Name:
Student number:

University of Toronto
Faculty of Applied Science and Engineering

MIDTERM EXAMINATION 1
ECE462H1S, Multimedia Systems

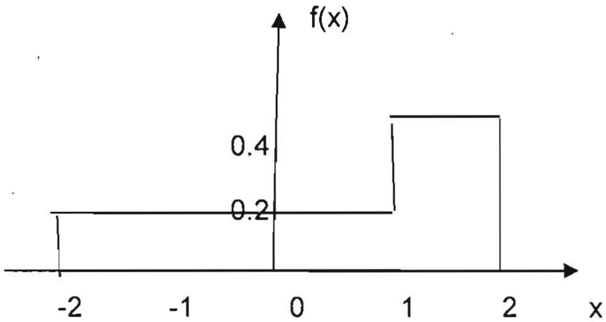
February 17, 2022, 5:00 - 6 pm
Instructor: D. Hatzinakos

Instructions:

1. The exam counts for 15% of overall mark.
2. Please solve all problems. Do not show only final answers. You should demonstrate how the answer has been obtained by including intermediate results and explanations wherever needed.
3. Use the blank space provided in this handout to record your answers.
4. Write your name and/or student number on top of all submitted pages.

QUESTIONS.

1. A signal x has the following pdf *f (x)*.



Use the Max-Lloyd algorithm to design a 2-bit non-uniform quantizer. Use the settings for a uniform quantizer to initialize the algorithm. What are the decision boundaries and reconstruction levels at the end of recursion (iteration) 1. What is the corresponding MSE distortion for this quantizer? (3 points)

2. How many bits will you assign to two values A and B so that the average distortion (MSE of quantization error) is at maximum MSE=0.01?. Describe a procedure to achieve this if you know that the variances of the values A and B are Var(A)=3.5 and Var(B)=1.5. How many bits will you assign to A and how many bits will you assign to B? (3 points)

3. Consider compressing the following 4x4 image using JPEG (2 points)

1	1	1	1
1	0	0	0
1	0	0	0
1	0	0	0

   Assuming equal size blocks how will you segment the image for best performance? Why?
   Assuming non equal size blocks how will you segment the image for best performance? Why?
   Is JPEG the best choice for compression?

4. An alien species has a different colour visual spectrum than the human range. Can they still perceive human generated images generated by JPEG encoding? (1 point)

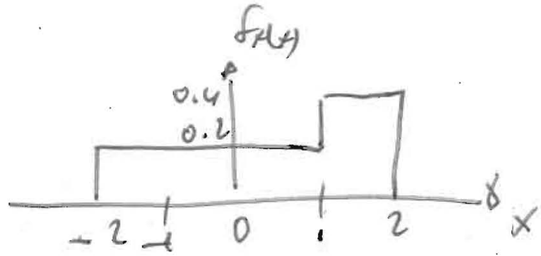
5. Given the sequence of 8 values for a signal x(n), , 0,,...7, that is 1, 1.2, 1.4,1.2, 1.4,1.6,1.4,1.6 design a MSE predictor of length 1 and estimate the value of x(8). (2 points)

6. Now many different Hufman codes can you design for the word TAKATAKA . What is th e rate for these codes? (2 points)

7. A signal x(n) is uniformly distributed between -0.5 and 0.5 and is uniformly quantized using 8 bits/sample. A second signal y(n) uniformly distributed between -1 and 1 is also uniformly

quantized using b bits /sample. Find b so that the Signal to Quantization noise ratio (SQNR) is the same in both cases. (2 points)

(1)



2 bit quantizer → 4 levels

$\Delta = \frac{2-(-2)}{4} = 1$

Initialization. Uniform quantizer

5 boundaries $b_0 = -2$, $b_1 = -1$, $b_2 = 0$, $b_3 = 1$, $b_4 = 2$

4 reconstruction level $u_0 = -1.5$, $u_1 = -0.5$, $u_2 = 0.5$, $u_3 = 1.5$

Looking at the graph we realize that because the pdf is flat ~~between~~ between all boundaries The uniform solution is the optimum sol. and therefore there is no need to run the Lloyd-max algorithm

Then we know that $MSE = \frac{\Delta^2}{12} = \frac{1}{12}$ ✓

(2)

	A	B
Variance	3.5	1.5
1 bit A, 1 bit B	$D_A = 0.875$	$D_B = 0.375$
2 bit A, 2 bit B	0.218	0.0937
3 bit A, 3 bit B	0.0542	0.023
4 bit A, 4 bit B	0.0136	0.0058

$D_{A,B} = \sigma^2_{A,B}\, 2^{-2b}$

$D_{av.} = \frac{D_A + D_B}{2}$

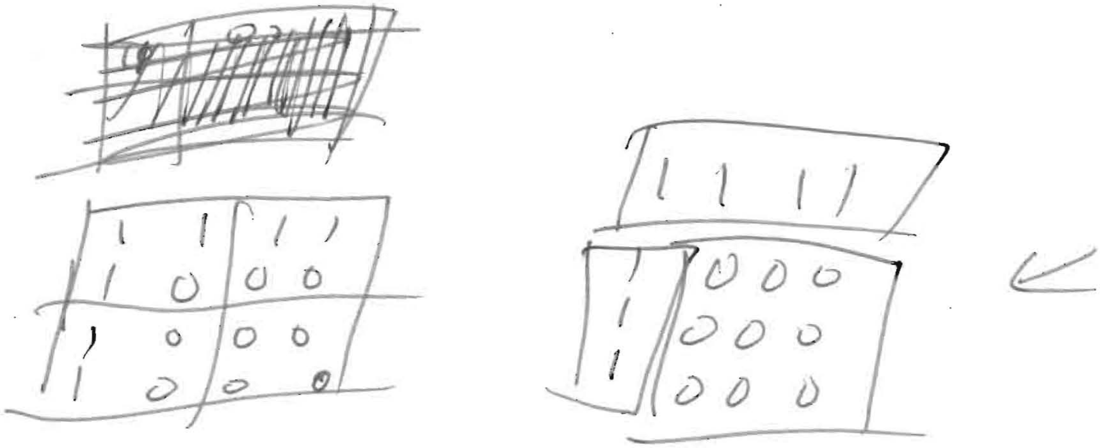
$D_{ave} = \frac{0.0136 + 0.0058}{2} < 0.01$

So at least 4 bits for A and 4 bits to B achieves the objective.

(3) To achieve the best JPEG compression we must remove maximum possible correlation from the data therefore it seems the 6 library subarray would work better

equal blocks

Not-equal blocks



JPEG is not the best compression scheme for binary images

(4) Since the visual ranges (frequencies) are different nothing can be done with JPEG that can make the aliens see the images.

(5) $x(0)$ … $x(7)$: 1 1.2 1.4 1.2 1.4 1.6 1.6 1.6 $\hat{x}(8) = a\,x(7)$

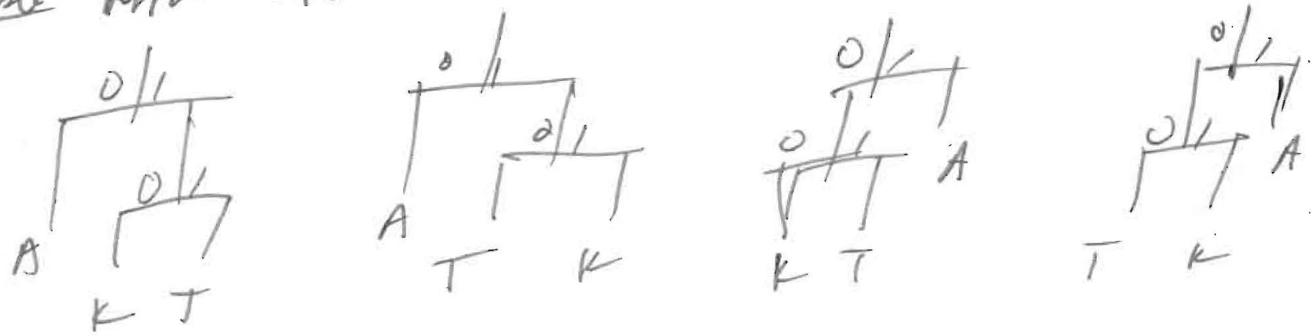
One step predictor: $a = \frac{R(1)}{R(0)} = \frac{\frac{1}{8}(1\times1.2+1.2\times1.4+\dots1.6\times1.6)}{\frac{1}{8}(1^2+1.2^2+1.4^2+\dots+1.6^2)}$

$\Rightarrow a = 0.8756 \Rightarrow \hat{x}(8) = 0.8256\times1.6 \simeq 1.4$

(6) TAKATAKA

	A	K	T
freq.	4	2	2
prob	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

So possible Huffman codes



Since probability are all powers of $\frac{1}{2}$ the codes of all these Huffman codes achieve the entropy of the source

$H = \frac{1}{2}\log_2 2 + 2\,\frac{1}{4}\log_2 4 = \frac{1}{2} + 1 = \frac{3}{2}$

(7) 1st solution: Assuming ideal conditions: SQNR = 6.02 b + 10.8 so b = always

2nd solution $D_1 = \sigma_x^2 \cdot 2^{-2\cdot8} = \sigma_y^2 \cdot e^{-2b} \Rightarrow \frac{1}{12}2^{-2\cdot8} = \frac{4}{12}e^{-2b}$

$\Rightarrow e^{-2b} = \frac{1}{4}2^{-16} = 2^{-18} \Rightarrow b = 9$

Student name: Student number:

Aid sheet (Useful relations)

- 4-DCT Matrix

$$T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{pmatrix},$$

- 2-DCT matrix

$$T=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- MSE prediction optimization : $\begin{pmatrix} R(0) & R(1) & R(2) \\ R(1) & R(0) & R(1) \\ R(2) & R(1) & R(0) \end{pmatrix}\begin{pmatrix} a1 \\ a2 \\ a3 \end{pmatrix} = \begin{pmatrix} R(1) \\ R(2) \\ R(3) \end{pmatrix}$

- Distortion $D \sim \sigma^2 2^{-2R}$

- Max-Lloyd relations:

$$b_i = \frac{\hat{x}_{i-1} + \hat{x}_i}{2},$$

$$\hat{x}_i = \frac{\int_{b_i}^{b_{i+1}} x f_X(x) dx}{\int_{b_i}^{b_{i+1}} f_X(x) dx}$$