Student name: Student number:

with brief solutions

MIDTERM EXAMINATION
ECE462H1S, Multimedia Systems

Thursday February 27, 2020
Examiner: D. Hatzinakos

Time: 2:00-3:00 pm
Room: HA-316

- This is a closed book exam (type A). You may use non-programmable calculators. No additional aids are permitted.
- An aid sheet with formulas you may use is attached (last page).
- All sub-questions in each question are equally weighted
- This test counts for 30% of the final mark.
- Please answer all questions. Use only the space provided in these sheets.

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Exam questions

Question 1. (10 points)

You are given the following Huffman code information

Code 1. (Symbols, rate, code word) : (H,6,0), (A,2,10), (E,2,110), (O,1,1110), (I,1,1111)
Code 2. (Symbols, rate, code word) : (H,6,0), (A,2,100), (E,2,101), (O,1,110), (I,1,1111)
Code 3. (Symbols, rate, code word) : (H,6,0), (A,2,10), (E,2,101), (O,1,1110), (I,1,1111)

1) Decode the received Huffman encoded signal: 0100100111101110011001110
2) Can you tell which one of the three code versions is the true one?
3) What is the entropy of the source?
4) What are the rates of the codes?

Code 1

	rate	
H	6	0
A	2	10
E	2	110
O	1	1110
I	1	1111

Code 2

H	6	0
A	2	100
E	2	101
O	1	110
I	1	1111

Code 3

H	6	0
A	2	10
E	2	101
O	1	1111
I	1	1111

not a valid Huffman Code

(1) 0|10|0|10|0|1111|0|111|0|0|11|0|0|110
H A H A H I H O H E H E

0|100|100|1111|0|111 0011 00110
H A A I H

(2) So the first code is the true one

(3) $H = \frac{6}{12}\log_2\frac{12}{6} + 2\frac{2}{12}\log_2\frac{12}{2} + 2\frac{1}{12}\log_2\frac{12}{1} = \frac{6}{12}\cdot 1 + \frac{1}{3}\log_2 6 + \frac{1}{6}\log_2 12 = (1.9$

$= \frac{1}{2} + \frac{1}{3}\,2.58\ldots + \frac{1}{6}\,3.58$

(4) $C1: \frac{6}{12}\times 1 + \frac{2}{12}\,2 + \frac{2}{12}\times 3 + 2\frac{1}{12}\,4 = \frac{1}{2} + \frac{1}{6}\,2 + \frac{1}{6}\times 3 + \frac{1}{6}\times 4 = \frac{1}{2} + \frac{1}{3} + \frac{1}{2} + \frac{2}{3} = 1 + \frac{4}{3} = \frac{7}{3} = 2.3$

So C1 has rate 2.3 bits > 1.96 = H

~~C2 -~~ ~~C3 =~~

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Question 2 (10 points)

1) A signal x(n) has autocorrelation function R(k)=$0.7^k, k = 0,1,2 \ldots$, R(-k)=R(k). Design an optimum MSE predictor $\hat{x}$(n)=a1 x(n-1) (i.e. determine the value a1)

$$R(0)\cdot a_1 = R(1) \Rightarrow \quad a_1 = \frac{R(1)}{R(0)} = \frac{0.7}{1} = \underline{\underline{0.7}}$$

2. The zig-zag scanning of a 8x8 DCT produces the following values

(8,0,6,0,2,0,0,-1,0,0, 1,0,0,0,…,0,0)

Provide the RLC coding for this set of values.

(0,8)(1,6), (1,2), (2,-1) (2,1) (0,0)

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3. The SQNR in a decoded image is 128 dB. Can you estimate the average number of bits used to represent each pixel?

Assuming the quantization is uniform and the dynamic range of the quantizer ≃ dynamic range of input signal:

$$128 \cong 6 \cdot b + 10.8 \rightarrow b = \frac{128 - 10.8}{6} \rightarrow b \simeq \underline{\underline{20}}$$

4. What is the basic principle behind linear predictive coding and DPCM in JPEG?

Reduce dynamic range of signal prior to quantization in order to utilize less bits to represent the signal. (for the same degree of distortion)

Question 3 (10 points)

Consider the following 4x4 image

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We want to compress this image using JPEG.

1) Divide the image in 2x2 blocks and calculate 2d-DCT transforms
2) Calculate the 4-DCT for the entire image.
3) Which of the above procedures provides a better compressible image in the DCT domain?

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$$\left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ \hline 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

2-PCTs

$$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} \ldots = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$$

4-PCT

$$\begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & 0.25 & -0.25 \\ 0.2 & -0.4 & 0.4 & -0.4 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0.25 & 0.4 & 0.25 & 0.2 \\ 0.25 & 0.2 & -0.25 & -0.4 \\ 0.25 & -0.2 & 0.25 & 0.4 \\ 0.25 & -0.4 & -0.25 & -0.2 \end{pmatrix} =$$

$$= \begin{bmatrix} 0.25 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The 4-PCT will have (provide) better compression

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Aid sheet (Useful relations)

- 4-DCT Matrix

$$T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{pmatrix},$$

- 2-DCT matrix

$$T=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- MSE prediction optimization : $\begin{pmatrix} R(0) & R(1) & R(2) \\ R(1) & R(0) & R(1) \\ R(2) & R(1) & R(0) \end{pmatrix}\begin{pmatrix} a1 \\ a2 \\ a3 \end{pmatrix} = \begin{pmatrix} R(1) \\ R(2) \\ R(3) \end{pmatrix}$

- Distortion D$\sim\sigma^2 2^{-2R}$

- Max-Lloyd relations:

$$b_i = \frac{\hat{x}_{i-1} + \hat{x}_i}{2},$$

$$\hat{x}_i = \frac{\int_{b_i}^{b_{i+1}} x f_X(x)dx}{\int_{b_i}^{b_{i+1}} f_X(x)dx}$$