Student name: Student number:

MIDTERM EXAMINATION
ECE462H1S, Multimedia Systems

Thursday February 28, 2019
Examiner: D. Hatzinakos

Time: 11:10-12:00 pm
Room: GB404

- This is a closed book exam (type A). You may use non-programmable calculators. No additional aids are permitted.
- An aid sheet with formulas you may use is attached (last page).
- All sub-questions in each question are equally weighted
- This test counts for 30% of the final mark.
- Please answer all questions. Use only the space provided in these sheets.

Student name: Student number:

Exam questions

Question 1. (10 points)

You are given the following Huffman code information

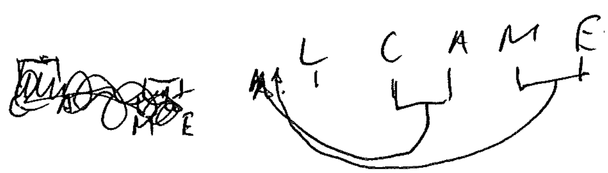
(Symbols, rate, code word) : (L,2,11), (C,1, 00), (A,1,01), (M,1, 100), (E,1, 101)

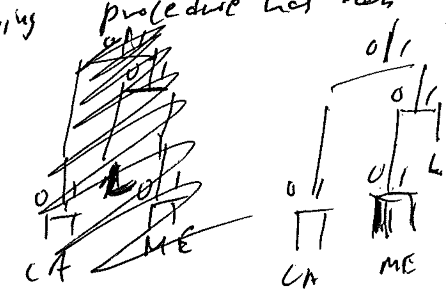
1) Decode the received Huffman encoded signal: 10010100110111
2) You realize that the recovered message does not make sense because the assumed Huffman tree is wrong. Based on the given information can you design a different tree that will provide a more meaningful decoded message?
3) What is the entropy of the source?
4) What is the rate of the code?

L 2 11
C 1 00
A 1 01
M 1 100
E 1 101

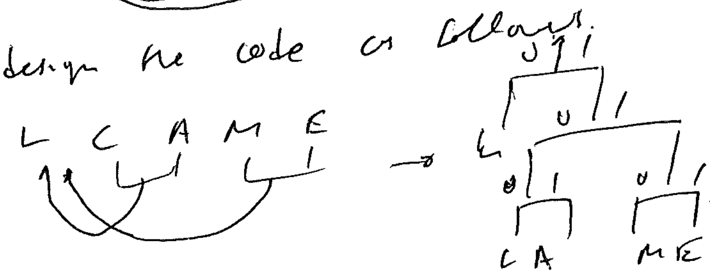
(1) 100|101|00|1|01|11
M E C L A L

(2) Looking at the code we realize that the following procedure has been followed





Instead we can design the code as follows:



L	2	0
C	1	100
A	1	101
M	1	110
E	1	111

Using this code book we decode the signal as follow

100|101|0|0|110|111
C A L L M E which make sense.

(3) Entropy of the source ; $H = \sum p_i \log_2 \frac{1}{p_i} = \frac{2}{6} \log_2 \frac{6}{2} + 4 \frac{1}{6} \log_2 6 = \frac{1}{3} \log_2 3 + \frac{4}{6} \log_2 6$

(use the formula $\log_2 x = \frac{\log_{10} x}{\log_{10} 2}$) $\rightarrow$ $= \frac{1}{3} \log_2 3 + \frac{2}{3} \log_2 6$

$= \frac{1}{3} \times 1.584 + \frac{2}{3}\, 2.585$

$= \underline{2.25}$

(4) Code rate ; $(2 \cdot 1 + 4 \cdot 3)/6 = \frac{2+12}{6} = \frac{14}{6} = \underline{2.33}$

Student name: Student number:

Question 2 (10 points)

1) A signal has autocorrelation function R(k)=$0.8^{k}, k = 0, \pm 1, \pm 2, \ldots$. Design an optimum MSE predictor $\hat{x}$(n)=a1 x(n-1) +a2 x(n-2) (i.e. determine the values a1 and a2)

$$\begin{bmatrix} R(0) & R(1) \\ R(1) & R(0)\end{bmatrix}\begin{bmatrix}a_1\\a_2\end{bmatrix} = \begin{bmatrix}R(1)\\R(2)\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 0.8\\0.8 & 1\end{bmatrix}\begin{bmatrix}a_1\\a_2\end{bmatrix} = \begin{bmatrix}0.8\\0.64\end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix}a_1\\a_2\end{bmatrix} = \frac{\begin{bmatrix}1 & -0.8\\-0.8 & 1\end{bmatrix}}{1-0.8^2}\cdot\begin{bmatrix}0.8\\0.64\end{bmatrix} = \frac{1}{0.36}\begin{bmatrix}0.8-0.8\cdot0.64\\-0.8^2+0.64\end{bmatrix} = \begin{bmatrix}0.8\\0\end{bmatrix}$$

Then the R predictor is $\underline{\hat{x}(n) = 0.8\,x(n-1) + 0\,x(n-2)}$
$= 0.8\,x(n-1)$

2. The zig-zag scanning of a 8x8 DCT produces the following values

(7,0,-6,0,1,0,0,0,-1,0,0, 1,0,0,0,…,0,0)

Provide the RLC coding for this set of values.

(0 7) (1 -6) (1 1)(3 -1) (2 1) (0 0)

# of zeros prior to number

number

Student name: Student number:

3. An alien race is colour blind. How will you modify a JPEG coder to provide best compression performance in this case?

After the RGB − Y Cb Cr transformation you retain only the Y section.

4. The ~~PSNR~~ SQNR in a decoded image is 64 dB. Can you estimate the average number of bits used to represent each pixel?

$SQNR \cong 6b + 10.8$

Assuming dynamic range of signal matches the dynamic range of the "quantizer" (equivalent)

$\rightarrow 64 - 10.8 = 6b \rightarrow$

$\rightarrow 53.2 = 6b \rightarrow b \cong 8.86 = 9$ bits

(does not make sense since images are represented with less than 8 bits/pixel, on the average)

5. What is the basic principle behind Transform coding? How does this relate to the concept of removing spatial redundancy during compression?

By energy compaction in the DCT domain we are able to remove significant high frequency content. This is equivalent to removing redundant information from the signal in the time (space) domain.

Student name: Student number:

Question 3 (10 points)

Consider the following 4x4 image

$$\begin{pmatrix} 9 & 8 & 3 & 2 \\ 8 & 8 & 5 & 3 \\ 4 & 2 & 1 & 2 \\ 3 & 5 & 2 & 3 \end{pmatrix}$$

We want to compress this image using JPEG.

1) What is the best block size NxN to divide the image in order to calculate 2d-DCT transform ((i.e., 16 1x1 blocks, 4 2x2 blocks or 1 4x4 block?). Justify your choice.

2) Calculate the DCT (s) for each NxN block

3) Calculate the variance of the transformed coefficients in each block in the DCT domain and describe a procedure to assign 32 bits to the 16 transformed values so that the distortion (MSE) is minimized.

(1) Blocks 2x2 should be chosen. The values of 2x2 blocks are more similar to each other so higher energy compaction is expected in the corresponding DCT.

(2) DCT 1 $T\begin{bmatrix} 9 & 8 \\ 8 & 8 \end{bmatrix}T^H = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 9 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{pmatrix} 17 & 16 \\ 1 & 0 \end{pmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 33 & 1 \\ 1 & 1 \end{bmatrix}$

DCT 2 $T\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}T^H = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 8 & 5 \\ -2 & -1 \end{bmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{bmatrix} 13 & 3 \\ -3 & -1 \end{bmatrix}$

DCT 3 $T\begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}T^H = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 7 & 7 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{bmatrix} 14 & 0 \\ -2 & 4 \end{bmatrix}$

DCT 4 $T\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}T^H = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{pmatrix} 3 & 5 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{bmatrix} 8 & -2 \\ 2 & 0 \end{bmatrix}$

(3) Var 1 $\frac{1}{4}\cdot\frac{1}{4}(33^2 + 13^2 + 14^2 + 8^2) - \left(\frac{33+13+14+8}{8}\right)^2 = 94.87 - \cancel{72.25} = \cancel{90.625}$ $\frac{68}{8}$ 18 $= 76.87$

Var 2 $\frac{1}{16}(1^2 + 3^2 + 0 + (-2)^2) - \left(\frac{1+3+0-2}{8}\right)^2 = 0.875 - 0.25 = 0.625$

Var 3 $\frac{1}{16}(4^2 + 2^2 + 3^2 + 5^2) - \left(\frac{4+2+3+5}{8}\right)^2 = 3.375 - 3.06 = 0.3125$

Var 4 $\frac{1}{16}(1^2 + 1^2 + 2^2 + ...$ $(1+1+2 ...)$ 8 bits to allocate $= 1.125 - 1 = 0.125$

Assume bits are allocated

$\begin{pmatrix} y(0) & y(1) \\ y(2) & y(3) \end{pmatrix} : \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$

or $\begin{pmatrix} 5 & 1 \\ 1 & 1 \end{pmatrix}$

Student name: Student number:

Aid sheet (Useful relations)

- 4-DCT Matrix

$$T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{pmatrix},$$

- 2-DCT matrix

$$T=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- MSE prediction optimization : $\begin{pmatrix} R(0) & R(1) & R(2) \\ R(1) & R(0) & R(1) \\ R(2) & R(1) & R(0) \end{pmatrix}\begin{pmatrix} a1 \\ a2 \\ a3 \end{pmatrix} = \begin{pmatrix} R(1) \\ R(2) \\ R(3) \end{pmatrix}$

- Distortion $D \sim \sigma^2 2^{-2R}$