Student name: Student number: Solutions

MIDTERM EXAMINATION
ECE462H1S, Multimedia Systems

Tuesday February 23, 2016
Examiner: D. Hatzinakos

Time: 5-6 pm
Room: SF3202

- This is a closed book exam (type A). All type calculators are allowed. No additional aids are permitted.
- An aid sheet with formulas you may use is attached (last page).
- All sub-questions in each question are equally weighted
- This test counts for 30% of the final mark.
- Please answer all questions. Use only the space provided in these sheets.

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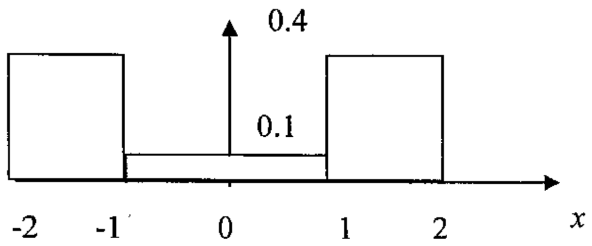
Exam questions
Answer all of the following questions by providing sufficient explanation:

1. A signal X has the following pdf $f_X(x)$:



- Use the Max-Lloyd algorithm to design a 1-bit non-uniform quantizer (see last page for useful formulas). Use a uniform quantizer to initialize the algorithm. Use at least one iteration of the algorithm and comment on the final decision boundaries and reconstruction values. (5 points)

Initialization: boundaries $b_0 = -2$, $b_1^{(0)} = 0$, $b_2 = 2$

reconstruction levels $x_0^{(0)} = -1$, $x_1^{(0)} = 1$

only boundary $b_1^{(0)}$ will be adjusted by Max-Lloyd

iteration 1

$$b_1^{(1)} = \frac{x_0^{(0)} + x_1^{(0)}}{2} = \frac{-1+1}{2} = 0$$

and

$$x_0^{(1)} = \frac{\int_{-2}^{0} x f_X(x)\,dx}{\int_{-2}^{0} f_X(x)\,dx} = \frac{\int_{-2}^{-1} x \cdot 0.4\,dx + \int_{-1}^{0} x \cdot 0.1\,dx}{\int_{-2}^{-1} 0.4\,dx + \int_{-1}^{0} 0.1\,dx} = \frac{0.4\frac{x^2}{2}\Big|_{-2}^{-1} + 0.1\frac{x^2}{2}\Big|_{-1}^{0}}{0.5} =$$

$$= \frac{\frac{0.4}{2}(1-4) + \frac{0.1}{2}(0-1)}{0.5} = \frac{(-0.6)+(-0.05)}{0.5} = \frac{-0.65}{0.5} = -1.3$$

$$x_1^{(1)} = \frac{\int_{0}^{2} x f_X(x)\,dx}{\int_{0}^{2} f_X(x)\,dx} = \dots = 1.3$$

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2. A string of characters was Huffman coded as "0001100010011000001011 1". Is the decoding unique? (2 points)

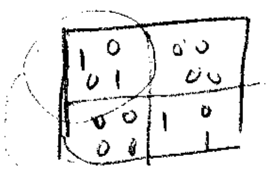
No it is not unique. it depends on the Hufman tree used for encoding

3. Calculate the 2 level Haar wavelet transform of a 4x4 Identity matrix I. Clearly identify all the wavelet sub- bands in the wavelet domain. (2 points)

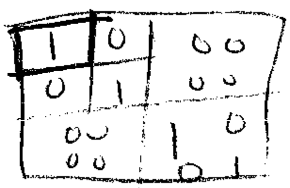
Let M_4 - 4x4 Haar matrix

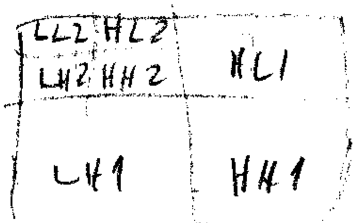
M_2 2x2 Haar matrix.

Then 1st level WT = $M_4 \cdot I_4 M_4^T = M_4 \cdot M_4^T = I_4$



2nd level WT (LL band) → $M_2 I_2 M_2^T = M_2 M_2^T = I_2$





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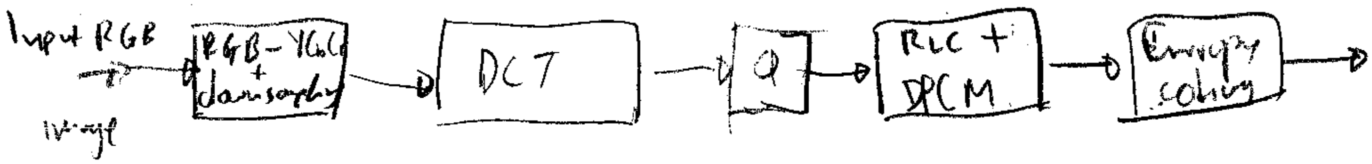
4. Briefly describe the major stages of a JPEG coder. Comment on the effectiveness of the JPEG in compressing black and white images , greyscale images and color image (4 points)



JPEG → Not very effective in B&W images

→ Effective in compressing smooth greyscale and colour images

5. The autocorrelation sequence of a signal x[n], takes the values R(0)=1, R(1)=0.6 Given that x[4]=0.5 and assuming a minimum mean square error predictor of length 1, what will be your prediction for the signal sample x[5]? (4 points)

$\hat{x}(5) = a\,x(4)$ where $a = \frac{R(1)}{R(0)} = 0.6$

Thus $\hat{x}(5) = 0.6 \cdot 0.5 = \underline{\underline{0.3}}$

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6. Does the output of a quantizer have higher or lower entropy than the input to the quantizer? Explain your answer. (2 poits)

The output has lower entropy than the input since the possible outcomes (values) of the output are much less than the input values

7. Consider a compact disk that uses binary encoding to record audio signals whose bandwidth is w=15 kHz. The quantization process is uniform with 512 levels. What is the minimum permissible bit rate? (2 points)

Sampling rate = $2w = 2 \times 15 = 30$ kHz

512 levels = 2^9 thus 9 bits/sample

Thus the rate is Sampling rate × bits/sample = $30 \times 9 = 270$ Kbits/sec

8. Suppose we are coding a binary source with p(0)=7/8. What is the entropy? (log2(7)=2.8074) (2 points)

$$H = p(0)\log_2\frac{1}{p(0)} + p(1)\log_2\frac{1}{p(1)} = \frac{7}{8}\log_2\frac{8}{7} + \frac{1}{8}\log_2 8$$

$$= \frac{7}{8}(3-2.8074) + \frac{1}{8}3 = 0.168525 + 0.375 = 0.543525$$

9. What is the set of Huffman codes in question 8? And what is the average bit rate? (2 points

Set of H. Codes 0, 1

Average bit rate $= \frac{7}{8} \times 1 + \frac{1}{8}1 = 1$

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10. A signal x[n] is divided into segments of 4 samples and then a 4-DCT is applied to each segment . Assume that the 4x4 autocorrelation matrix of x[n] is $R_x = I_{(4)}$ Using the 4-DCT matrix given (last page) calculate the 4x4 autocorrelation matrix of the DCT transformed signal y(n). Does the transform provide energy compaction? Assuming that you are given 8 bits / segment, how would you allocate these bits to the DCT coefficients? (5 points)

$R_x = I$

$R_y = T R_x T^T = T T^T = \ldots = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.125 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix}$

because the given T is not a pure DCT matrix, otherwise $R_y = I$

	σ_1^2	σ_2^2	σ_3^2	σ_4^2
	0.25	0.4	0.25	0.4
1 bit	0.0375	0.1	0.0375	0.1
2 bits	0.956	0.025	0.0156	0.25

$D_i = \sigma_i^2 \cdot 2^{-2b_i}$

... performance in terms of D_i

So assign 2 bits to each value (that is no compression)

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Aid sheet (Useful relations)

- Entropy

$$H=\sum_i p_i \log_2 \frac{1}{p_i}$$

- Max-Lloyd relations:

$$b_i = \frac{\hat{x}_{i-1} + \hat{x}_i}{2},$$

$$\hat{x}_i = \frac{\int_{b_i}^{b_{i+1}} x f_X(x) dx}{\int_{b_i}^{b_{i+1}} f_X(x) dx}$$

- 4-DCT Matrix

$$T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{pmatrix},$$