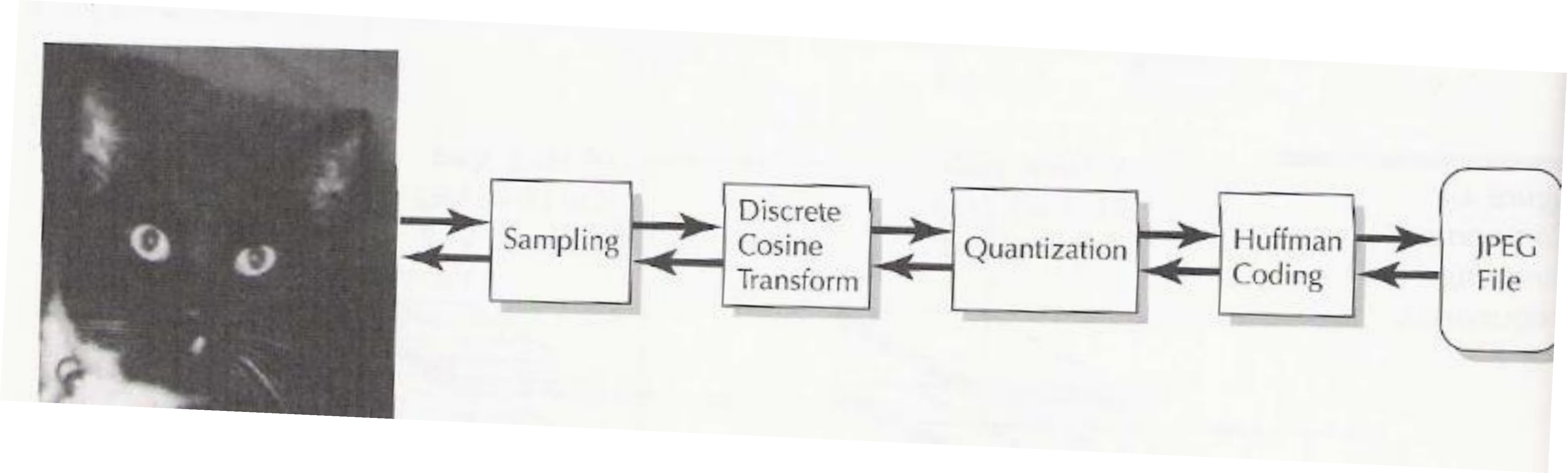
ECE462 – Lecture 6

Introduction to JPEG

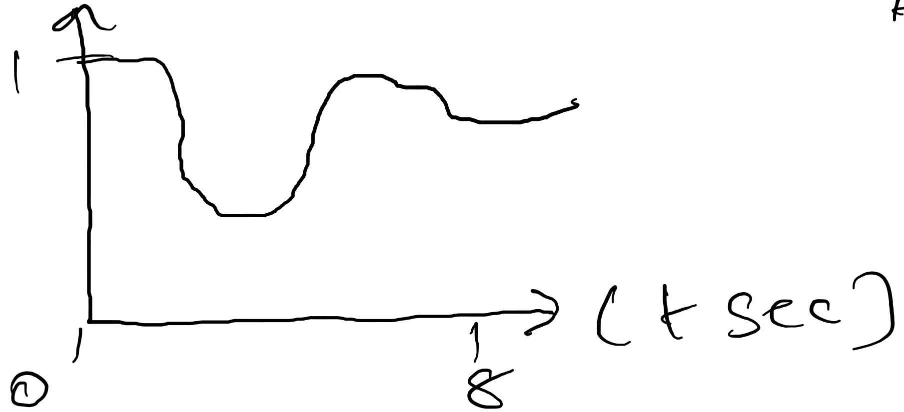
- <u>Sampling:</u> Pixel data are converted from RGB to YC_bC_r color space and down sampling is applied.



Introduction to JPEG

- **Discrete Cosine Transform (DCT):** The image is divided into 8x8 pixel blocks. The DCT converts each block values into a sum of cosine functions.
- **Quantization:** DCT Coefficients that are not essential receive less dense quantization than essential coefficients. A large number of coefficients are eliminated.
- **Huffman Coding:** Quantized DCT coefficients are entropy encoded.

Sampling and down sampling



Analog Signal : $x_a(t)$

$0 \le t \le 8$ sec

Discrete Signal: $x[n] = x_a(nT)$

$n = 0, 1, 2, 3 \ldots$

$T = 1$ sec, $f_s = \frac{1}{T} = 1$ Hz

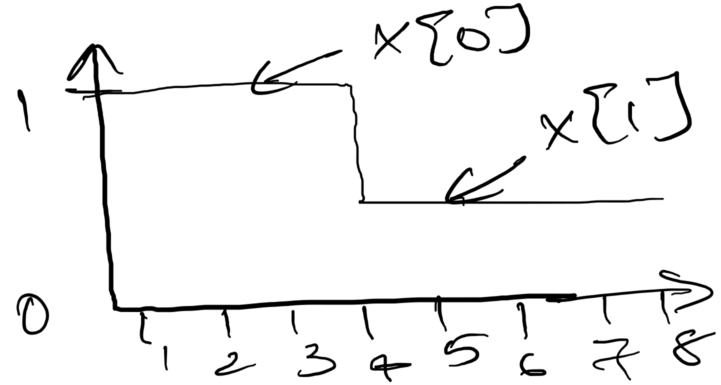
$x[n] = 1, 0.8, 0.65, 0.4,$
$0.6, 0.7, 0.8, 0.7$

Sampling and down sampling

$T = 2$ sec, $f_s = \frac{1}{T} = 0.5$ Hz

$n = 0 \quad 1 \quad 2 \quad 3$

$x[n] = 1,\ 0.65,\ 0.6,\ 0.8$

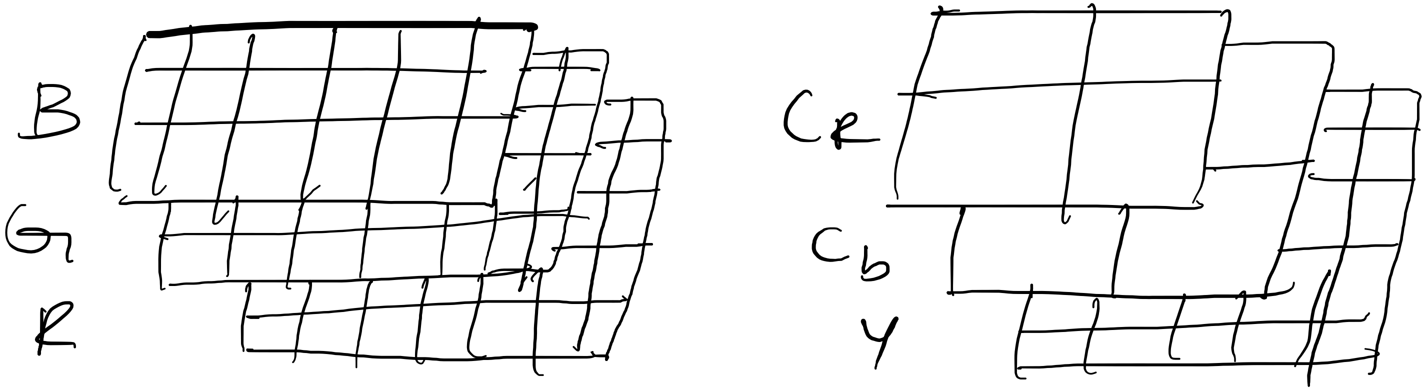


$T = 4$ sec, $f_s = \frac{1}{T} = 0.25$ Hz

$n = 0,\ 1$

$x[n] = 1,\ 0.6$

Sampling and down sampling



- While R,G,B components are usually equally important (intensity wise) Y, C_b, C_r components are not. In this case Y is more important than C_b, C_r.

Thus, C_b, C_r may be downsampled so that higher compression may be possible.

Sampling and down sampling

- Above: All R, G, B and Y components (channels) are represented by a higher number of pixels compared to the

C_b, C_r components (channels) usually down sampled by 2 in both directions (4 times less “resolution”)

- At decoder up-sampling + filtering “restores” all pixels.

Chroma Subsampling

(4 : 4 : 4) No chroma subsampling (Same # of pixels for Y, C_b, C_r)

(4 : 2 : 2) Horizontal subsampling of C_b, C_r by a factor of 2

(4 : 1 : 1) Horizontal subsampling of C_b, C_r by a factor of 4

(4 : 2 : 0) Subsampling of C_b, C_r both horizontally and vertically by a factor of 2.

(4 : 2 : 0) is used in JPEG, MPEG.

The Discrete Cosine Transform (DCT)

The DCT is the “heart” of JPEG compression.

It is a mapping of values from time or space domain to an equal number of values in the frequency domain.

1-D DCT (One dimensional DCT)

Given a signal x[n], n=0,1,…,N-1

Write x[n] as:

$$x[n] = \sum_{k=0}^{N-1} y(k)\, Cos\, \frac{(2n+1)k\pi}{2N}$$

IDCT

$n = 0, 1, \ldots, N-1$

$k = 0, 1, \ldots, N-1$

Solution

DCT

$$y[k] = C_k \sum_{n=0}^{N-1} x[n]\, Cos\, \frac{(2n+1)k\pi}{2N}$$

$$C_0 = \sqrt{\frac{1}{N}}, C_k = \sqrt{\frac{2}{N}}, k \neq 0$$

Example

Let x[n] be a signal of length N=8

n : 0, 1, 2, 3, 4, 5, 6, 7

x[n] : 190, 184, 186, 182, 167, 123, 63, 38

DCT:

k : 0, 1, 2, 3, 4, 5, 6, 7

Y[k] : 38.5, 143.81, −67.76, −16.33, 7.42, −4.73, 5.49, 0.05

Example

y[0]: DC DCT Coefficient

y[k], $k \neq 0$: AC DCT Coefficients

Note:

$$x[n] \underset{\text{IDCT}}{\overset{\text{DCT}}{\rightleftarrows}} y[k]$$

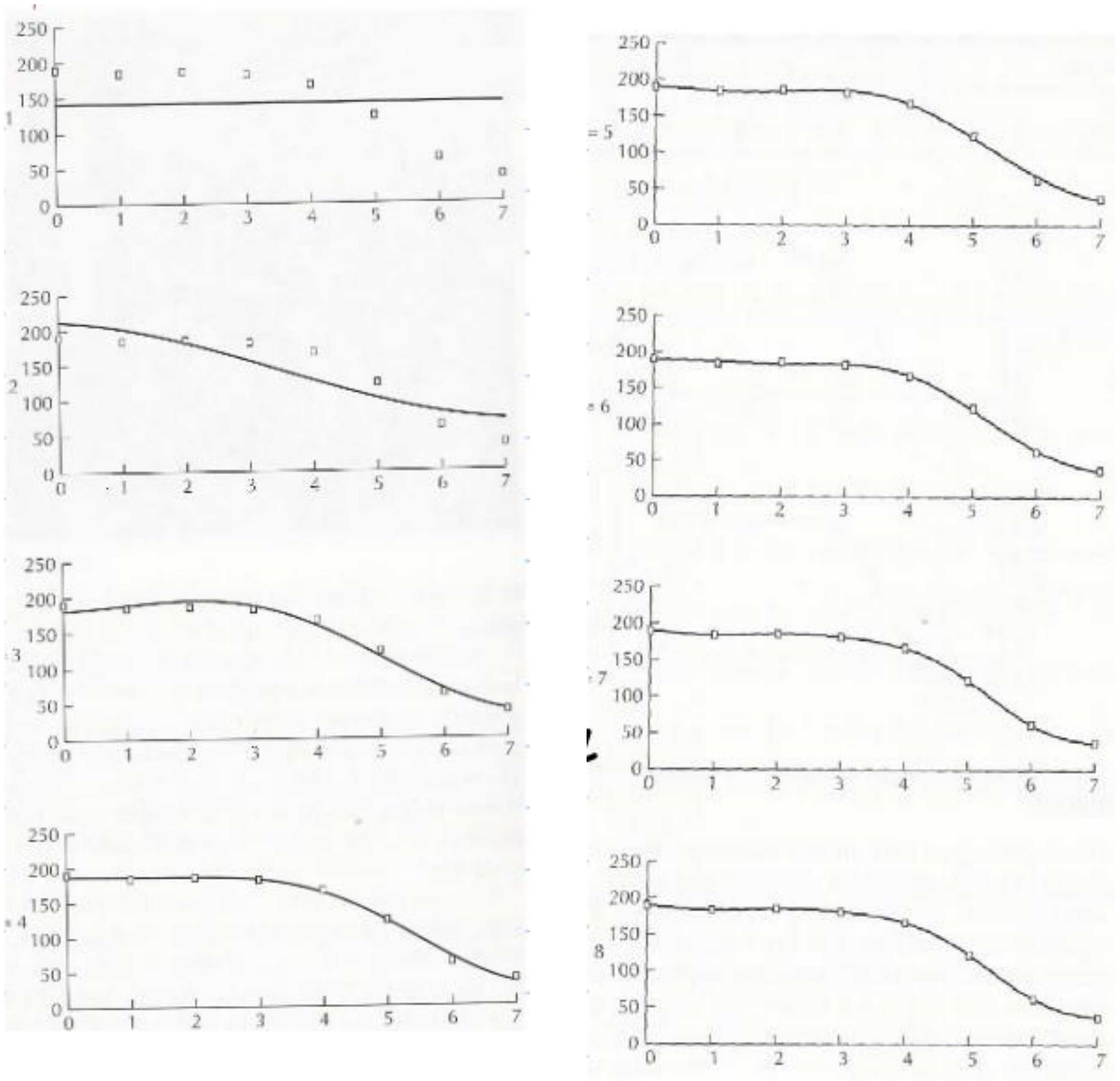
$n = 0, 1, \ldots, 7$

$k = 0, 1, \ldots, 7$

Example

IDCT using reduced number of DCT coefficients

Note: The first 3 DCT coefficients y(k), $k = 0,1,2$ are sufficient to express x[n] n = 0,1,2,..,7.



Example

- Dotted lines are the true x[n] values.
- Solid line shows the values of x[n] reconstructed from y[k] via IDCT by using only 1-DCT coefficient, only 2-DCT coefficients, only 3-DCT coefficients and so on…

Matrix notation for 1D-DCT

Given a signal x[n] of length N samples and corresponding DCT of N samples we can write in vector form :

$$Y = [Y[0], Y[1], Y[2] \ldots Y[N-1]]^T$$

$$X = [x[0], \ldots \ldots x[N-1]]^T$$

and then **Y=M X**

where T denotes transpose of a vector and M is an NxN matrix of cosines.

Matrix notation for 1D-DCT

For example, for N=8, M takes the from

$$
\begin{bmatrix}
\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} \\
\frac{1}{2}\cos\frac{1}{16}\pi & \frac{1}{2}\cos\frac{3}{16}\pi & \frac{1}{2}\cos\frac{5}{16}\pi & \frac{1}{2}\cos\frac{7}{16}\pi & \frac{1}{2}\cos\frac{9}{16}\pi & \frac{1}{2}\cos\frac{11}{16}\pi & \frac{1}{2}\cos\frac{13}{16}\pi & \frac{1}{2}\cos\frac{15}{16}\pi \\
\frac{1}{2}\cos\frac{2}{16}\pi & \frac{1}{2}\cos\frac{6}{16}\pi & \frac{1}{2}\cos\frac{10}{16}\pi & \frac{1}{2}\cos\frac{14}{16}\pi & \frac{1}{2}\cos\frac{18}{16}\pi & \frac{1}{2}\cos\frac{22}{16}\pi & \frac{1}{2}\cos\frac{26}{16}\pi & \frac{1}{2}\cos\frac{30}{16}\pi \\
\frac{1}{2}\cos\frac{3}{16}\pi & \frac{1}{2}\cos\frac{9}{16}\pi & \frac{1}{2}\cos\frac{15}{16}\pi & \frac{1}{2}\cos\frac{21}{16}\pi & \frac{1}{2}\cos\frac{27}{16}\pi & \frac{1}{2}\cos\frac{33}{16}\pi & \frac{1}{2}\cos\frac{39}{16}\pi & \frac{1}{2}\cos\frac{45}{16}\pi \\
\frac{1}{2}\cos\frac{4}{16}\pi & \frac{1}{2}\cos\frac{12}{16}\pi & \frac{1}{2}\cos\frac{20}{16}\pi & \frac{1}{2}\cos\frac{28}{16}\pi & \frac{1}{2}\cos\frac{36}{16}\pi & \frac{1}{2}\cos\frac{44}{16}\pi & \frac{1}{2}\cos\frac{52}{16}\pi & \frac{1}{2}\cos\frac{60}{16}\pi \\
\frac{1}{2}\cos\frac{5}{16}\pi & \frac{1}{2}\cos\frac{15}{16}\pi & \frac{1}{2}\cos\frac{25}{16}\pi & \frac{1}{2}\cos\frac{35}{16}\pi & \frac{1}{2}\cos\frac{45}{16}\pi & \frac{1}{2}\cos\frac{55}{16}\pi & \frac{1}{2}\cos\frac{65}{16}\pi & \frac{1}{2}\cos\frac{75}{16}\pi \\
\frac{1}{2}\cos\frac{6}{16}\pi & \frac{1}{2}\cos\frac{18}{16}\pi & \frac{1}{2}\cos\frac{30}{16}\pi & \frac{1}{2}\cos\frac{42}{16}\pi & \frac{1}{2}\cos\frac{54}{16}\pi & \frac{1}{2}\cos\frac{66}{16}\pi & \frac{1}{2}\cos\frac{78}{16}\pi & \frac{1}{2}\cos\frac{90}{16}\pi \\
\frac{1}{2}\cos\frac{7}{16}\pi & \frac{1}{2}\cos\frac{21}{16}\pi & \frac{1}{2}\cos\frac{35}{16}\pi & \frac{1}{2}\cos\frac{49}{16}\pi & \frac{1}{2}\cos\frac{63}{16}\pi & \frac{1}{2}\cos\frac{77}{16}\pi & \frac{1}{2}\cos\frac{91}{16}\pi & \frac{1}{2}\cos\frac{105}{16}\pi
\end{bmatrix}
$$

Matrix notation for 1D-DCT

Note that the matrix M has the property:

$$M.M^T = M^T.M = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & 1 & \vdots \\ 0 & \cdots & 1 \end{bmatrix} = I$$

Where I is the identity matrix (NxN) and T denotes Matrix transposition.

In other words, the DCT transform is an **“orthonormal transform” or also denoted as Unitary transform.**

Orthonormal transforms are important because they distribute the energy of the signal into “independent” or “uncorrelated” basis functions which then, can be treated separately.

Unitary Transform and Energy preservation

Total energy is preserved via a unitary transformation and is the same in the two domains that is

total energy in time domain=total energy in DCT domain