruct $x_1[n]$ and $x_2[n]$ in each case by means of a block diagram. (10 pts)

$X_1(e^{j\omega})$

$X_2(e^{j(\omega-\pi)})$

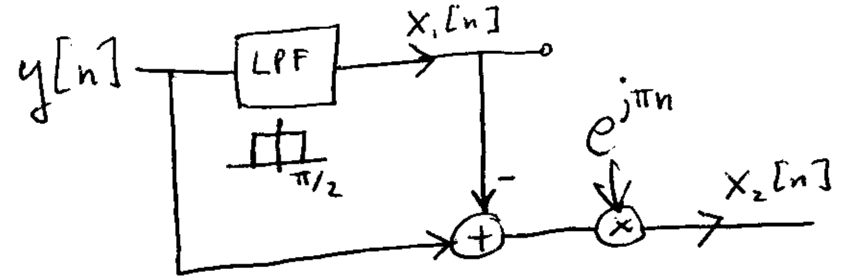
1. $y[n]$ IS FORMED AS FOLLOWS (SINCE: $(-1)^n = e^{j\pi n}$)





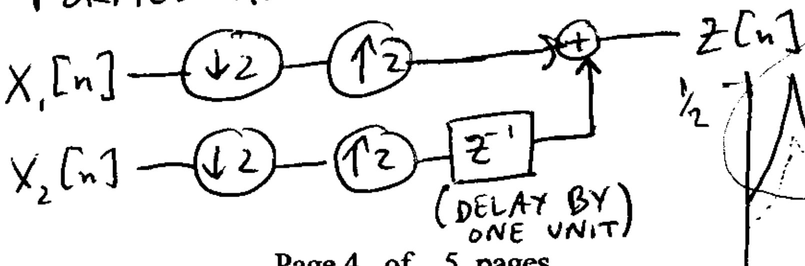
SINCE BOTH $x_1[n]$ AND $x_2[n]$ ARE APPROXIMATELY BANDLIMITED BY $\pi/2$ rad; $Y(e^{j\omega}) = X_1(e^{j\omega}) + X_2(e^{j(\omega-\pi)})$ CAN BE USED TO RECONSTRUCT $x_1[n]$ AND $x_2[n]$. SO THE PROCEDURE IS REVERSIBLE (APPROXIMATELY):

1

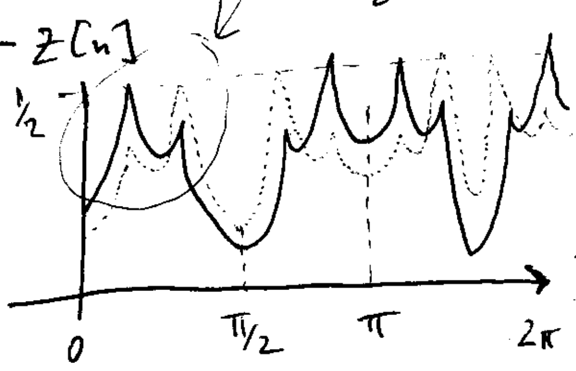


2

2. $z[n]$ IS FORMED AS FOLLOWS:



$\frac{1}{2}X_1(e^{j\omega}) + \frac{1}{2}X_2(e^{j\omega})$



2

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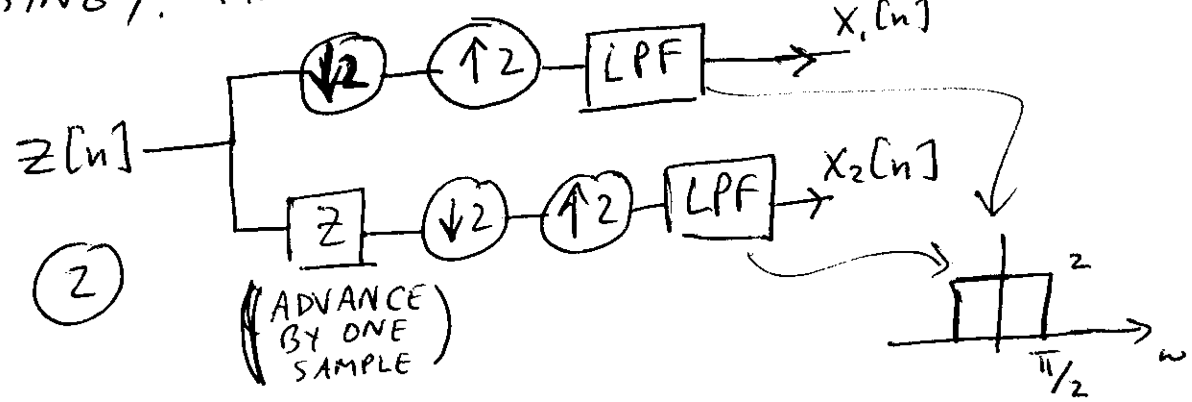
(1) EVEN THOUGH $X_1[n]$ AND $X_2[n]$ NOW OVERLAP IN FREQUENCY THEY ARE SEPARATED IN TIME SO RECOVERY IS POSSIBLE USING INTERPOLATION (THE SIGNALS ARE APPROX. BANDLIMITED BY $\pi/2$ SO DOWNSAMPLING WILL NOT INTRODUCE ALIASING). THIS IS ACHIEVED AS FOLLOWS:



(2)

f) Assuming that the same quantizer has been used for both signals in part (e), what is the resulting Signal-to-Quantization noise ratio per signal in the two cases considered? Assuming that we tolerate no loss of important signal's frequency content, how much could you have improved the Signal-to-Quantization noise ratio by low pass filtering each individual signal before forming any combination of the signals? (3 pts)

EACH SIGNAL CAN BE WRITTEN AS

$$X_1[n] = \tilde{X}_1[n] + V_1[n]$$
$$X_2[n] = \tilde{X}_2[n] + V_2[n]$$

WHERE $V_1[n]$, $V_2[n]$ is NOISE (FROM QUANTIZATION) AND $\tilde{X}_1[n]$, $\tilde{X}_2[n]$ ARE THE NOISELESS SIGNALS. ASSUMING INDEPENDENT SIGNAL AND NOISE COMPONENTS, AND ASSUMING THE NOISE POWER SPECTRAL DENSITY IS WHITE, THE SIGNAL-TO-QUANTIZATION NOISE RATIO OF $Y[n]$ AND $Z[n]$ IS THE SAME AS THAT OF $X_1[n]$ AND $X_2[n]$ (i.e. there is no improvement). HOWEVER, SINCE $\tilde{X}_1[n]$ AND $\tilde{X}_2[n]$ ARE APPROXIMATELY BANDLIMITED BY $\pi/2$, FILTERING $X_1[n]$ AND $X_2[n]$ BY A LPF WITH A BANDWIDTH OF $\pi/2$ (PRIOR TO FORMING $Y[n]$ AND $Z[n]$) WILL IMPROVE THE SNR BY 3 dB AND THERE WILL BE NO LOSS OF IMPORTANT SIGNAL FREQUENCY CONTENT.