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Solutions

MIDTERM EXAMINATION
ECE462H1S, Multimedia Systems

Tuesday February 26, 2013
Examiner: D. Hatzinakos

Time: 9:30-11:00 am (90 minutes)
Room: BA2155

- This is a closed book exam (type A). You may use non-programmable calculators. No additional aids are permitted.
- An aid sheet with formulas you may use is attached (last page).
- All sub-questions in each question are equally weighted
- This test counts for 30% of the final mark.
- Please answer all questions. Use only the space provided in these sheets.

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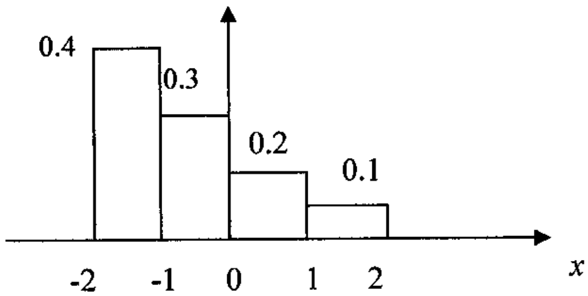
Exam questions

Question 1. (10 points)
A signal X has the following pdf $f_X(x)$:



- Design a 1-bit uniform quantizer for this signal. Specify the decision boundaries and reconstruction levels.

$\Delta = \frac{2-(-2)}{2} = 2$ 1 bit quantizer $\rightarrow$ 2 reconstruction levels, 1 threshold

Boundaries: $b_0 = -2$, $b_1 = 0$, $b_2 = 2$

Reconstruction levels: $x_0 = -1$, $x_1 = 1$

- Use the Max-Lloyd algorithm to design in a non-uniform quantizer (ee last page for useful formulas). Use the previous settings for a uniform quantizer to initialize the algorithm. Use at least two iterations of the algorithm and comment on the final decision boundaries and reconstruction values.

Initial values (uniform quantizer)

$b_1^{(0)} = 0$, $x_0^{(0)} = -1$, $x_1^{(0)} = 1$

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First iteration (1):

$$b_0^{(1)} = \frac{x_0^{(0)} + x_1^{(0)}}{2} = \frac{1-1}{2} = 0$$

$$x_0^{(1)} = \frac{\int_{-2}^{0} x f_x(x)\,dx}{\int_{-2}^{0} f_x(x)\,dx} = \frac{\int_{-2}^{-1} x\,0.4\,dx + \int_{-1}^{0} x\,0.3\,dx}{0.7} = \frac{0.4\frac{x^2}{2}\Big|_{-2}^{-1} + 0.3\frac{x^2}{2}\Big|_{-1}^{0}}{0.7} = -1.07$$

$$x_1^{(1)} = \frac{\int_0^2 x f_x(x)\,dx}{\int_0^2 f_x(x)\,dx} = \frac{\int_0^1 x\,0.2\,dx + \int_1^2 x\,0.1\,dx}{0.3} = \frac{0.2\frac{x^2}{2}\Big|_0^1 + 0.1\frac{x^2}{2}\Big|_1^2}{0.3} = \frac{5}{6} = 0.833$$

<u>2nd iteration (2)</u>

$$b_0^{(2)} = \frac{x_0^{(1)} + x_1^{(1)}}{2} = \frac{-1.07 + 0.833}{2} = -0.1185$$

$$x_0^{(2)} = \frac{\int_{-2}^{-0.1185} x\, f_x(x)\,dx}{\int_{-2}^{-0.1185} f_x(x)\,dx} = \frac{\int_{-2}^{-1} x\,0.4\,dx + \int_{-1}^{-0.1185} x\,0.3\,dx}{\int_{-2}^{-1} 0.4\,dx + \int_{-1}^{-0.1185} 0.3\,dx} = \cdots = \underline{\underline{-1.125}}$$

$$x_1^{(2)} = \frac{\int_{-0.1185}^{0} x\,0.3\,dx + \int_0^1 x\,0.2\,dx + \int_1^2 x\,0.1\,dx}{\int_{-0.1185}^{0} 0.3\,dx + \int_0^1 0.2\,dx + \int_1^2 0.1\,dx} = \cdots = 0.59$$

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Question 2 (10 points)

1. The following values are observed:

n	0	1	2	3	4	5	6	7
x[n]	3	2	- 3	- 2	3	2	-3	

Can you estimate the missing value for n=7 from the value of x(6) based on linear prediction?

We want $\hat{x}(7) = a_1 x(6)$

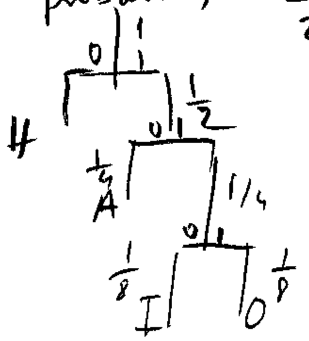
Then, $R_x(0)\, a_1 = R_x(1) \Rightarrow a_1 = \frac{R_x(1)}{R_x(0)}$

$$R_x(1) = \sum_n x(n)\,x(n+1) = 3\cdot 2 + 2(-3) + (-3)(-2) + (-2)\cdot 3 + 3\cdot 2 + 2(-3)$$
$$= 6 - 6 + 6 - 6 + 6 - 6 = 0$$

Since $R_x(1) = 0$ prediction of $x(7)$ based on $x(6)$ is not possible

2. Derive a Huffman code for the word HAHAHIHO. What is the rate of the code?

Symbols	H	A	I	O
probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$



Symbol	Code	probability
H	0	$\frac{1}{2}$
A	10	$\frac{1}{4}$
I	110	$\frac{1}{8}$
O	111	$\frac{1}{8}$

$$\text{Rate} = \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8} 3 + \frac{1}{8} 3$$
$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{8} + \frac{1}{8} = 1 + \frac{1}{4} = 1.25$$

This is also the entropy of the source

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3. Why transformation of a signal is an effective procedure in signal compression?

Depending on the signal characteristics the transformation to a new domain may cause energy compaction which will allow more efficient quantization of the signal and therefore better (higher) compression

4. Is high image entropy desirable in image compression? Please explain your answer.

No:
High entropy means high uncertainty or high degree of randomness in the signal values. This will imply that transformation of the signal (like DCT) will not provide high energy compaction and therefore compression will not be efficient

5. Given an image does it make more sense to apply Huffman coding to the image itself or to its DCT transform? Explain your answer.

It may not make much difference unless the image has special structure (eg binary) in which case Huffman coding may have some advantage in the image domain

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Question 3 (10 points)

12 values of a signal x[n] are observed:

n	0	1	2	3	4	5	6	7	8	9	10	11
x[n]	0	0	0	0	1	9	-1	0	0	0	0	0

The observed values undergo the following operations:

1. Segmentation into segments of 4 values followed by 4-DCT in each segment
2. ~~Segmentation into segments of 4 values followed by DPCM in each segment~~
2. Segmentation into segments of 4 values followed by a two level DWT using Haar filters in each segment

Calculate the variance of the transformed coefficients in each case and describe a procedure to assign 36 bits to the 12 transformed values so that the distortion (MSE) is minimized. Which transform in your opinion gives the best performance?

(DCT)

S1 $\quad Y_1^T = T X_1^T = T\begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = [y_1(1), y_1(2), y_1(3), y_1(4)]^T$

S2 $\quad Y_2^T = T X_2^T = T\begin{bmatrix}1\\9\\-1\\0\end{bmatrix} = \begin{bmatrix}2.25\\1.08\\-2.25\\-3.8\end{bmatrix} = (y_2(1), y_2(2), y_2(3), y_2(4))^T$

S3 $\quad Y_3^T = T X_3^T = T\begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = [y_3(1), y_3(2), y_3(3), y_3(4)]^T$

Variance for $\quad \sigma^2_{y(i)} = \frac{1}{3}\sum_{j=1}^{3} y_j^2(i) - \left(\frac{1}{3}\sum_{j=1}^{3} y_j(i)\right)^2$, then $D_i = \sigma^2_{y(i)} \cdot 2^{-2b}$

Thus for DCT: $\quad \sigma^2_{y(1)} = \frac{1}{3}\cdot 2.25^2 - \left(\frac{1}{3}\cdot 2.25\right)^2 = 1.125 \quad$ (2 bits / value) × 4 = 8 bits

$\sigma^2_{y(2)} = \frac{1}{3} 1.08^2 - \left(\frac{1}{3} 1.08\right)^2 = 0.26 \quad$ (1 bit / value) · 4 = 4 bits

$\sigma^2_{y(3)} = 1.125 \quad$ (2 bits / value) × 4 = 8 bits

$\sigma^2_{y(4)} = \frac{1}{3} 3.8^2 - \left(\frac{1}{3} 3.8\right)^2 = 3.2 \quad$ (4 bits / value) × 4 = 16 bits

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WT (Haar): S1 $[0,0,0,0] \xrightarrow{\text{1st level}} [0\ 0 \mid 0\ 0] \xrightarrow{\text{2nd level}} [0|0|00]$

S2 $[1, 9, -1, 0] \xrightarrow{\text{1st level WT}} \left[\frac{10}{\sqrt2}, \frac{-1}{\sqrt2}, +\frac{8}{\sqrt2}, -\frac{1}{\sqrt2}\right] \xrightarrow{\text{2nd level}} \left(\left[\frac{9}{2}, \frac{+11}{2}, +\frac{8}{\sqrt2}, \frac{1}{\sqrt2}\right)\right.$

S3 $[0,0,0,0] \longrightarrow [0|0|00]$

Variance for subbands $\quad$ L2 : $\sigma^2_{L2} = \frac{1}{3}\left(\frac{9}{2}\right)^2 - \left(\frac{1}{3}\frac{9}{2}\right)^2 = 6.68$

H2 : $\sigma^2_{H2} = \frac{1}{3}\frac{11}{2} - \left(\frac{1}{3}\frac{11}{2}\right)^2 = 6.72$

[L2 | H2 | H1] (4 bits / value) = 24 bits

(SIMILAR PERFORMANCE)

H1 : $\sigma^2_{H1} = \frac{1}{6}\left[\left(\frac{8}{\sqrt2}\right)^2 + \left(\frac{1}{\sqrt2}\right)^2\right] - \left(\frac{1}{6}\frac{9}{\sqrt2}\right)^2 = 4.29 \quad$ (2 bit / value) × 6 = 12 bits

total = 36 bits

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Aid sheet (Useful relations)

- Distortion

$$D = E[(x-\hat{x})^2] = \sum_{i=0}^{N-1} \int_{b_i}^{b_{i+1}} (x-\hat{x}_i)^2 f_X(x)dx$$

- Max-Lloyd relations:

$$b_i = \frac{\hat{x}_{i-1}+\hat{x}_i}{2},$$

$$\hat{x}_i = \frac{\int_{b_i}^{b_{i+1}} x f_X(x)dx}{\int_{b_i}^{b_{i+1}} f_X(x)dx}$$

- 4-DCT Matrix

$$T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{pmatrix},$$

- Haar Filters

$$h_0 = \frac{1}{\sqrt{2}}(\delta(n)+\delta(n-1))$$

$$h_1 = \frac{1}{\sqrt{2}}(\delta(n)-\delta(n-1)$$

- Autocorrelation $R(k) = \frac{1}{N}\sum_n x(n)x(n+k)$