Name:
Student number:

University of Toronto
Faculty of Applied Science and Engineering

MIDTERM EXAMINATION 1
ECE462H1S, Multimedia Systems

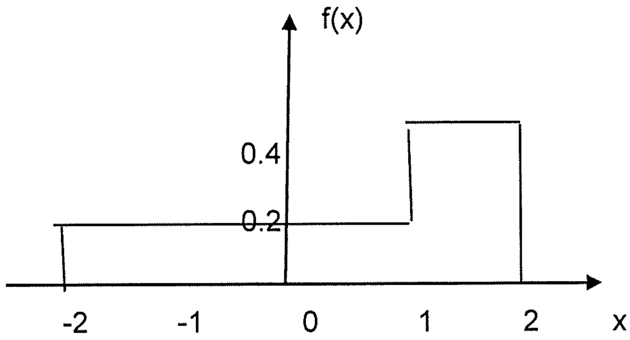
February 16, 2024, 10:10-11:00 am
Instructor: D. Hatzinakos

Instructions:

1. The exam counts for 15% of overall mark.
2. Please solve all problems. Do not show only final answers. You should demonstrate how the answer has been obtained by including intermediate results and explanations wherever needed.
3. Use the blank space provided in this handout to record your answers.
4. Write your name and/or student number on top of all submitted pages.

QUESTIONS.

1. A signal x has the following pdf *f (x).*



Use the Max-Lloyd algorithm to design a B-bit non-uniform quantizer. Use the settings for a uniform quantizer to initialize the algorithm. What are the decision boundaries and reconstruction levels at the end of recursion (iteration) 1. What is the corresponding MSE distortion for this quantizer? Please consider the following cases given in table to record your results (4 points)

Number of bits (B)	Step size (Delta)	Quantization Noise power (MSE)	Type of optimum quantizer (U or NU)
1			
2			
3			
4			

2. Assuming you are asked to assign 5 quantization levels for the signal of question 1 and design a 5 level nonuniform quantizer empirically. What will be your choice of levels to minimize the quantization error power? (2 points)
3. Assuming you are considering the following events associated with the figure of question 1. A=(-2< x<-1), B=(-1<x<0), C=(0<x<1), D=(1<x<2) . Given the events A,B,C,D design a Hufman code and provide its codewords, rate and its efficiency compared to other possible codes and the entropy of the source . (2 points)
4. Why is block coding used in JPEG standard? Will JPEG blocking artifacts be more or less pronounced as the block coding size increases? (2 points)
5. The autocorrelation functions of 2 signals x(n) and y(n) satisfy the following properties Rx(0)=Ry(0)=1, Rx(k)>Ry(k) , k=0,1,2, ... Which of the following statements are correct and why? (2 points)
   a) The sequence x(n) can achieve higher compression than y(n) using JPEG.
   b) The sequence y(n) is more efficiently coded via DPCM
   c) MSE prediction is better for x(n)
   d) The KLT transform provides better JPEG type compression for y(n) (2 points)
6. The zig-zag vector after quantization of DCT coefficients takes the values 0,8,0,0,6,0,3,0,0,-3, 0,0,0,1,0,0,....0,0,0. Provide the RLC coding for this sequence. (2 points)
7. Is transform based coding compression more efficient (that is achieving lower bit rate) for signals with dominant low frequency or dominant high frequency content? (1 point)

① 1 bit quantizer $\Delta = \frac{2-(-2)}{2^1} = \frac{4}{2} = 2$ so

uniform quantizer
$b_0 = -2$
$b_1 = 0$
$b_2 = 2$
$u_0 = -1$
$u_1 = 1$

looking at pdf over the intervals $(-2, 0)$ $(0, 2)$ we conclude that a non-uniform quantizer is optimum.

Max Loyd initialization: as above

$u_0 =$

$b_1^{(1)} = \frac{u_0^{(0)} + u_1^{(0)}}{2} = \frac{u_0 + u_1}{2} = \frac{-1+1}{2} = 0$

$b_0(1) = b_0$

$b_2(1) = b_2$

also, $u_0^{(1)} = \frac{\int_{-2}^{0} x f(x) dx}{\int_{-2}^{0} f_x(x) dx} = \frac{\int_{-2}^{0} x 0.2 dx}{\int_{-2}^{0} 0.4 dx} = \frac{0.4}{0.4} = 1$ ✓

$u_1^{(1)} = \frac{\int_0^2 x f_X(x) dx}{\int_0^2 f_x(x) dx} = \frac{\int_0^1 x\, 0.2 dx + \int_1^2 x\, 0.4 dx}{\int_0^1 0.2 dx + \int_1^2 0.4 dx} = \dots = \frac{7}{6} = 1.16$

MSE $= \int_{-2}^{1} (x-(-1))^2 0.4 dx + \int_{0}^{1} (x-1.16)^2 0.2 dx$

$+ \int_1^2 (x-1.16)^2 0.4 dx = \dots = 3.93$

All higher, 1 bit designs are uniform quantizer with higher than 1 bit $MSE = \frac{\Delta^2}{12}$ because the pdf is flat over the corresponding intervals.

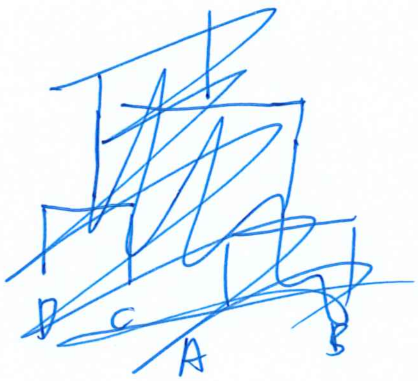
(2) five quantization levels: by inspection the optimum values are $u_0 = -\frac{3}{2}$, $u_1 = -\frac{1}{2}$ $u_2 = \frac{1}{2}$

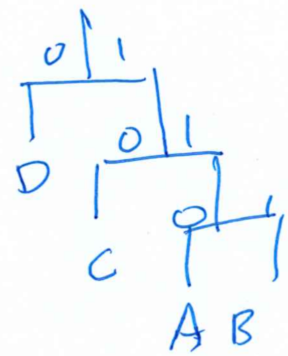
$u_3 = 1 + \frac{1}{3}$ $u_4 = 1 + \frac{2}{3}$. V.

(3)

	prob.
A	0.2
B	0.2
C	0.2
D	0.4

so





So

	code	# bits
A	110	3
B	111	3
C	10	2
D	0	1

code rate $= 0.4 \cdot 1 + 3 \cdot 0.2 \cdot 2 + 2 \cdot 0.2$

$= 0.4 + 1.2 + 0.4 = \underline{2}$

Entropy $= 0.4 \log_2 \frac{1}{0.4} + 3 \cdot 0.2 \log_2 \frac{1}{0.2} = \ldots = \underline{1.92}$

The code is not efficient. You do not save any bits over constant word length case.

(4) Blocks are considered JPEG in order to exploit the correlation between block values and then achieve energy compaction in the DCT domain. The correlation indicates that there is redundancy within a block and by removing it we achieve compression.

The larger the blocks ~~the more pronounced~~ the more pronounced will be the blocking artifacts

(5) a) true the higher the correlation the higher the energy compaction in DCT domain → higher compression

b) ~~The~~ false the smaller the correlation the higher the prediction error → DPCM less efficient.

c) true the lower the correlation the better the prediction

d) We cannot say for sure. Since there is no direct (obvious) correlation between eigenvalues of correlation function

(6) (1, 8) (2, 6) (1, 3) (2, -3) (3, 1) (0, 0)

(7) Not ~~necessarily~~ necessarily: It does not matter what frequency content is dominant what it matters is the degree of energy compaction over the entire spectrum. of frequencies