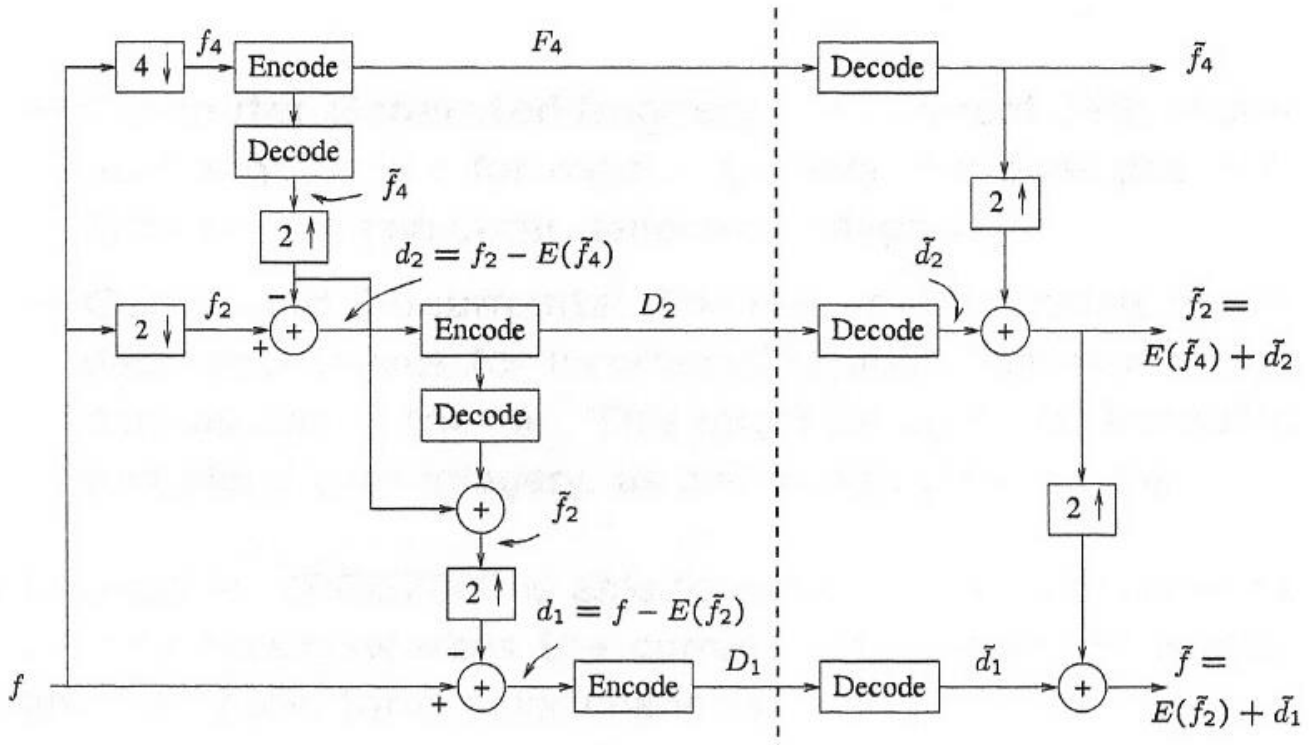
ECE462 – Lecture 13

JPEG Hierarchical Mode

JPEG Hierarchical Mode

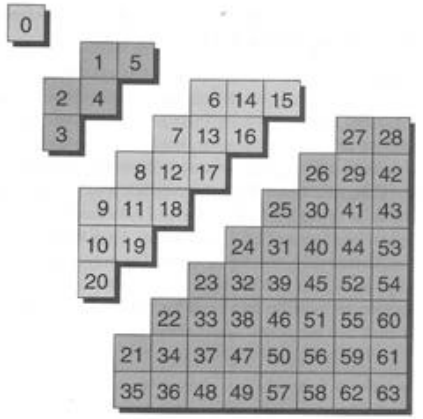
- An image is broken down into a number of sub-images called “frames”.
- A frame is a collection of one or more scans.
- The first frame creates a low resolution version of the image.
- The remaining frames refine the image by increasing the resolution.
- High complexity and larger amount of data to be transmitted.
- Not in general use.

JPEG Hierarchical Mode

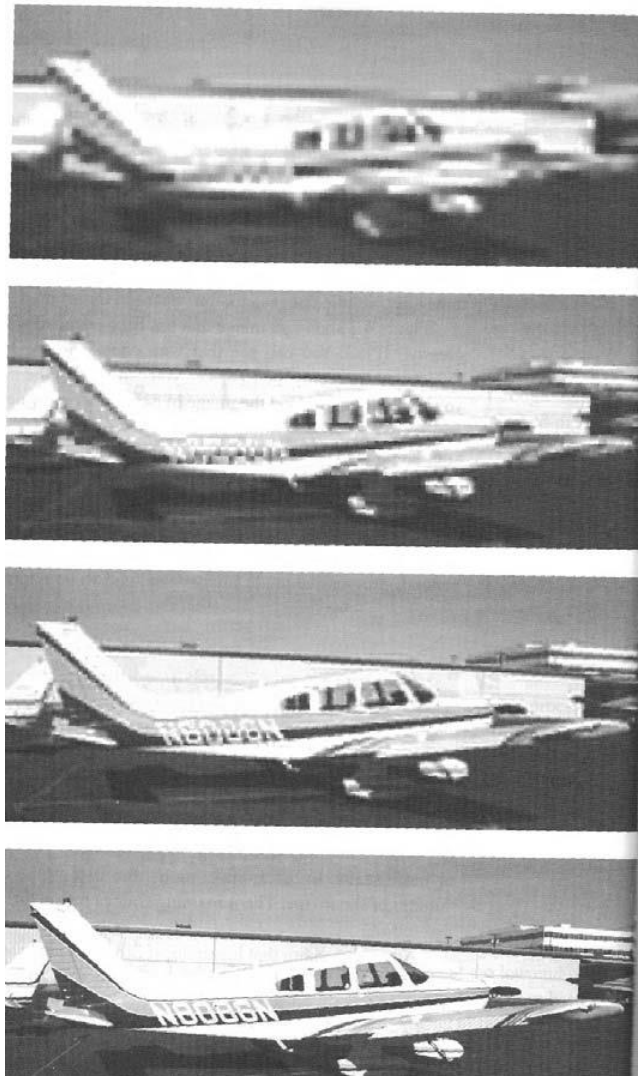


JPEG Progressive Mode

- The image is encoded in successive multiple scans and decoded progressively (quality increases as more and more high frequency content is added in decoded image)
- Spectral selection example
  - Encode DC and first AC coefficients
  - Encode a few more AC coefficients
  - More AC coefficients etc.
- Progressive mode is suitable when processing is faster than the transmission speed of image.
- Rarely used in practice.
- Image files of same size as in sequential JPEG.



JPEG Progressive Mode



Lossless JPEG-LS

- It utilizes spatial prediction and context recognition to compress more effectively an image.
- However, compression ratios achieved much lower than regular JPEG.
- Interest for applications remains to be seen.

The Karhunen-Loeve Transform (KLT)

Given a vector $X = [X_0, X_1, X_2, \dots, X_{N-1}]^T : N \times 1$

1. Calculate the autocovariance matrix

$$C_x = E[XX^T] - E[X]E[X^T]$$

2. Find the eigenvalues $\lambda_0, \lambda_1, \dots, \lambda_{N-1}$ and the eigenvectors $u_0, u_1, \dots, u_{N-1}$ of C_x.

Then the KLT is Y = FX

Where F = $[u_0, u_1, \dots, u_{N-1}]$

Important properties of KLT

- $FF^T = I$ i.e., orthonormal.

$$R_y = E[y y^T] = \begin{bmatrix} \lambda_0 & & 0 \\ & \lambda_1 & \\ & & \ddots \\ 0 & & \lambda_{n-1} \end{bmatrix}$$ diagnol matrix

$$\text{i.e. } E[y_i y_j] = 0 \quad \text{to} \quad i \neq j$$

- The KLT is the optimum decorrelation transform.

Ex. 8: Ex. 8.5, p.221 in text Note: there is an inconsistency in text: they describe the use of the autocorrelation matrix R_X for deriving the KLT, but then use the autocovariance matrix C_X in the example. The KLT does in fact use C_X (though, some people define R_X as we define C_X!), but we often have/assume zero-mean processes ($\mu_X = 0$), therefore $R_X = C_X$.

Ex. 8.5 $\Rightarrow$ Let us assume X is a zero-mean process

We have $N = 4$ sample sequences of length 3 ($X(i)$, $i = \{0, 1, 2\}$):
$x_0 = [4, 4, 5]$, $x_1 = [3, 2, 5]$, $x_2 = [5, 7, 6]$, $x_4 = [6, 7, 7]$

$$R_X = E[XX^T] = E\left(\begin{bmatrix} X(0) \\ X(1) \\ X(2) \end{bmatrix} [X(0)\, X(1)\, X(2)]\right)$$

$$= \begin{bmatrix} E[X(0)X(0)] & E[X(1)X(0)] & E[X(2)X(0)] \\ E[X(0)X(1)] & E[X(1)X(1)] & E[X(2)X(1)] \\ E[X(0)X(2)] & E[X(1)X(2)] & E[X(2)X(2)] \end{bmatrix}$$

Note: diagonal of R_X, $R_X(i,i) = E[X(i)^2] = \sigma^2_{X(i)}$ (when $\mu_X = 0$)

$$\Rightarrow \text{ Estimate } R_X(i,j) = \frac{1}{N-1} \sum_{n=0}^{N-1} x_n(i) x_n(j)$$

$$= \begin{bmatrix} 28.67 & 33 & 35.67 \\ 33 & 39.33 & 40.33 \\ 35.67 & 40.33 & 45 \end{bmatrix}$$

Find the eigenvalues of R_X:

$$\Rightarrow |\lambda I - R_X| = 0 \Rightarrow \lambda = \{111.2052, 1.7485, 0.0463\}$$

where $|\cdot|$ is the determinant operator.

And the eigenvectors:

$$R_X u = \lambda u \Rightarrow \begin{aligned} u_o &= [0.5073, 0.5870, 0.6301] \\ u_1 &= [0.0794, -0.7609, 0.6440] \\ u_2 &= [-0.8581, 0.2766, 0.4326] \end{aligned}$$

These eigenvectors from R_X are the bases of the KLT, and thus are assembled as the rows of the transform matrix F:

$$y = Fx \Rightarrow F = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix}$$

Remember:

$$\begin{aligned} R_Y &= E[YY^T] \\ &= E[FXX^TF^T] \\ &= F\,E[XX^T]\,F^T \\ &= FR_XF^T \end{aligned}$$

For KLT:

$$\begin{aligned} R_Y &= \begin{bmatrix} \lambda & & 0 \\ & \ddots & \\ 0 & & \lambda_{N-1} \end{bmatrix} \\ \therefore \lambda_i &= \sigma^2_{Y(i)} \quad (\text{for } \mu_Y = 0) \end{aligned}$$

Note: λ_0 is large compared to others $\Rightarrow$ good energy compaction.

We can easily calculate

$$\begin{aligned} G_{TC_Y} &= \frac{\frac{1}{N}\sum_{i=0}^{N-1}\sigma^2_{Y(i)}}{\left(\prod_{i=0}^{N-1}\sigma^2_{Y(i)}\right)^{1/N}} = 18.1082 \\ \Rightarrow G_{TC_X} &= 1.0174 \end{aligned}$$

Note: $G_{TC_X} = 1$ if $\sigma^2_{X(i)} = \sigma^2_X$ for all i

Also note that the transform is energy preserving $\sum \sigma^2_X = \sum \sigma^2_Y$ (since it is orthonormal).

Probability Definitions

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Name	Definition (Continuous r.v.)	Definition (Discrete r.v.)	Estimate from Samples
Mean (μ_X)	$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx$	$E[X] = \sum_k x p_X(x_k)$	$\overline{x} = \frac{1}{N} \sum_{n=0}^{N-1} x_n$
Variance (σ_X^2)	$E[(X-\mu_X)^2] = \int_{-\infty}^{\infty} (x-\mu_X)^2 f_X(x) dx$	$E[(X-\mu_X)^2] = \sum_k (x-\mu_X)^2 p_X(x_k)$	$\frac{1}{N-1} \sum_{n=0}^{N-1} (x_n - \overline{x})^2$
Correlation	$E[XY] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_{X,Y}(x,y) dx dy$	$E[XY] = \sum_k \sum_l xy p_{X,Y}(x_k, y_l)$	$\frac{1}{N-1} \sum_{n=0}^{N-1} x_n y_n$
Covariance ($\mathrm{Cov}(X,Y)$)	$E[(X-\mu_X)(Y-\mu_Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x-\mu_X)(y-\mu_Y) f_{X,Y}(x,y) dx dy$	$E[(X-\mu_X)(Y-\mu_Y)] = \sum_k \sum_l (x-\mu_X)(y-\mu_Y) p_{X,Y}(x_k, y_l)$	$\frac{1}{N-1} \sum_{n=0}^{N-1} (x_n - \overline{x})(y_n - \overline{y})$
Autocorrelation ($R_X(i,j)$)	$E[X(i)X(j)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_{X(i),X(j)}(x,y) dx dy$	$E[X(i)X(j)] = \sum_k \sum_l xy p_{X(i),X(j)}(x_k, y_l)$	$\frac{1}{N-1} \sum_{n=0}^{N-1} x_n(i) x_n(j)$
Autocovariance ($C_X(i,j)$)	$E[(X(i)-\mu_{X(i)})(X(j)-\mu_{X(j)})] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \widetilde{x}\widetilde{y} f_{X(i),X(j)}(x,y) dx dy$ where $\widetilde{x} = (x - \mu_{X(i)})$ and $\widetilde{y} = (y - \mu_{X(j)})$	$E[(X(i)-\mu_{X(i)})(X(j)-\mu_{X(j)})] = \sum_k \sum_l \widetilde{x}\widetilde{y} p_{X(i),X(j)}(x_k, y_l)$ where $\widetilde{x} = (x - \mu_{X(i)})$ and $\widetilde{y} = (y - \mu_{X(j)})$	$\frac{1}{N-1} \sum_{n=0}^{N-1} (x_n(i) - \overline{x(i)})(x_n(j) - \overline{x(j)})$

See important notes on next page

NOTES:

1. $f_X(x)$ is the probability density function (p.d.f.) of continuous random variable (r.v.) X; the probability that X will take on values in the range $a \leq x \leq b$ is $\int_a^b f_X(x)dx$.

2. $p_X(x_k)$ is the probability mass function (p.m.f.) of discrete r.v. X; the probability that X will take on the value $x = x_k$ is $p_X(x_k)$.

3. All definitions assume that the random variables are real-valued.

4. For estimates using sample values (last column in the table), x_n refers to the n^{th} sample. For example, if you are estimating the mean of a pixel and you have 3 sample pixel values $\{5, 9, 2\}$, you estimate the mean $\overline{x} = (5+9+2)/3$.

5. Many of the estimates using samples are scaled by $N-1$ (rather than N). This makes them *unbiased* estimates of their respective statistic. In practice you may see either version; in this course either version will be accepted.

6. Correlation and covariance is calculated for a pair of random variables, X and Y, where $f_{X,Y}(x,y)$ is the *joint* p.d.f. (for continuous random variables), and $p_{X,Y}(x_k, y_l)$ is the *joint* p.m.f. (for discrete random variables).

7. Autocorrelation and autocovariance are calculated for a random process, $X(i)$, where i is the time/position at which the process is sampled. For example, we may consider a row of 5 pixels in an image a random process, where $0 \leq i < 5$ is the position index from left to right. Each $X(i)$ (pixel) can be considered an individual r.v.

8. The estimates for autocorrelation and autocovariance require multiple samples for each of the time/positions i and j. For example, say we have 3 sample sequences, $x_0 = [1, 4, 7, 10]$, $x_1 = [2, 5, 8, 11]$, and $x_3 = [3, 6, 9, 12]$; to estimate the autocorrelation at time/position points 0 and 2:

$$\begin{aligned} R_X(0,2) &\approx \frac{1}{N-1}\sum_{n=0}^{N-1} x_n(0)x_n(2) \\ &= \frac{1}{3-1}(1\cdot 7 + 2\cdot 8 + 3\cdot 9) \\ &= \frac{1}{2}\cdot 50 = 25 \end{aligned}$$