SHORT TIME FOURIER TRANSFORM (OR WINDOWED FOURIER TRANSFORM

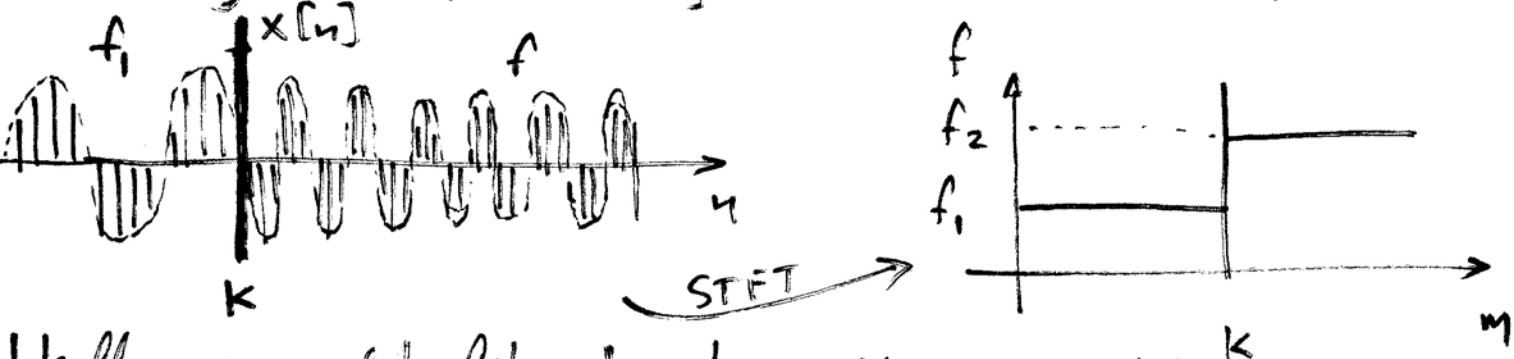
(10) The DTFT (Discrete time Fourier Transform $X(f)=\sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi fn}$ looks at a "global" characteristic of the signal. by summing over the entire time $n=-\infty,\dots,0,\dots\infty$. This is reflected by the fact that the basis function $e^{-j2\pi fn}$ are spread over the whole time axis. Thus $X(f)$ cannot analyze local behavior of the signal.

(11) The short time FT (STFT) or windowed FT (WFT)

$$X(m,f)=\sum_{n=-\infty}^{\infty} x[n]\cdot w[n-m]\, e^{-j2\pi fn}$$

$$=\text{DTFT}\left[x[n]\,w[n-m]\right]$$

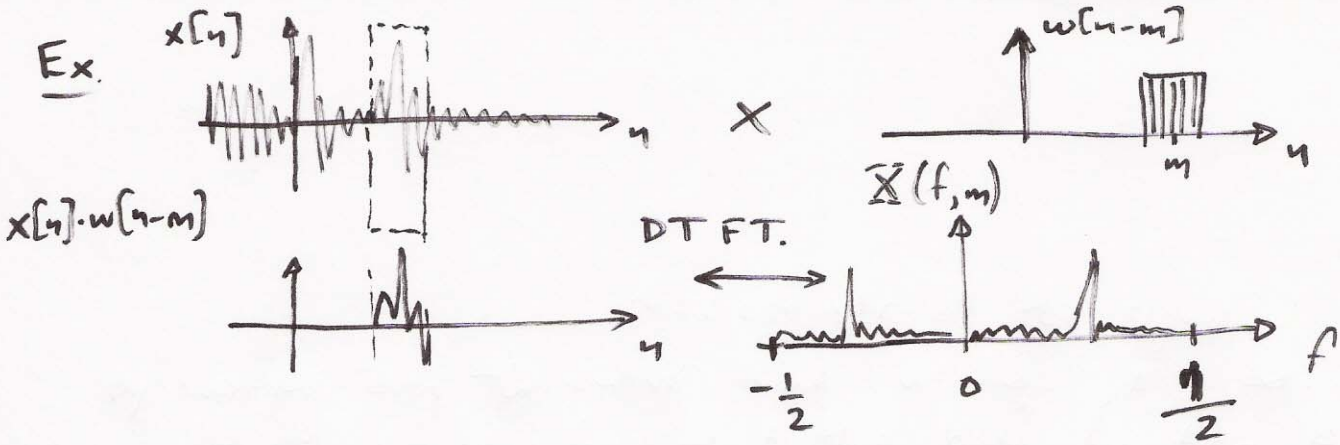
with $w[n-m]$ a window placed (or centered) around the variable m provides a better local description of the frequency content of $x[n]$ at time instant m.



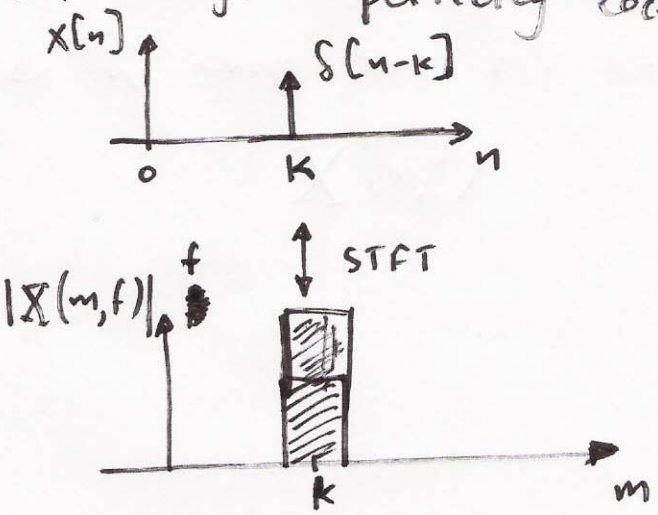
Ideally we would like to have the representation shown in the picture.

The quantity $|X(m,f)|^2$ is called the <u>spectogram</u>. Usually, we choose a window (real) with unit energy that is $\sum_n |w[n]|^2=1$

Ex.

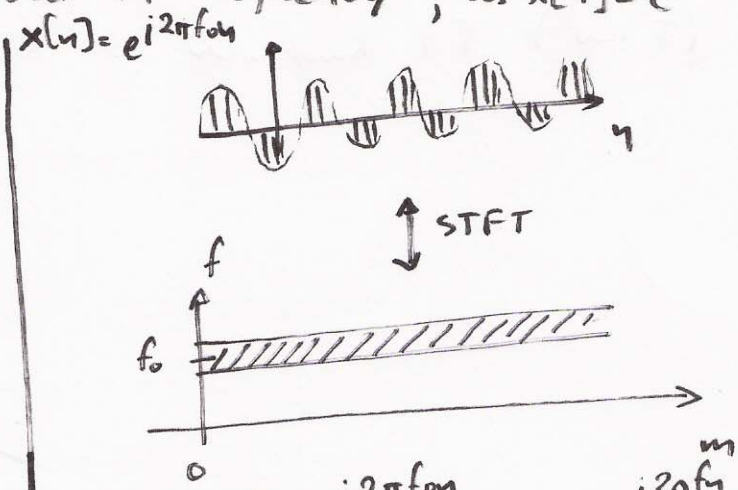


In order to see the behaviour of STFT let us look at two examples. A signal perfectly localized in time, as $x[n]=\delta[n-k]$ and a signal perfectly localized in frequency, as $x[n]=e^{j2\pi f_0 n}$



$$X(m,f)=\sum_{n=-\infty}^{\infty}\delta[n-k]\,w[n-m]\,e^{-j2\pi f n}$$
$$=w[k-m]\,e^{-j2\pi f k}$$

so $|X(m,f)|=|w(k-m)|$ for all f



$$X(m,f)=\sum_{n=-\infty}^{\infty}e^{j2\pi f_0 n}\,w[n-m]\,e^{-j2\pi f n}=$$
$$=W(f-f_0)\,e^{-j2\pi(f-f_0)m}$$

so $|X(m,f)|=|W(f-f_0)|$

The resolution of estimation (degree of localization in time or frequency) is determined by the width of the window in time or frequency.

$$w[n] \overset{DTFT}{\longleftrightarrow} W(f)$$

The longer the window in time domain the better the frequency resolution and vise versa.

Student name:

ECE 431111S, Digital Signal Processing

Quiz 3
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Assume that the signal $x[n]$ in fig. 1 measures the displacement of a bridge when a car is passing by (e.g., negative spike)

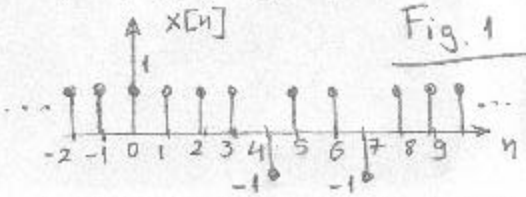


Fig. 1

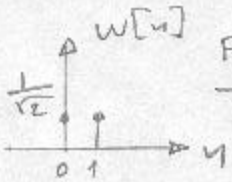


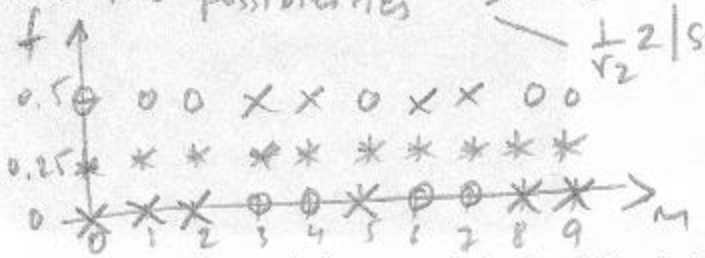
Fig. 2

1. Compute the spectrogram $|X(m,f)|$ for $m = 0,1,\ldots,9$ and $f = -0.5,-0.25,0,0.25,0.5$ by using the window in fig. 2. Place the calculated values in a two dimensional graph.

Hint: $\left[\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix} \xleftrightarrow{DTFT} 2e^{-j\frac{w}{2}}\cos\frac{w}{2} \;,\; \begin{matrix} 1 & 1 \\ 0 & -1 \end{matrix} \xleftrightarrow{DTFT} 2je^{-j\frac{w}{2}}\sin\frac{w}{2} \right]$

$$|X(m,f)| = |DTFT(x[n]\cdot w[n-m])|$$

there two possibilities $\begin{cases} \frac{1}{\sqrt{2}}\cdot 2|\cos \pi f| & , m = 0,1,2,5,8,9 \\ \frac{1}{\sqrt{2}} 2|\sin \pi f| & , m = 3,4,6,7 \end{cases}$



$X = \frac{2}{\sqrt{2}}$
$0 = 0$
$* = 1$

2. What conclusions can we derive from $|X(m, f = 0)|$?

One conclusion is that the signal has variable DC component

3. Can you think how $|X(m,f)|$ can help you design the bridge by properly selecting the "natural" vibrating frequency of the bridge?

at $f = 0.25$ the response of the bridge remains constant over time and thus not affected by the load. This is a good choice