with solutions

Name:
Student number:

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MIDTERM EXAMINATION 1
ECE462H1S, Multimedia Systems

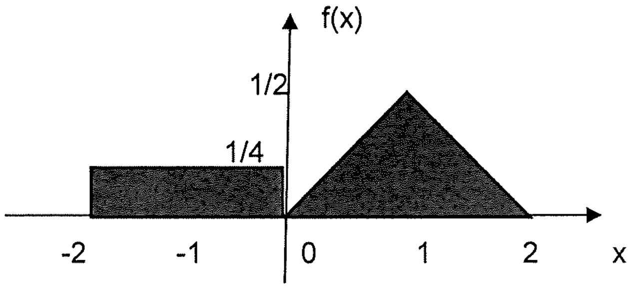
February 13, 2025, 10:10-11:00 am
Instructor: D. Hatzinakos

Instructions:

1. The exam counts for 15% of overall mark.
2. Please solve all problems. Do not show only final answers. You should demonstrate how the answer has been obtained by including intermediate results and explanations wherever needed.
3. Use the blank space provided in this handout to record your answers.
4. Write your name and/or student number on top of all submitted pages.

QUESTIONS.

1. A signal x has the following pdf *f (x).*



Use the Max-Lloyd algorithm to design a 2-bit non-uniform quantizer. Use the settings for a uniform quantizer to initialize the algorithm. What are the decision boundaries and reconstruction levels at the end of recursion (iteration) 1. The equations for the two sides of the triangle are x/2 and x/2+1, respectively. (3 points)

2. What is the basic principle (idea) for achieving compression using a transform based algorithm? (1 point)

3. A signal x(n), n=0,1,...is divided into segments of 3 samples and two different 3-point transforms are applied to each segment. Assume that the 3 samples of transform 1 have variances: sigma1=0.5, sigma2=0. 1, sigma3=0.05 while the 3 samples of transform 2 have variances: sigma1=0.5, sigma2=0.3, sigma 3=0.005 . Which of the two transforms will allow higher compression gain? (1 point)

4. We receive the Hufman binary coded signal 100001011111. We want to decode the signal and recover the message encoded. The only information we have is that the letters involved are: c,s,a,e with frequencies 2,1,1,1, respectively. Describe a strategy to / and recover the message. (2 points)

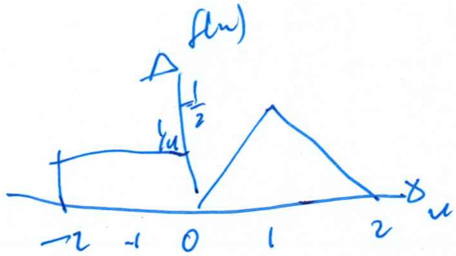
5. Design a linear predictor of length equal to 2 for a signal x(n) having the autocorrelation values R(0)=1, R(1)=0, R(2)=-1. Then assuming that x(0)=1 and x(1)=1, derive the entire signal x(n), n=2,3,4,.... (3 points).

6. An alien species is colour blind. Can you design a JPEG that is more efficient for that species? (1 point)

7. What properties of the human visual system justify the use of nonuniform quantizers? (2 points)

8. The PSNR of a quantized signal is 27 dB. If the dynamic range of the signal is (-2,2) how many bits have been approximately used in the quantization? (2 points)

(1)



$N = 2^2 = 4$ reconstruction levels

$$\Delta = \frac{2-(-2)}{4} = 1.$$

Iteration 0 : uniform quantizer → boundaries $u_0=-2,\ u_1=-1,\ u_2=0,\ u_3=1,\ u_4=2$
→ reconstruction levels $V_0 = -\frac{3}{2},\ V_1 = -\frac{1}{2},\ V_2 = \frac{1}{2},\ V_3 = \frac{3}{2}$

Iteration 1 :

$$u_1^{(1)} = \frac{V_0+V_1}{2} = \frac{-\frac{3}{2}-\frac{1}{2}}{2} = \frac{-2}{2} = -1$$

$$u_2^{(1)} = \frac{V_1+V_2}{2} = \frac{-\frac{1}{2}+\frac{1}{2}}{2} = 0$$

$$u_3^{(1)} = \frac{V_2+V_3}{2} = \frac{\frac{1}{2}+\frac{3}{2}}{2} = 1.$$

Also,

$$V_0^{(1)} = \frac{\int_{-2}^{-1} u \cdot \frac{1}{4}\, du}{\int_{-2}^{-1} \frac{1}{4}\, du} = \dots -\frac{3}{2} \quad ✓$$

$$V_1^{(1)} = \frac{\int_{-1}^{0} u \frac{1}{4}\, du}{\int_{-1}^{0} \frac{1}{4}\, du} = \dots = -\frac{1}{2} \quad ✓$$

$$V_2^{(1)} = \frac{\int_0^1 u(u/2)\, du}{\int_0^1 \frac{u}{2}\, du} = \frac{\left.\frac{u^3}{6}\right|_0^1}{1/4} = \dots \frac{2}{3} \quad ✓$$

$$V_3^{(1)} = \frac{\int_1^2 u\left(-\frac{u}{2}+1\right) du}{\int_1^2 \left(-\frac{u}{2}+1\right) du} = \frac{\left.-\frac{u^3}{6}+\frac{u^2}{2}\right|_1^2}{1/4} = \dots = \frac{4}{3} \quad ✓$$

So reconstructed values in the triangular region are closer to the value 1 as expected...

(2) The basic principle behind transform based compression is the expectation that transformation of a signal from time/space to frequency domain will create energy compaction which will further allow to represent frequency components with less bits and achieve compression.

(3) We use the formula $G_{TC} = \dfrac{\frac{1}{3}\sum_{i=0}^{2} \sigma_i^2}{\left(\prod_{i=0}^{2} \sigma_i^2\right)^{1/3}}$

Thus

$$G_{TC1} = \frac{\frac{1}{3}(0.5^2+0.1^2+0.05^2)}{(0.5^2\cdot 0.1^2\cdot 0.05^2)^{1/3}} = \ldots\ 14.44$$

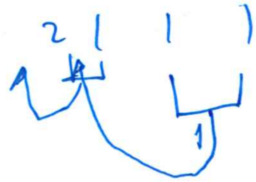
$$G_{TC2} = \frac{\frac{1}{3}(0.5^2+0.3^2+0.005^2)}{(0.5^2\cdot 0.1^2\cdot 0.005^2)^{\frac{1}{3}}} = \ldots\ 41.46$$

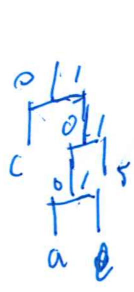
Therefore the transform gain for case 2 is much larger & transformer 2 more efficient (higher energy compaction).

(4) Generate all Huffman code trees, use them to reconstruct corresponding messages and pick the one that makes more sense.

e.g.

c	s	a	e
2	1	1	1





c	0
s	11
a	100
e	101

1000010 11111
a c c e s s

the message is "access"

(5) $$\begin{bmatrix} R(0) & R(1) \\ R(1) & R(0)\end{bmatrix}\begin{bmatrix}a_1\\a_2\end{bmatrix} = \begin{bmatrix}R(1)\\R(2)\end{bmatrix} \Rightarrow \begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$$

$$\Rightarrow \begin{matrix} a_1 + 0 = 0 \\ 0 + a_2 = -1 \end{matrix} \Rightarrow \begin{matrix} a_1 = 0 \\ a_2 = -1 \end{matrix}$$

predictor: $x(n) = a_1 x(n-1) + a_2 x(n-2) = 0 - x(n-2) \Rightarrow \boxed{x(n) = -x(n-2)}$

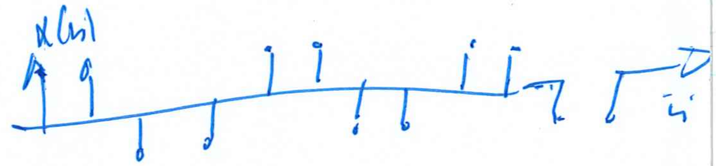
So, assume $x(0) = x(1) = 1$

then $x(2) = x(3) = -1$

$x(4) = x(5) = 1$

$x(6) = x(7) = -1$

⋮



(6) RGB → Y Cr Cb → keep only Y component, which represents greyscale information only.

(7) The Weber law indicating logarithmic visual properties and the fact that lower level values occur more often than higher values signals in nature are the motivation for non-uniform quantizers.

(8) $$PSNR = 10\log_{10}\frac{(2^b)^2}{D} = 10\log_{10}\frac{(2^b)^2}{\left(\frac{\Delta^2}{12}\right)} = 10\log_{10}\frac{(2^b)^2}{\frac{(2-(-2))^2}{2^{2b}}/12} = 27\ dB$$

$$\Rightarrow \frac{12\cdot 2^{2b}}{4^2/2^{2b}} = 10^{2.7} \Rightarrow \frac{12}{16}2^{4b} = 10^{2.7} \Rightarrow \frac{4}{3}10^{2.7} = 2^{4b}$$

$$\Rightarrow 4b = \log_2\left(\frac{4}{3}\cdot 10^{2.7}\right) \Rightarrow b = \frac{1}{4}\log_2\left(\frac{4}{3}\cdot 10^{2.7}\right) = 2\ bits$$