University of Toronto
Faculty of Applied Science and Engineering

FINAL EXAMINATION
ECE462H1S, Multimedia Systems

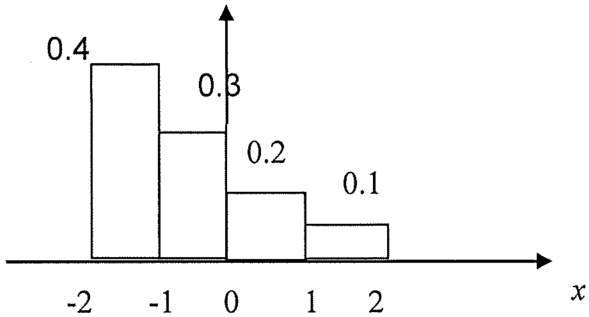
Tuesday, April 14, 2015
2:00-4:30 pm
Instructor: D. Hatzinakos

Instructions:

1. Type A exam
2. Non-programmable calculators are allowed
3. Please solve all four problems.
4. All answers must be written in the examination booklet. Do not write any answers in this problem handout.

QUESTION 1. (10 points)

A signal X has the following pdf $f_X(x)$:



a) Design a 1-bit uniform quantizer for this signal. Specify the decision boundaries and reconstruction levels.
b) Use the Max-Lloyd algorithm to design a non-uniform quantizer (see below for useful formulas). Use the previous settings for a uniform quantizer to initialize the algorithm. Use at least two iterations of the algorithm and comment on the final decision boundaries and reconstruction values.

Hint:

- Max-Lloyd relations:

$$b_i = \frac{\hat{x}_{i-1} + \hat{x}_i}{2},$$

$$\hat{x}_i = \frac{\int_{b_i}^{b_{i+1}} x f_X(x) dx}{\int_{b_i}^{b_{i+1}} f_X(x) dx}$$

QUESTION 2. (10 points)
Assume that in an EZW based compression of a 4x4 image f(x,y), the decoder receives the following information:

2 level Haar-based wavelet transform
To=16
Do=11000000
So=1
D1=10000011110000
S1=010

Assuming that the following codes have been used

Zerotree root	t	00
Significant positive	p	11
Significant negative	n	01
Isolated zero	z	10

1. What is the reconstructed 2-pass wavelet transform?

2. What is the decoded (reconstructed) image?

   (The 4-point Haar matrices and 2-point Haar matrices are defined below☺

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad H^{-1} = H^T, \quad H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad H_1^{-1} = H_1^T$$

3. Generate a Huffman code for the dominant pass symbols. Use the two coding iterations D0, D1 to estimate the probability of occurrence for each symbol. Show the final code for each symbol and the average code rate.
4. What is the entropy of the dominant pass signal? Are there any benefits compared to fixed length coding?

QUESTION 3 (20 points)

Answer all of the following questions by providing sufficient explanation:
(2 points for each question)

1. What is frequency masking and what is its significance in audio coding?

2. Given the autocorrelation matrix $R = \begin{bmatrix} 1 & 0.9 & 0.8 \\ 0.9 & 1 & 0.9 \\ 0.8 & 0.9 & 1 \end{bmatrix}$ of an audio signal , what is the maximum linear (MSE optimum) predictor you can design? Calculate the coefficients of this predictor.

3. Given a binary signal with three times as many ones than zeros, what is the maximum lossless compression that we can achieve?

4. Given the sequence 31.5, 16.2, 5.6, 3.1, 1.1 and a quantizer round(x/q) where x is the signal value and q a constant scale, describe a two level scalable-SNR process to obtain an average error of less than 0.01.

5. What are the 4 different forms of redundancy used in signal compression applications?

6. A black and white image has entropy less than 1 bit. What can we conclude for the image?

7. Given an image does it make more sense to apply Huffman coding to the image itself or to its DCT transform? Explain your answer.

8. The following code is considered for the four characters A,B,C,D
   a. A: 0, B: 01, C:10, D:110
   b. Can this be a Huffman Code?

9. A random signal having Gaussian pdf (probability density function) is a) non-uniformly quantized to 256 levels , and b) uniformly quantized to 512 levels. Which one do you think this is a better strategy?

10. Suppose you have a video sequence coded using the following GOP display order:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
I	B	B	P	B	B					I	B	B	P

What is the transmitted bit stream order?

QUESTION 4 (10 points)

In a data compression system the data is first transformed and then the transform values are either kept or placed to zero by using a threshold of 0.375. Two transforms are under consideration. The Discrete Cosine Transform (DCT) and the Discrete Walsh Transform (DWALT). The matrices for 1-dimensional 4-point DCT and 4-point DWALT are given below:

DCT matrices (These are normalized approximations to DCT matrices and are used in practice)

$$T = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{bmatrix} \qquad T^{-1} = \begin{bmatrix} 1 & 1 & 1 & 0.5 \\ 1 & 0.5 & -1 & -1 \\ 1 & -0.5 & -1 & 1 \\ 1 & -1 & 1 & -0.5 \end{bmatrix}$$

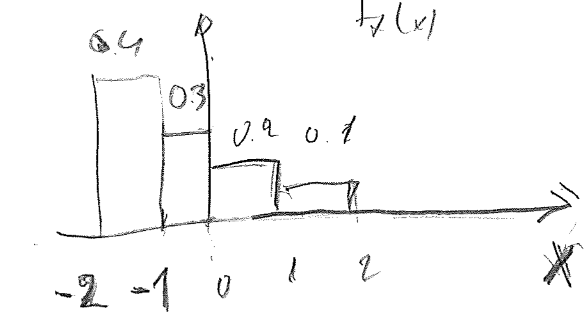
DWALT matrix (forward transform)

$$W = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

1. Assume you are given the data sequence {1,2,3,4}. Which transform is more efficient for data compression in this case?
2. Test whether the Walsh transform is Unitary. What is the inverse Walsh transform?
3. Given the thresholded transforms , reconstruct the data sequence by applying the inverse transforms and compare to the original data
4. Can you see any advantage in using the DWALT over the DCT ?

Solutions

Question 1



$\underline{b=1}$ $\quad M = 2^1 = 2$ levels

a) $$\Delta = \frac{2-(-2)}{2} = \frac{4}{2} = 2$$

Therefore the two reconstruction levels by inspection are $\underline{V_0 = -1,\ V_1 = 1}$

boundaries are $b_0 = -2$
$b_1 = 0$
$b_2 = 2$

b) Initialization

$V_0^0 = -1$, $V_1^0 = 1$, $\quad b_0 = -2$ (do not change)
$b_1^0 = 0$ (changes in every iteration)
$b_2 = 2$ (do not change)

1st iteration: $$b_1^1(0) = \frac{V_0^0 + V_1^0}{2} = \frac{-1+1}{2} = 0$$

$$V_0^1 = \frac{\int_{-2}^{-1} x\, 0.4\, dx + \int_{-1}^{0} x\, 0.3\, dx}{\int_{-2}^{-1} 0.4\, dx + \int_{-1}^{0} 0.3\, dx} = \frac{0.4 \frac{x^2}{2}\Big|_{-2}^{-1} + 0.3\frac{x^2}{2}\Big|_{-1}^{0}}{0.7} = \frac{-0.4\cdot\frac{3}{2} + 0.3\cdot\frac{1}{2}}{0.7} =$$

$$= -\frac{1.2+0.3}{1.4} = -\frac{1.5}{1.4} = -1.071$$

$$V_1^1 = \frac{\int_0^1 x\, 0.2\, dx + \int_1^2 x\, 0.1\, dx}{\int_0^1 0.2\, dx + \int_1^2 0.1\, dx} = \frac{0.2\frac{x^2}{2}\Big|_0^1 + 0.1\frac{x^2}{2}\Big|_1^2}{0.3} = \frac{0.2+0.3}{0.6} = \frac{0.5}{0.6} = 0.8333$$

2nd iteration

$$b_1^2(0) = \frac{V_0^1 + V_1^1}{2} = \frac{-1.071 + 0.8333}{2} = -0.119$$

$$V_0^2 = \frac{\int_{-2}^{-0.119} x f_X(x)\, dx}{\int_{-2}^{-0.119} f_X(x)\, dx}$$

$$V_1^2 = \frac{\int_{-0.119}^{2} x f_X(x)\, dx}{\int_{-0.119}^{2} f_X(x)\, dx}$$

QUESTION 2

2 level Haar transform 4x4 image

$T_0 = 16$

$D_0 = 11000000$

$S_0 = 1$

$D_1 = 10000\cancel{\#}111\ 0000$

$S_1 = 010$

1

a) $T_0 = 16$

$D_0 = 11|00|00|00$

sp t t t

16+8=24

$$\begin{bmatrix} 24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$S_0 = 1$

$\cancel{x} = 24 + 4 = 28$

$$\begin{bmatrix} 28 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$T_1 = 8$

b) $D1 = 1\,0|00|00|11|11|00\ 00$

z t t p p t t

$$\begin{bmatrix} 28 & 0 & 12 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$S1 = 010$

$$\begin{bmatrix} 26 & 0 & 14 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2

first let $F_1 = \begin{pmatrix} 26 & 0 \\ 0 & 0 \end{pmatrix}$ and $H_1 = \begin{bmatrix} +\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} =$

Then $f_1 = H_1^T F_1 H_1 = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{bmatrix} 26 & 0 \\ 0 & 0 \end{bmatrix}\frac{\sqrt{2}}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{bmatrix} 26 & 26 \\ 0 & 0 \end{bmatrix}$

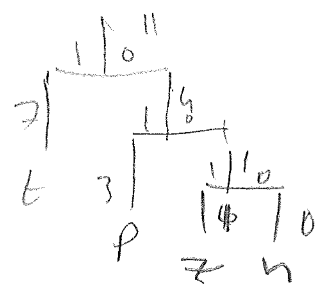
$$= \frac{1}{2}\begin{bmatrix} 26 & 26 \\ 26 & 26 \end{bmatrix}$$

Then. $f = H^T \begin{bmatrix} 26 & 26 & 14 & 10 \\ 26 & 26 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} H = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 13 & 13 & 14 & 10 \\ 13 & 13 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 27 & -1 & 23 & 3 \\ 13 & 13 & 13 & 13 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 27 & -1 & 23 & 3 \\ 27 & -1 & 23 & 3 \\ 13 & 13 & 13 & 13 \\ 13 & 13 & 13 & 13 \end{bmatrix}$

(3) $p(sp) = \frac{3}{11}$, $p(t) = \frac{7}{11}$, $p(z) = \frac{1}{11}$, $p(h) = 0$

HC



Symbol	Code	p
t	1	7/11
p	01	3/11
z	001	1/11
h	000	0

Average code rate

$= \frac{7}{11} + 2\frac{3}{11} + 3 \cdot \frac{1}{11} =$

$= \frac{16}{11} = 1.4545$

(4)

$H = \frac{7}{11}\log_2\frac{11}{7} + \frac{3}{11}\log_2\frac{11}{3} + \frac{1}{11}\log_2 11 = \frac{7}{11} \cdot 0.65 + \frac{3}{11} \cdot 1.874 + \frac{1}{11} 3.459$

$\log_2 x = \frac{\log_{10} x}{\log_{10} 2}$

$= \underline{1.256}$

QUESTION 3

1. A weak frequency within the critical range of a strong frequency in the spectrum of an audio signal is not audible. Therefore, it can be eliminated from the data and provide compression

2. $R = \begin{bmatrix} 1 & 0.9 & 0.8 \\ 0.9 & 1 & 0.9 \\ 0.8 & 0.9 & 1 \end{bmatrix}$ $3 \times 3 \rightarrow$ 2 length predictor

two coefficients a_1, a_2

$$\begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.8 \end{bmatrix} \Rightarrow \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix}^{-1}\begin{bmatrix} 0.9 \\ 0.8 \end{bmatrix} =$$

$$= \frac{1}{1.23}\begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix}\begin{pmatrix} 0.9 \\ 0.8 \end{pmatrix} = \frac{1}{1.23}\begin{bmatrix} 0.18 \\ -0.01 \end{bmatrix} = \begin{pmatrix} 0.146 \\ -0.008 \end{pmatrix}$$

3. $p(1) = \frac{3}{4}$ $p(0) = \frac{1}{4}$ So $H = \frac{3}{4}\log_2\frac{4}{3} + \frac{1}{4}\log_2 4 =$

$= 0.75 \cdot 0.415 + 0.5 \cdot 2 = \underline{\underline{1.31 \text{ bits}}}$

4. x | 31.5 , 16.2 , 5.6 , 3.1 , 1.1 (let $q = 8$)

$\text{round}\left(\frac{x}{8}\right)$ 4 , 2 , 1 , 0 , 0

$\hat{x}_1 = 8x$ 32 , 16 , 8 , 0 , 0

$MSE_1 = \frac{1}{5}(0.5^2 + 0.2^2 + 2.4^2 + 3.1^2 + 1.1^2) = 3.374$

Error = 0.5 0.2 −2.4 3.1 1.1

round(e) = 1 0 −2 3 1

now $\hat{x}_2 = \hat{x}_1 + \hat{e}$ = 33 16 6 3 1

$\Rightarrow MSE_2 = \frac{1}{5}(1.5^2 + 0.2^2 + 0.4^2 + 0.1^2 + 0.1^2)$

$= 0.494$

(5) 1. Spatial
2. Frequency
3. Temporal
4. Physiological

(6) The image is not random

(7) We cannot say for sure

(8) No, because 0 (A) is a prefix of 0.1 (B)

(9) Most probably the nonuniform quantization is better given the non-uniform nature of the Gaussian PDF

(10) 1 4 2 3 11 5 6 14 12 13

QUESTION 4

(1) $T \cdot X = \begin{bmatrix} 2.5 \\ -1.4 \\ 0 \\ -0.2 \end{bmatrix} \xrightarrow{\text{thresh}} \begin{bmatrix} 2.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ similar compression performance

$W \cdot X = \begin{bmatrix} 2.5 \\ -1 \\ 0 \\ -.5 \end{bmatrix} \xrightarrow{\text{thresh}} \begin{bmatrix} 2.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(2) Test whether $WW^T = I \Rightarrow \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = \frac{I}{4}$

not unitary

$W^{-1} = 4W^T$

(3)

$$\hat{x} = T^{-1}\begin{bmatrix} 2.5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2.5 \\ 2.5 \\ 2.5 \end{bmatrix}$$

$$\hat{x} = W^{-1}\begin{bmatrix} 2.5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2.5 \\ 2.5 \\ 2.5 \end{bmatrix}$$

(4) The DWHT has integer values so less floating point arithmetic errors in calculations and rounding of results