Midterm 2 , 2025
Solutions

QUESTIONS

1. A 4x4 image f(x,y) has the following orthonormal 2-level Haar wavelet transform.

16	10	9	9
-10	5	9	9
9	9	0	0
9	9	0	0

You are asked to apply a two pass EZW coding in compressing the image under the following assumptions

- We can send the initial threshold value T_0 separately without effecting the bit budget
- We use the following codes

Zerotree root	zr	00
Significant positive	sp	11
Significant negative	sn	01
Isolated zero	iz	10

Show all the steps of the EZW coding and the final transmitted bitstream. (4-points)

2. Continuing question 1, and assuming that we have available only 16 bits to compress the image, find the reconstructed image corresponding to the 16 bit EZW by using the inverse Haar wavelet transform and corresponding matrices. (4-points)

3. Given the wavelet transform of Figure 1. , describe a process to assign bits to the different subbands with an overall bit count of Rc=1 bit/pixel. (3-points)

4. Consider a 4 level wavelet transform of a 256x256 image where all the sub bands are of equal resolution. What is the size (resolution) of each sub band? What is the total number of subbands? (2-points)

5. An alien race is color blind. What parts of a JPEG-2000 coder scan be modified or removed without affecting the efficiency of the standard? (1-point)

6. What are fundamental differences between H261 and MPEG-2 compression standards? (1-point)

5140.90

① $\begin{pmatrix} 16 & 10 & 9 & 9 \\ 10 & 5 & 9 & 9 \\ 9 & 9 & 0 & 0 \\ 9 & 9 & 0 & 0 \end{pmatrix}$

First pass $T_0 = 16$ $\quad 1.5T_0 = 24, \quad \frac{T_0}{4} = 4$

Dominant pass: $D_0 = sp, zr, zr, zr$

Correction: $S_0 = 0 \quad (16-24) < 0$

$\begin{pmatrix} 24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Second pass: $T_1 = \frac{T_0}{2} = 8, \quad 1.5T_1 = 12, \quad \frac{T_1}{4} = 2$

Dominant pass $D_1 = sp, sn, zr, sp, sp, sp, sp, sp, sp, sp, sp)$

$\begin{pmatrix} * & 12 & 12 & 12 \\ 12 & 0 & 12 & 12 \\ 12 & 12 & 0 & 0 \\ 12 & 12 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 18 & 10 & 10 & 10 \\ 10 & 0 & 10 & 10 \\ 10 & 10 & 0 & 0 \\ 10 & 10 & 0 & 0 \end{pmatrix}$

Correction: $S_1 = 0010000000$

So total of $9 + 33 = 42$ bits

$D_{final} = (11|00|00|00 \,/\, 11|01|00 / 111111 11 / 11111111/$

$S_{final} = (0\,0010000000\,000)$

15 bits

② 9 bits from pass 1, we have only 7 bits to encode pass 2. So the reconstructed must look like

$$\begin{pmatrix} 20 & 12 & 0 & 0 \\ 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

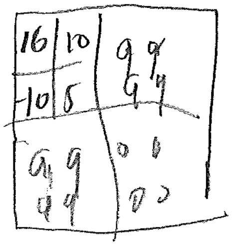
Apply 2 level inverse Haar transform: $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 20 & 12 \\ 12 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 10 & -2 \\ 22 & 10 \end{pmatrix}$

then:

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}^T \begin{pmatrix} 10 & -2 & 0 & 0 \\ 22 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} = \ldots$$

$$= \begin{pmatrix} 5 & 5 & -1 & -1 \\ 5 & 5 & -1 & -1 \\ 11 & 11 & 5 & 5 \\ 11 & 11 & 5 & 5 \end{pmatrix}$$

(3)



R=4 bit/pixel

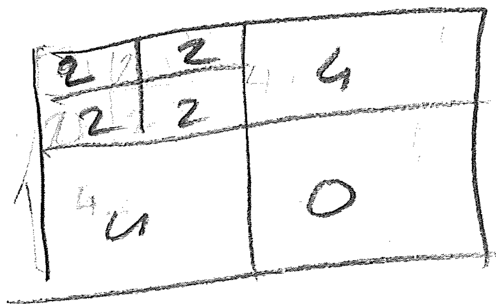
16 bits total

7 subbands in total

If we try to estimate variances σ_k^2 for each subband we end up having $\sigma_k^2 = 0$ for k=9,…,7

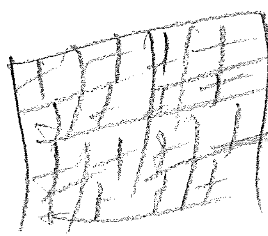
So clearly using the bit allocation formula is not the proper way

We will do it empirically, The HH1 subband will receive 0 bits/pixel So we can assign more bits to the LL2, HL2, LH2, HH2 and less bits to the HL1, LH1, subbands. For example we can have the following set up



one possible bit allocation

(4)



At each level i a total of 4^i subbands

$N=256$, $L=4$ Each subband has size $\frac{N}{2^i} \times \frac{N}{2^i}$

so at level 4 we have $\frac{256}{2^4} \times \frac{256}{2^4} = 16 \times 16$

Total # of subbands $= 4^4 = 256$

(5) Same approach as in JPEG case. Ignore the chrominance components in the Y Cr Cb domain → only Y

(6) H261 is symmetrical, MPEG2 is asymmetrical

H261 has only I, P frames MPEG2 has I, P, B frames.