University of Toronto
Faculty of Applied Science and Engineering

FINAL EXAMINATION
ECE462H1S, Multimedia Systems

Thursday, April 15, 2010,
9:30 am-12:00 pm
Instructor: D. Hatzinakos

Instructions:

1. Type A exam
2. Non-programmable calculators are allowed
3. Please solve all four problems.
4. All answers must be written in the examination booklet. Do not write any answers in this problem handout.

QUESTION 1. (10 points)

a) Assume that in an EZW based compression of a 4x4 image f(x,y), the decoder receives the following information:

2 level Haar-based wavelet transform
To=32
Do=11011000000000000000001100
So=110
D1=100011000000110000
S1=01001

Assuming that the following codes have been used

Zerotree root	zr	00
Significant positive	sp	11
Significant negative	sn	01
Isolated zero	iz	10

What is the reconstructed 2-pass wavelet transform? (5 points)

b) What is the decoded (reconstructed) image? (5 points)

The 4-point Haar matrices (you may derive similarly a 2-point Haar matrix) are defined below:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \qquad H^{-1} = H^{T}$$

QUESTION 2 (10 points)

Given a 4x4 image (8 bits/pixel)

$$I = \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 256 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

Calculate the one level Haar wavelet transform of the image and then encode to an average of 2 bits /pixel as follows:

a. Encode the LL subband with 8 bits/pixel and the rest of the subbands with 0 bits/pixel using scalar uniform quantization.
b. Encode all 4 subbands with 2 bits/pixel using scalar uniform quantization
c. Encode all 4 subbands using DPCM followed by 2 bit/pixel scalar uniform quantization.

Compare the reconstructions of the image I, corresponding to these different encoding schemes.

,

QUESTION 3 (20 points)

Answer all of the following questions by providing sufficient explanation:
(2 points for each question)

1. Thinking about my large collection of .jpg images, I decide to unify them and make them more accessible by simply combine them into a big .mpg file by simply treating them as frames in a video: my reasoning is that I can simply use a viewer to step through the file , thus making a cohesive whole out of my collection. Comment on the utility of this idea in terms of the compression ratio achievable for the set of images.

2. (a) Please define “motion estimation”, (b) Please define “motion compensation”.

3. (a) What is Signal to Quantization Noise Ratio (SQNR)? (b) How does an additional 2 bits affect the SQNR? (c) Explain why the worst SQNR occurs when the sample equals half of the interval.

4. Suppose we decide to quantize an 8-bit gray scale image down to just 2 bits of accuracy. What is the simplest way to do that? What ranges of byte values in the original image are mapped to what quantized values?

5. Suppose that we acquire a video which has been compressed using motion-JPEG, and import it into Adobe Premiere (or a similar program). Then we create a movie using an MPEG 2 coder. Comment on (a) the compression ratio, (b) appearance of the resuting video.

6. Given the autocorrelation matrix $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ of an audio signal , calculate the optimal first and second order predictors.

7. Suppose an alphabet consists of six symbols, and the probability of each symbol is 1/6. What is the entropy for this set?

8. Draw the Huffman tree for the set of question 7. What is the average bit rate?

9. What is true color and what is indexed color?

10. In transform coding, why is it important the transform matrix be Unitary?

QUESTION 4 (10 points)

Consider a 4 level scalar quantizer of a continuous source x having a symmetric triangular probability density function between [-1, 1].

a) Assume that the quantizer output values y(i), i=1,2,3,4 are equally spaced over [-1,1] and calculate the mean square error introduced by the quantizer.
b) Move the higher level y(4) only to the origin leaving the other three output levels y(i) , i=1,2,3 as were before. Calculate the mean square error introduced by this nonuniform quantizer.
c) Compare the two quantizers and comment accordingly.

ECE462 final S2000

Quick Solutions

Q1

a)

$T_1 = 32$

D_0 = sp/sn/iz/zt/zt/zt/zt/zt/zt/zt/sp/zt

S_0 = 1 1 0

48	-48	0	0
0	0	0	0
0	0	0	0
48	0	0	0

$T_0 \cdot 1.5 = 48$

$\frac{T_0}{4} = 8$

→

56	-40	0	0
0	0	0	0
0	0	0	0
40	0	0	0

D_1 = iz/zt/sp/zt/zt/zt/sp/zt/zt

S_0 = 010 0 1

$T_1 = 16$ $T_1 \cdot 1.5 = 24$

$\frac{T_1}{4} = 4$

*	*	24	0
0	0	0	0
24	0	0	0
*	0	0	0

↓

56	-40	24	0
0	0	0	0
24	0	0	0
40	0	0	0

→

52	-36	20	0
0	0	0	0
20	0	0	0
44	0	0	0

$H_1^T \begin{bmatrix} 52 & -36 \\ 0 & 0 \end{bmatrix} H_1 = \begin{bmatrix} 8 & 44 \\ 8 & 44 \end{bmatrix}$

$H^T \begin{bmatrix} & \\ & \end{bmatrix} H = \begin{bmatrix} 24 & 4 & 22 & 22 \\ 4 & -16 & 22 & 22 \\ 26 & 26 & 22 & 22 \\ 18 & 18 & 22 & 22 \end{bmatrix}$

Q.7

$$I = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 256 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$w.T = H \cdot I \cdot H^T = \left(\begin{array}{cc|cc} 128 & 0 & -128 & 0 \\ 0 & 0 & 0 & 0 \\ \hline -128 & 0 & 128 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

All sub blocks of equal importance

a) 8 blk / pixel → 256 levels

$$A = \left(\begin{array}{cc|cc} 128 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad H^T A H = \begin{pmatrix} 64 & 64 & 0 & 0 \\ 64 & 64 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

b) 2 blk / 4 levels /

$$\left(\begin{array}{cc|cc} * & 0 & * & 0 \\ 0 & 0 & 0 & 0 \\ \hline * & 0 & * & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow H^T A H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

c) Same → (b)

Image ⇒ DPCM

Q.3

(1) There will be no benefit since MC will not work effectively in these unrelated frames

(2) Motion estimation: Determine motion vectors between frames
Motion compensation: Use motion vectors to predict next frame and subtract it from target frame

(3.) a) $SNR = 10\log_{10}\left(\frac{\text{Signal power}}{\text{Channel noise power}}\right)$.

b) $SQNR = 6B + 10.7$ → 2 bits increase or decrease SNR by 12 dB

c) $-\frac{\Delta}{2} \le e(n) \le \frac{\Delta}{2}$ so $\frac{\Delta}{2}$ is the maximum error.

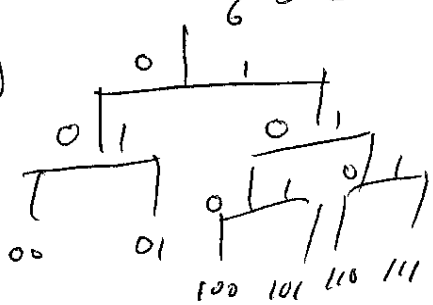
(4.) Keep the 2 MSB

(5) Higher CR, Worst appearance

(6) Prediction is not possible since underlying signal values are uncorrelated ($R = I$)

(7) $H = 6 \times \frac{1}{6}\log_2 6 = 2.585$ bits

(8)

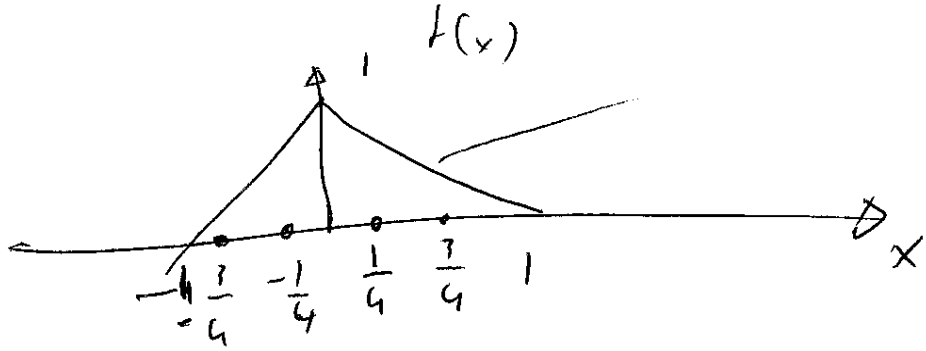


$rate = \frac{1}{6} \times (2\times 2 + 4\times 3) = 2.667$ bits

(9.) Av. C.R. = 24 bits/pixel ; C.R. = table

(10) Energy is preserved between time and DCT domains.

(4)



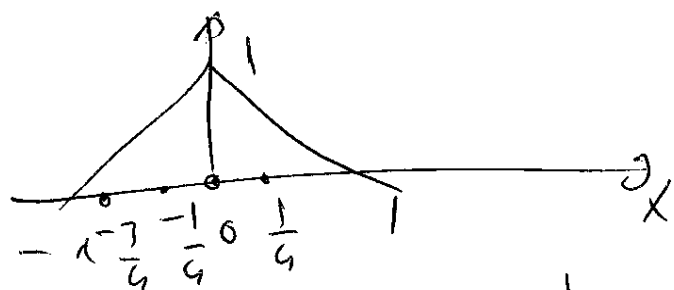
$\Delta = \frac{2}{4} = \frac{1}{2}$

a) $MSE = \int_{-1}^{-\frac{1}{2}} \left(x + \frac{3}{4}\right)(x+1)\,dx + \int_{-\frac{1}{2}}^{0} \left(x+\frac{1}{4}\right)(x+1)\,dx$

$+ \int_{0}^{\frac{1}{2}} \left(x - \frac{1}{4}\right)(1-x)\,dx + \int_{\frac{1}{2}}^{1} \left(x - \frac{3}{4}\right)(1-x)\,dx$

$= \cdots$

b)



$MSE = \int_{-1}^{-\frac{1}{2}} \left(x + \frac{3}{4}\right)(x+1)\,dx + \int_{-\frac{1}{2}}^{-\frac{1}{8}} \left(x+\frac{1}{4}\right)(x+1)\,dx$

$+ \int_{-\frac{1}{8}}^{0} (x+0)(x+1)\,dx + \int_{0}^{\frac{1}{8}} (x-0)(1-x)\,dx$

$+ \int_{\frac{1}{8}}^{1} x\left(-\frac{1}{4}\right)(1-x)\,dx = \cdots$

→ Lower MSE due to high curvs introduced by hyper values in the vicinity of 1