Student name: Student number:

Brief solutions

MIDTERM EXAMINATION
ECE462H1S, Multimedia Systems

Thursday February 15, 2018
Examiner: D. Hatzinakos

Time: 2:10-3:00 pm
Room: BA2139

- This is a closed book exam (type A). You may use non-programmable calculators. No additional aids are permitted.
- An aid sheet with formulas you may use is attached (last page).
- All sub-questions in each question are equally weighted
- This test counts for 30% of the final mark.
- Please answer all questions. Use only the space provided in these sheets.

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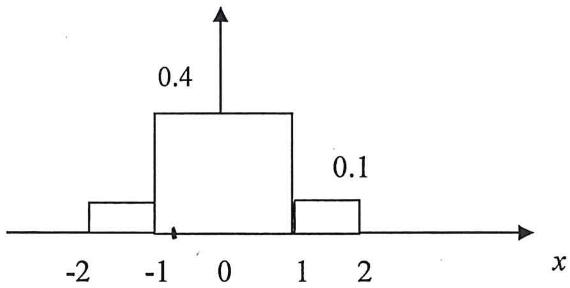
Exam questions

Question 1. (10 points)
A signal X has the following pdf $f_X(x)$:



Use the Max-Lloyd algorithm to design a 1 bit non-uniform quantizer (see last page for useful formulas). Use the settings for a uniform quantizer to initialize the algorithm. Use at least one iterations of the algorithm and comment on the final decision boundaries and reconstruction values.

$N = 2^1 = 2$, $\Delta = \dfrac{x_{max} - x_{min}}{2} = \dfrac{2-(-2)}{2} = 2$

Initialization: Uniform quantizer: $b_0^{(0)} = -2$, $b_1^{(0)} = 0$, $b_2^{(0)} = 2$

quantization levels $x_0^{(0)} = -1$, $x_1^{(0)} = 1$

1st iteration Max Lloyd: $b_1^{(1)} = \dfrac{x_0^{(0)} + x_1^{(0)}}{2} = \dfrac{-1+1}{2} = 0$

$$x_0^{(1)} = \frac{\int_{b_0}^{b_1} x f_x(x)dx}{\int_{b_0}^{b_1} f_x(x)dx} = \frac{\int_{-2}^{-1} x\cdot 0.1\,dx + \int_{-1}^{0} x \cdot 0.4\,dx}{\int_{-2}^{-1} 0.1\,dx + \int_{-1}^{0} 0.4\,dx} = \frac{0.1 \frac{x^2}{2}\Big|_{-2}^{-1} + 0.4\frac{x^2}{2}\Big|_{-1}^{0}}{0.1\,x\Big|_{-2}^{-1} + 0.4\,x\Big|_{-1}^{0}} =$$

$$= \frac{0.1 \cdot \frac{1-4}{2} + 0.4\,\frac{0-1}{2}}{0.1 + 0.4} = \frac{\frac{-0.3}{2} - \frac{0.4}{2}}{0.5} = \frac{\frac{-0.7}{2}}{0.5} = \frac{-0.7}{1} = -0.7$$

From symmetry.

$x_1^{(1)} = \ldots = 0.7$ ✓

This is final and optimum solution

Question 2 (10 points)

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1. The following values are observed:

n	0	1	2	3	4	5	6	7
x[n]	3	2	3	2	3	2	3	

What is the estimate of the missing value for n=7 based on optimum MSE linear prediction?

1 step predictor.

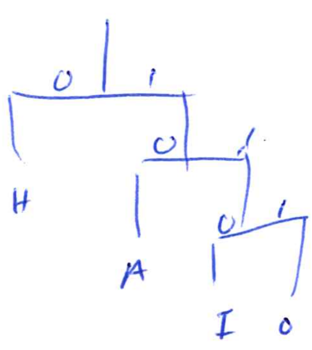
$x(7) = a \cdot x(6)$

$$a = \frac{R_x(1)}{R_x(0)} = \frac{\frac{1}{8}(3\cdot2+2\cdot3+3\cdot2+2\cdot3+3\cdot2+2\cdot3)}{\frac{1}{8}(3^2+2^2+3^2+2^2+3^2+2^2+3^2)} = \frac{36}{48} = \frac{3}{4} = 0.75$$

so $x(7) = \frac{3}{4} \times 3 = \frac{9}{4} = 2.25$

2. Derive a Huffman code for the word HAHAHIHO. What is the rate of the code? Is this the best possible rate of an entropy encoder? Is there a unique decoder?

Symbols	H	A	I	O
frequency	4	2	1	1
p( )	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$



Symbol	code
H	0
A	10
I	110
O	111

Rate: $= \frac{1}{2}\cdot 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{8}\times 3$

$= 1.75$

This is best possible encoder since it achieves entropy (all p( ) are powers of $\frac{1}{2}$)

The decoder is ~~not~~ unique.

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3. What is the principle behind DPCM signal encoding?

The prediction error sequence has lower dynamic range than the actual signal sequence. Therefore it can utilize less bits for representation → Compression.

4. Explain how RLC coding works and why is effective in reducing bit rates.

By grouping together long sequences of 0's in the transform domain is able to use less bits than representing all symbols separately.

5. Given an image does it make more sense to apply Huffman coding to the image itself or to its DCT transform? Explain your answer.

It depends on the image. In general no significant advantage exists in either domain.

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Question 3 (10 points)

12 values of a signal x[n] are observed:

n	0	1	2	3	4	5	6	7	8	9	10	11
x[n]	0	1	2	3	2	1	0	1	2	3	2	1

$y_0(i)$ $y_1(i)$ $y_2(i)$

The observed values undergo the following operation: Segmentation into segments of 4 values followed by 4-DCT in each segment. Then bit assignment.
Calculate the variance of the transformed coefficients in each case and describe a procedure to assign 24 bits to the 12 transformed values so that the distortion (MSE) is minimized.

$$\begin{bmatrix} y_0(0) \\ y_0(1) \\ y_0(2) \\ y_0(3) \end{bmatrix} = T \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.5 \\ -1.4 \\ 0 \\ -0.2 \end{bmatrix}$$

$$\sigma_0^2 = \frac{1}{3}(1.5^2 + 1^2 + 2^2) - \left(\frac{1}{3}(1.5+1+2)\right)^2 = 0.17$$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \end{bmatrix} = T \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.6 \\ 1 \\ -0.2 \end{bmatrix}$$

$$\sigma_1^2 = \ldots = 0.31$$

$$\sigma_2^2 = \ldots = 0.16$$

$$\begin{pmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \end{pmatrix} = T \begin{pmatrix} 2 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0.6 \\ -0.5 \\ 0.2 \end{pmatrix}$$

$$\sigma_3^2 = \ldots = 0.03$$

We need to assign 8 bits to ~~each~~ the 4 values of each sequence. Given the above variances the bit allocation will be

2 bits,	3 bits,	1 bit,	1 bit
$y_0(0)$	$y_0(1)$	$y_0(2)$	$y_0(3)$
$y_1(0)$	$y_1(1)$	$y_1(2)$	$y_1(3)$
$y_2(0)$	$y_2(1)$	$y_2(2)$	$y_2(3)$

Aid sheet (Useful relations)

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Aid sheet (Useful relations)

- Distortion

$$D = E[(x-\hat{x})^2] = \sum_{i=0}^{N-1} \int_{b_i}^{b_{i+1}} (x-\hat{x}_i)^2 f_X(x)dx$$

- Max-Lloyd relations:

$$b_i = \frac{\hat{x}_{i-1} + \hat{x}_i}{2},$$

$$\hat{x}_i = \frac{\int_{b_i}^{b_{i+1}} x f_X(x)dx}{\int_{b_i}^{b_{i+1}} f_X(x)dx}$$

- Coding Gain

$$G_{TC_Y} = \frac{\frac{1}{N}\sum_{i=0}^{N-1} \sigma_{Y(i)}^2}{\left(\prod_{i=0}^{N-1} \sigma_{Y(i)}^2\right)^{1/N}}$$

- 4-DCT Matrices

$$T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 1 & 1 & 1 & 0.5 \\ 1 & 0.5 & -1 & -1 \\ 1 & -0.5 & -1 & 1 \\ 1 & -1 & 1 & -0.5 \end{pmatrix}$$

- $$R(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)x(n+k)$$

- $$\begin{bmatrix} R(0) & R(1) & R(2) \\ R(1) & R(0) & R(1) \\ R(2) & R(1) & R(0) \end{bmatrix} \begin{bmatrix} a1 \\ a2 \\ a3 \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \\ R(3) \end{bmatrix}$$