QUESTIONS (2 points each, total 20 points).

1. An image f(x,y) has the following orthonormal Haar wavelet transform.

20	**10**	**12**	**7**
-10	**5**	**4**	**3**
10	**6**	**3**	**-1**
-2	**0**	**2**	**0**

You are asked to apply EZW coding in compressing the image under the following assumptions

- We have a budget of 16 bits.
- We can send the initial threshold value T_0 separately without effecting the bit budget
- We use the following codes

Zerotree root	zr	00
Significant positive	sp	11
Significant negative	sn	01
Isolated zero	iz	10

Show all the steps of the EZW coding and the final transmitted bitstream.

2. Continuing question 1, find the reconstructed image at the 16 bit wavelet transform by using the inverse Haar wavelet transform and corresponding matrices.
    3. In a two level 1-D wavelet transform, sub-band 1 is uniformly distributed between 1 and 3 while sub-band 2 is Gaussian distributed with standard deviation sigma_2=0.5 and sub-band 3 is also Gaussian distributed but with standard deviation sigma_3=0.2. Allocate bits to the three sub-bands in an optimum way so that the overall bit count is Rc=1bit/pixel.
4. Provide the NxN Haar Wavelet transform matrix A for N=6. Verify that the matrix is unitary.
5. In a 6 level 2-D Wavelet transform NxN what is the size (resolution) of the largest sub-band and what is the size (resolution of the smallest sub-band? What is the overall number of sub-bands?
6. In a data hiding color image application provide one reason on why we may wish to hide the information in the lowest resolution sub-bad and one reason why may not.
7. What are fundamental differences between JPEG and JPEG-2000? Why one may prefer JPEG over JPEG-2000?
8. Is a wavelet transform coder lossy or lossless process? Please explain.
9. A texture image is coded via a wavelet transform based coder. What will be your strategy in allocating bits?
10. Assume that a signal is decomposed into a multiple resolution wavelet transform (different resolution sub-bands) or a wavelet transform with the same resolution in all sub-bands. Will your strategy in allocating bits to each sub-band differ in the two cases?
    Please explain.

(1)

16 bits budget 2021 MIDTERM 2 Solutions

20	10	12	7
-10	5	4	3
10	6	3	-1
-2	0	2	0

$T_0 = 16 = 2^4$, $1.5T_0 = 24$

pass 1

24	0	0
0	0	
0		0

$20 \rightarrow sp$

$10 \rightarrow zr$

$-10 \rightarrow zr$

$5 \rightarrow zr$

$20 - 24 < 0 \rightarrow 24 \rightarrow 24 - \frac{T_0}{4} = 24 - 4 = 20$

20	0	0
0	0	
0		0

so bitstream:

11 00 00 00/ 0 9 bits

pass 2

*	10	12	7
-10	5	4	3
10	6	3	-1
-2	0	2	0

$T_1 = \frac{T_0}{2} = 2^3 = 8$, $1.5T_0 = 12$

$10 \rightarrow sp$

$-10 \rightarrow sn$

$5 \rightarrow zr$ } 6 bits

stop here due to limited bit budget

$12 \rightarrow sp$

$7 \rightarrow zr$

$4 \rightarrow zr$

$3 \rightarrow zr$

20	12	0
-12	0	
0		0

Final.

bitstream $\boxed{11\ 00\ 00\ 00/0/11\ 01\ 00}$ → 15 bits in total

(2) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\boxed{\begin{matrix}20 & 12\\ -12 & 0\end{matrix}}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}8 & 12\\ 32 & 12\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}20 & -4\\ 44 & 20\end{bmatrix}$

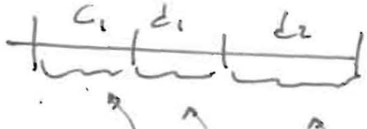
Then $A_1 = \frac{1}{2}\underbrace{\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & -1\end{bmatrix}}_{F}$ and $A_1^T F A_1 = \frac{1}{4}\begin{bmatrix}1 & 0 & \cancel{1} & 0\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & 1\\ 0 & 1 & 0 & -1\end{bmatrix}\left[\begin{array}{cc|cc}20 & -4 & 0 & 0\\ 44 & 20 & 0 & 0\\ \hline 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right]\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & -1\end{bmatrix}$

$$= \frac{1}{4}\begin{bmatrix}20 & -4 & 0 & 0\\ 20 & -4 & 0 & 0\\ 44 & 20 & 0 & 0\\ 44 & 20 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & -1\end{bmatrix} = \frac{1}{4}\begin{bmatrix}20 & 20 & -4 & -4\\ 20 & 20 & -4 & -4\\ 44 & 44 & 20 & 20\\ 44 & 44 & 20 & 20\end{bmatrix}$$

$$= \begin{bmatrix}5 & 5 & -1 & -1\\ 5 & 5 & -1 & -1\\ 11 & 11 & 5 & 5\\ 11 & 11 & 5 & 5\end{bmatrix}$$

(3)

Easy to show that



Also $a_1 = \frac{1}{4}$, $a_2 = \frac{1}{4}$, $a_3 = \frac{1}{2}$

$$\sigma_1^2 = \frac{(3-1)^2}{12} = \frac{4}{12} = \frac{1}{3}$$
$$\sigma_2^2 = 0.5^2 = 0.25$$
$$\sigma_3^2 = 0.2^2 = 0.04$$

$\underline{R_c = 1}$

Then using relation $$b_k = R_c + \frac{1}{2}\log_2 \frac{\sigma_k^2}{(\sigma_1^2)^{\frac{1}{4}}(\sigma_2^2)^{\frac{1}{4}}(\sigma_3^2)^{\frac{1}{2}}} =$$

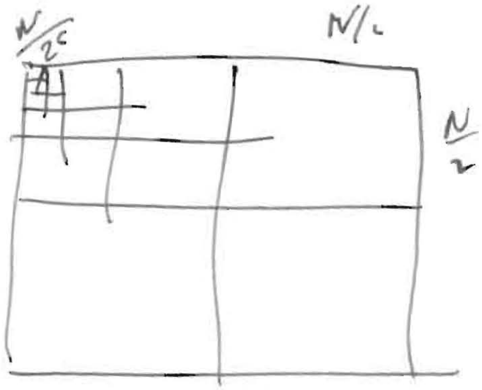
$$= R_c + \frac{1}{2}\log_2 \frac{\sigma_k^2}{\sqrt{\sigma_1\sigma_2}\,\sigma_3} = \ldots \begin{cases} b_1 = 2 \text{ bits} \\ b_2 = 2 \text{ bits} \\ b_3 = 0 \text{ bits} \end{cases}$$

So $R_c = \frac{1}{4}\cdot 2 + \frac{1}{4}2 + 0 = \frac{1}{2} + \frac{1}{2} = 1$ ✓ confirmed

(4)

$$\begin{bmatrix} 1\,1 & 0\,0 & 0\,0 \\ 0\,0 & 1\,1 & 0\,0 \\ 0\,0 & 0\,0 & 1\,1 \\ 1\,{-1} & 0\,0 & 0\,0 \\ 0\,0 & 1\,{-1} & 0\,0 \\ 0\,0 & 0\,0 & 1\,{-1} \end{bmatrix}$$

(5)



So largest subband size $\frac{N}{2} \times \frac{N}{2}$

smallest " size $\frac{N}{2^6} \times \frac{N}{2^6}$

Total # of subbands $3+3+3+3+3+3+1 =$

$= 3\cdot 6+1 = \underline{\underline{19}}$

(6)

WISH: Since usually the LL band is the most significant band

NOT distortion introduces by ~~...~~ hiding information may distort the image

WISH: Hidden information will be difficult to be removed by an attacker since this will distort the image

(7) JPEG — DCT based

JPEG-2000 — wavelet based, larger images, security, etc.

JPEG is simpler to implement.

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(8) Without quantization WT is a lossless process
when quantization takes place it becomes lossy

(9) Textured images contain high frequency information
So we allocate more bits to the HH subbands

(10) Resolution is irrelevant to assigning bits. What is important is
where most important information of image is
So the answer is NO

(11) $\Delta = \frac{2-(-2)}{4} = 1$

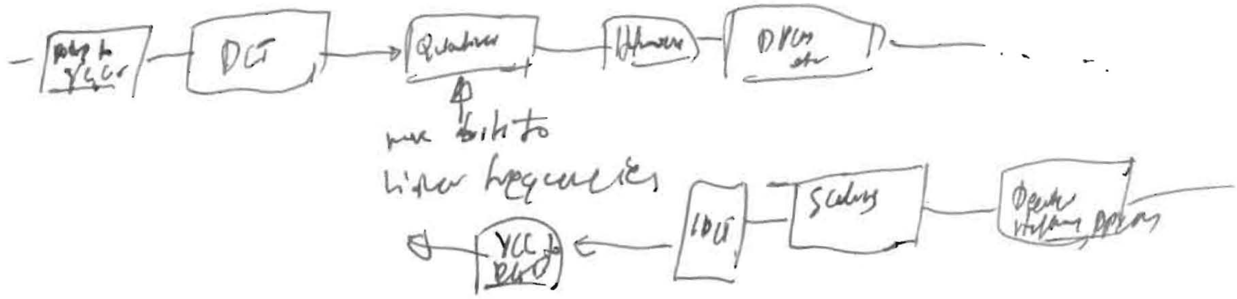
Boundaries: $-2, -1, 0, 1, 2$

Reconstruction levels: $-1.5, -0.5, 0.5, 1.5$

MSE: Given that PDF is flat over all intervals and reconstruction levels are in the centre of intervals $\rightarrow$ $MSE = \frac{\Delta^2}{12} = \frac{1}{12}$

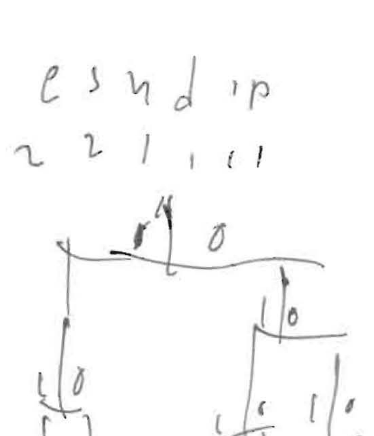
For the same reason the Max Lloyd algorithm will converge in one iteration $\rightarrow$ No improvement over uniform quantizer

(12)



(13) e s n d i p

2 2 1 1 1 1



			true bits
e	2	11	4
s	2	10	4
n	1	011	3
d	1	010	3
i	1	001	3
p	1	000	3
	8		20

rate $= \frac{20}{8} = \frac{10}{4} = 0.25$

Entropy $= 0.25$ as well

(14) Use formula $G_T = \frac{\frac{1}{3}\sum_{i=1}^{3} \sigma_{y_i}^2}{\left(\prod_{i=1}^{3} \sigma_{y_i}^2\right)^{\frac{1}{3}}} = \dots$ and pick the correct value

(15) ~~Image entropy is not ...~~ Image is not random