Solutions

Student name: Student number:

MIDTERM 1 EXAMINATION
ECE431H1F, Digital Signal Processing

October 16, 2025
Examiner: D. Hatzinakos

Time: 1:10-2:00 pm, Room HS106

This is a Type C exam. You may use non programmable calculators.

Exam questions

Pierre a piano tuner, has asked for yourhelp to tune a piano (apparently he suffers from a severe ear infection). You propose to employ frequency domain DSP operations. In particular, two tones (Frequencies) each from a different piano are recorder and their distance is estimated based on frequency analysis of the recorded signal (one of the pianos is tuned and used as a reference to tune the other piano) . Assuming the two frequencies are F1 and F2, where F2=F1+ΔF and F1, F2 ≤ 5kHz, the continuous time signal takes the form xa(t)=[sin(2πF1 t)+sin(2πF2 t), t=0,…, T sec. The objective is that |Δ| tends to zero. The recorder signal is ideally and uniformly sampled with a period of Ts seconds and then a N-point DFT is computed and plotted to estimate the distance between the two frequencies.

a) Let T/Ts=L. Assume that 1/Ts has been chosen at least four times greater than the highest possible frequency so that aliasing is negligible. Also assume that we require ΔF≤0.01 Hz in order to decide that the piano has been properly tuned. What should be the minimum value of L (or the corresponding length T) so that the estimator has sufficient resolution to decide whether tuning of the piano has been successful? What are the normalized frequencies f1 and f2 corresponding to the real frequencies F1 and F2? (3 points)

$T = LT_s$

$\Delta F \geq \frac{1}{LT_s} \Rightarrow LT_s \leq \frac{1}{\Delta F} \Rightarrow L \leq \frac{1}{\Delta F \cdot T_s}$

$F_1, F_2 \leq 5$ kHz close $F_s = 4.5$ kHz $= 20$ kHz $\Rightarrow \frac{1}{T_s} = 20$ kHz

Now $\Delta F \geq 0.01 \Rightarrow \frac{1}{\Delta F} \geq \frac{1}{0.01}$, so $L \leq \frac{1}{0.01} \cdot 20k \Rightarrow$

$L \geq 2 \cdot 10^6$

Also: $f_1 = F_1 T_s = \frac{F_1}{20k}$ $f_2 = F_2 T_s = \frac{F_2}{20k}$