SOLUTIONS

Student name: Student number:

MIDTERM EXAMINATION
ECE462H1S, Multimedia Systems

Tuesday February 25, 2014
Examiner: D. Hatzinakos

Time: 9:30-11:00 am (90 minutes)
Room: GB304

- This is a closed book exam (type A). You may use non-programmable calculators. No additional aids are permitted.
- An aid sheet with formulas you may use is attached (last page).
- All questions are equally weighted
- This test counts for 30% of the final mark.
- Please answer all questions. Use only the space provided in these sheets.

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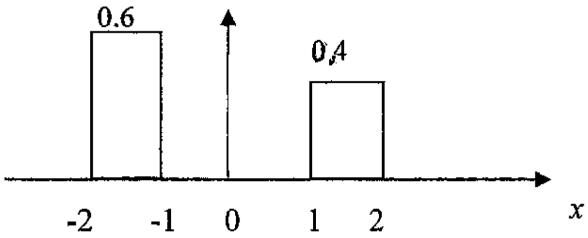
Exam questions

Question 1. (3 points)
A signal X has the following pdf $f_X(x)$:



- Design a 1-bit uniform quantizer for this signal. Specify the decision boundaries and reconstruction levels.

$\Delta = \frac{2-(-2)}{2} = 2$ 1 bit quantizer $\rightarrow$ 2 reconstruction levels, 1 threshold (boundary level)

Threshold: $b = 0$

Reconstruction levels: $x_0 = -1$, $x_1 = 1$

(That is it $x > 0 \rightarrow x_1 = 1$
$x < 0 \rightarrow x_0 = -1$

Question 2. (3 points)

- Design a Max-Lloyd quantizer for question 1 (see last page for useful formulas). Use the previous settings for a uniform quantizer to initialize the algorithm. Use one iteration of the algorithm and comment on the final decision boundaries and reconstruction values

Initial values (uniform quantizer): $b^{(0)} = 0$, $x_0^{(0)} = -\frac{3}{2}$, $x_1^{(0)} = \frac{3}{2}$

first iteration:

$$b^{(1)} = \frac{x_0^{(0)} + x_1^{(0)}}{2} = \frac{-\frac{3}{2} + \frac{3}{2}}{2} = 0$$

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Then $x_0^{(1)} = \dfrac{\int_{-2}^{0} x f_X(x)\,dx}{\int_{-2}^{0} f_X(x)\,dx} = \dfrac{\int_{-2}^{-1} x f_X(x)\,dx}{\int_{-2}^{-1} f_X(x)\,dx} = \dfrac{\int_{-2}^{-1} x \cdot 0.6\,dx}{0.6} = \dfrac{x^2}{2}\Big|_{-2}^{-1} =$

$= \frac{1}{2} - \frac{4}{2} = -\frac{3}{2}$

$x_1^{(1)} = \dfrac{\int_0^2 x f_X(x)\,dx}{\int_0^2 f_X(x)\,dx} = \dfrac{\int_1^2 x \cdot 0.4\,dx}{0.4} = \dfrac{x^2}{2}\Big|_1^2 = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

No change. Uniform quantizer is the optimum choice.

Question 3. (3 points)

Consider the following sequence of 1s and 0s

a) An alternating sequence of 1s and 0s.

b) A long sequence of 1s followed by a long sequence of 0s.

c) A long sequence of 0s followed by a single 1 and then a long sequence of 0s

Comment on the compressibility of the sequence in each case and describe in detail a method for compression.

a), b) Some variation of RLC can be designed to handle both repetitive values (a) and long sequences of 1s and 0s

c) Run length coding (RLC) can be applied as follows:

$(N, 1)(0, 0)$ ← end of block (remainder values are all zeros)

N ← number of zeros before 1

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Question 4. (3 points)

The sinusoidal wave m(t)=6 sin (2πt) volts is transmitted using 4-bit binary encoding. What is the expected MSE distortion D and what is the Signal to quantization noise ratio (SQNR)?

$$D = \frac{1}{12}\frac{(u_{max}-u_{min})^2}{2^{2B}} = \frac{1}{12}\frac{(6-(-6))^2}{2^{2\cdot 4}} = \frac{1}{12}\frac{12^2}{2^8} = \frac{12}{2^8} = \frac{3}{2^6} \cong 0.047$$

$$SQNR = 10\log_{10}\frac{\text{Signal Power}}{D} = 10\log_{10}\frac{\frac{6^2}{2}}{D} =$$

$$= 10\log_{10}\frac{36}{2\cdot 0.047} = 25.83 \text{ dB}$$

Question 5. (3 points)

Consider a compact disk that uses binary encoding to record audio signals whose bandwidth is w=15 Khz. The quantization process is uniform with 512 levels. What is the minimum permissible bit rate?

R = Number of bits/sample × Number of samples/sec

$$= \log_2 512 \times \underbrace{2w}_{\text{min sampling rate}} = 9 \times 30\,K = 270\,K$$

bits/sec

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Question 6. (3 points)

A sequence x[n] consists of the following 8 samples

n	0	1	2	3	4	5	6	7
x[n]	0.9	-2	1	-1.2	3	-1	2	-1.93

Round the values of x[n] to the closest integer and apply Huffman coding on the result. Construct the Huffman coding tree and show the resulting Huffman code. What is the average number of bits needed for each value using the Huffman code?

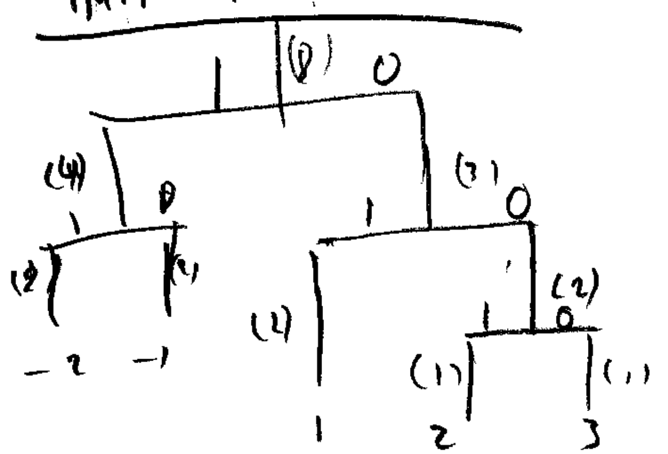
Let $x_q(n) = \text{round}[x(n)]$

Then $x_q(n) = 1, -2, 1, -1, 3, -1, 2, -2.$

Source symbols: $-2, -1, 1, 2, 3$

Frequency: $2 \quad 2 \quad 2 \quad 1 \quad 1$

Huffman tree



Huffman table

Symbol	Freq	Code	# of bits
-2	2	11	4
-1	2	10	4
1	2	01	4
2	1	001	3
3	1	000	3
	8		18

Huffman code rate: $\frac{18}{8} = 2.25$ bits/symbol

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Question 7. (3 points)

What is the entropy of the source before and after quantization in question 6? What is the average number of bits needed for each using fixed-length code and what is the Huffman coding gain?

before: $H_b = \sum_{i=0}^{7} \frac{1}{8} \log_2 8 = 8 \frac{1}{8} \log_2 8 = 3$ bits

After: $H_b = 3 \frac{1}{4} \log_2 4 + 2 \frac{1}{8} \log_2 8 = \frac{9}{4} = 2.25$ bits

$$\text{Gain} = \frac{3}{2.25} = 1.33$$

$Y_Q = \lfloor Y \rfloor = [-1, 0, 0, 1, 0, 1, 0, 2]^T$

source values: $-1, 0, 1, 2$

frequency: $1, 4, 2, 1$

$H_y = 2 \cdot \frac{1}{8} \log 8 + \frac{1}{2} \log 2 + \frac{1}{4} \log 4 = 1.75$

So Huffman coding is more effective in this case

Question 8. (3 points)

Divide the sequence in question 6 into two subsequences of length 4 samples and calculate the 4-point DCT.

What is the coding gain of the transform? Quantize the values to the lower integer and calculate the entropy of the resulting sequence. Comment on the effectiveness of applying a Huffman code in comparison to question 6.

$X_1 = [0.9 \quad -2 \quad 1 \quad -1.2]^T \qquad X_2 = [3 \quad -1 \quad 2 \quad -1.93]^T$

$DCT = T \cdot X_1 \qquad DCT = T \cdot X_2$

$Y_1 = [-0.325, 0.24, 0.175, 1.62]^T \qquad Y_2 = [0.5175, 1.372, 0.0175, 2.186]$

$\sigma_y^2(0) = \frac{1}{2}(0.325^2 + 0.5175^2) = 0.1867$

$\sigma_y^2(1) = \frac{1}{2}(0.24^2 + 1.375^2) = 0.974$

$\sigma_y^2(2) = \frac{1}{2}(0.175^2 + 0.0175^2) = 0.0154$

$\sigma_y^2(3) = \frac{1}{2}(1.62^2 + 2.186^2) = 3.7$

$$G_{TC} = \frac{\frac{1}{4}\left(\sigma_{y(0)}^2 + \sigma_{y(1)}^2 + \sigma_{y(2)}^2 + \sigma_{y(3)}^2\right)}{\sqrt[4]{\sigma_{y(0)}^2 \sigma_{y(1)}^2 \sigma_{y(2)}^2 \sigma_{y(3)}^2}} = \frac{1.22}{0.32} = 3.81$$

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Question 9. (3 points)

Given the autocorrelation matrix $R = \begin{bmatrix} 1 & 0.9 & 0 \\ 0.9 & 1 & 0.9 \\ 0. & 0.9 & 1 \end{bmatrix}$ of an audio signal,

calculate the optimal first and second order predictors.

first order: $R(0)\cdot a_1 = R(1) \Rightarrow a_1 = \frac{R(1)}{R(0)} = \frac{0.9}{1} = \boxed{0.9}$

So $\boxed{\hat{x}(n) = 0.9\,x(n-1)}$

2nd order: $\begin{bmatrix} R(0) & R(1) \\ R(1) & R(0) \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \end{bmatrix} \Rightarrow \begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{bmatrix} 0.9 \\ 0 \end{bmatrix} \Rightarrow$

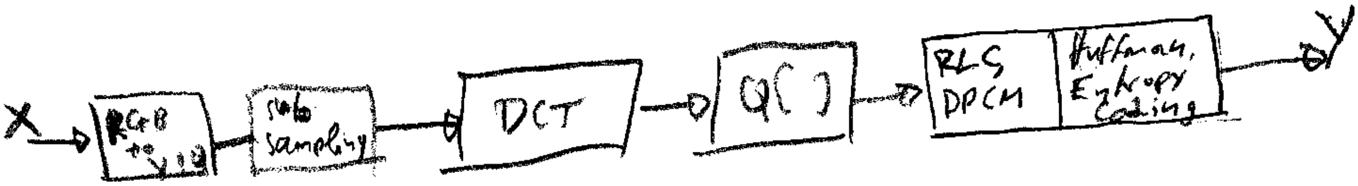
$\Rightarrow \left.\begin{array}{l} a_1 + 0.9\,a_2 = 0.9 \\ 0.9\,a_1 + a_2 = 0 \Rightarrow a_2 = -0.9\,a_1 \end{array}\right\} \Rightarrow a_1 + 0.9^2 a_1 = 0.9 \Rightarrow$

$\Rightarrow \boxed{\begin{array}{l} a_1 = \frac{0.9}{0.19} = 4.737 \\ a_2 = -0.9\times 4.737 = -4.263 \end{array}}$

So $\boxed{\hat{x}(n) = 0.497\,x(n-1) - 0.447\,x(n-2)}$

Question 10. (3 points)

Briefly describe the major stages of a JPEG coder. Comment on the effectiveness of the JPEG in compressing black and white images vs greyscale images and colour images



B & W images: Not very effective, other simpler schemes can be applied

$\left\{\begin{array}{l}\text{Greyscale images:} \\ \text{Colour images:}\end{array}\right\}$ Similar performance, effectiveness depends on the frequency content of image

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Aid sheet (Useful relations)

- Distortion

$$D = E[(x-\hat{x})^2] = \sum_{i=0}^{N-1} \int_{b_i}^{b_{i+1}} (x-\hat{x}_i)^2 f_X(x)dx \quad = \quad \frac{1}{12} \frac{(U_{max} - U_{min})^2}{2^{2R}}$$

- Max-Lloyd relations:

$$b_i = \frac{\hat{x}_{i-1} + \hat{x}_i}{2},$$

$$\hat{x}_i = \frac{\int_{b_i}^{b_{i+1}} x f_X(x)dx}{\int_{b_i}^{b_{i+1}} f_X(x)dx}$$

- Coding Gain

$$G_{TC_Y} = \frac{\frac{1}{N}\sum_{i=0}^{N-1} \sigma_{Y(i)}^2}{\left(\prod_{i=0}^{N-1} \sigma_{Y(i)}^2\right)^{1/N}}$$

- 4-DCT Matrices

$$T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 1 & 1 & 1 & 0.5 \\ 1 & 0.5 & -1 & -1 \\ 1 & -0.5 & -1 & 1 \\ 1 & -1 & 1 & -0.5 \end{pmatrix}$$