Name:
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University of Toronto
Faculty of Applied Science and Engineering

MIDTERM EXAMINATION 1
ECE462H1S, Multimedia Systems

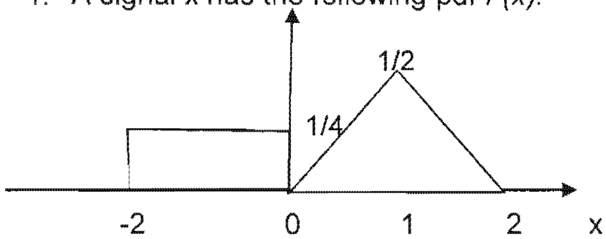
February 17, 2023, 12:10-1:00pm
Instructor: D. Hatzinakos

Instructions:

1. The exam counts for 15% of overall mark.
2. Please solve all problems. Do not show only final answers. You should demonstrate how the answer has been obtained by including intermediate results and explanations wherever needed.
3. Use the blank space provided in this handout to record your answers.
4. Write your name and/or student number on top of all submitted pages.

QUESTIONS.

1. A signal x has the following pdf *f (x)*.



   Use the Max-Lloyd algorithm to design a 1-bit non-uniform quantizer. Use the settings for a uniform quantizer to initialize the algorithm (iteration 0). What are the decision boundaries and reconstruction levels after 2 iterations of the algorithm ? (the equations for the two sides of the triangle are x/2 and –x/2+1, respectively) (3 points)

2. You are given 3 values A,B, and C of the Kahunen Loeve Transform (KLT) of a signal. The 3 eigenvalues of the corresponding autocovariance matrix of the signal take the values 3, 1.5 and 0.2, respectively. Assuming that you have 7 bits in total , how will you assign these bits to A,B and C So that the quantization mean square error is minimized? (2 points).

3. Two images A and B have concentration of their energy in different parts of their frequency spectrum. A has more concentration of energy in low frequencies and B has more concentration in high frequencies. How would you modify a typical JPEG encoder for A so that it can become equally efficient for image B? (2 points)

4. A real valued signal x(n), n=0,1,....,7 takes the following values:

   1, -1.2, 2, -1.8, 1.9, -1, 1.5, -1.1

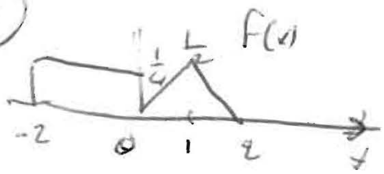
   Calculate the autocorrelation R(k) for k=0,1,2. Then use these values to design a length 2 MSE predictor. What will be the predicted value for x(8)? (2 points)

5. Design a Huffman code for the word ATAKATAK. How many different codes can you design? How efficient they are? (2 points)

6. A signal x(n) is uniformly distributed between -2 and 2. It is uniformly quantized using 8 bits/ sample. What is the corresponding QSNR? What is the corresponding PSNR? (2 points)

7. Calculate the 4-DCT y(k), k=0,1,2,3 of the signal x(0)=1, x(1)=3, x(2)=2, x(3)=1. Then, quantize using Round(y(k)/q(k)), where q(0)=10, q(1)=10, q(2)=50, q(3)=20. Write an RLC code for the quantized AC DCT coefficients. (2 points)

(1)



1 bit quantizer $\rightarrow$ $2^1 = 2$ reconstruction levels

$$\Delta = \frac{2-(-2)}{2} = \frac{4}{2} = 2$$

So boundaries at iteration 0 $\quad b_0^{(0)} = -2 \quad b_1^{(0)} = 0 \quad b_2^{(0)} = 2$

reconstruction levels at iteration 0 $\quad V_0^{(0)} = -1, \; V_1^{(0)} = 1$

$b_0^{(0)}$ and $b_2^{(0)}$ remain constant always. So we need to update $b_1^{(0)}$ & $V_0^{(0)}$ and $V_1^{(0)}$

First iteration:

$$b_1^{(1)} = \frac{V_0^{(0)} + V_1^{(0)}}{2} = \frac{-1+1}{2} = 0$$

then

$$V_0^{(1)} = \frac{\int_{-2}^{0} x f(x)\,dx}{\int_{-2}^{0} f(x)\,dx} = \frac{\int_{-2}^{0} x \cdot \frac{1}{4}\,dx}{\frac{1}{2}} = \frac{\frac{1}{4}\frac{x^2}{2}\Big|_{-2}^{0}}{\frac{1}{2}} = \frac{-\frac{1}{4}\frac{4}{2}}{\frac{1}{2}} = -1$$

also

$$V_1^{(1)} = \frac{\int_0^2 x f(x)\,dx}{\int_0^2 f(x)\,dx} = \frac{\int_0^1 x \frac{x}{2}\,dx + \int_1^2 x\left(-\frac{x}{2}+1\right)dx}{\frac{1}{2}} =$$

$$= \frac{\int_0^1 \frac{x^2}{2}\,dx + \int_1^2 \left(-\frac{x^2}{2} + x\right)dx}{\frac{1}{2}} = \frac{\frac{1}{2}\frac{x^3}{3}\Big|_0^1 + \left(-\frac{1}{2}\frac{x^3}{3} + \frac{x^2}{2}\right)\Big|_1^2}{\frac{1}{2}} =$$

$$= \frac{\frac{1}{6} + \left(-\frac{1}{2}\frac{8}{3} + \frac{4}{2}\right) - \left(-\frac{1}{2}\frac{1}{3} + \frac{1}{2}\right)}{1/2} = \frac{\frac{1}{6} - \frac{4}{3} + 2 + \frac{1}{6} - \frac{1}{2}}{\frac{1}{2}} =$$

$$= \frac{\frac{1}{3} - \frac{4}{3} + 2 - \frac{1}{2}}{\frac{1}{2}} = \frac{-1+2-\frac{1}{2}}{\frac{1}{2}} = \frac{1-\frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

Second iteration same as the first iteration

So the optimum Max Lloyd quantizer is the uniform quantizer after all in this case.

(2) In KLT the eigenvalues $L_A = 3 = \sigma_A^2$

$L_B = 1.5 = \sigma_B^2$

$L_C = 0.2 = \sigma_C^2$

So then

$D = \sigma^2 2^{-2b}$

(7 bits in total)

σ^2	A: 3	B: 1.5	C: 0.2
(1 bit)	0.75	0.375	0.05
(2 bits in A+B, 1 bit in C)	0.1875	0.0937	~~0.05~~
(3 bits A, 2 bits B, 1 bit C)	0.09375	0.0937	~~0.05~~ ✓
3 3 1	0.09325	0.023	~~0.05~~

So we will assign 3 bits in A, 3 bits in B, 1 bit in C

(3) Basically the only modification we need to do is change the values of the Q-tables needed to scale the DCT coefficients before rounding (quantization). All the other steps remain the same.

(4)

n	0	1	2	3	4	5	6	7
$x(n)$	1	-1.2	2	-1.8	1.9	-1	1.5	-1.1

$R(k) = \frac{1}{8} \sum_{k=0}^{7} x(n)\,x(n+k)$, Therefore $R(0) = \frac{1}{8}\left(1^2 + (-1.2)^2 + 2^2 + (-1.8)^2 + (1.9)^2 + (-1)^2 + (1.5)^2 + (-1.1)^2\right)$

$= \frac{17.54}{8} = 2.$

$R(1) = \frac{1}{8}\left(1\times(-1.2) + (-1.2\times 2) + 2\times(-1.8) + (-1.8\times 1.9) + (1.9\times -1) + (-1\times 1.5) + 1.5\times(-1.1)\right) = \frac{-15.5}{8} = -1.938$

$R(2) = \frac{1}{8}\left(1\times 2 + (-1.2\times 1.8) + 2\times 1.9 + 1.1\right) = \frac{9.06}{8} = 1.13$

So, for 2-length predictor we need to solve the system of equations

$$\begin{bmatrix} R(0) & R(1) \\ R(1) & R(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \end{bmatrix}$$

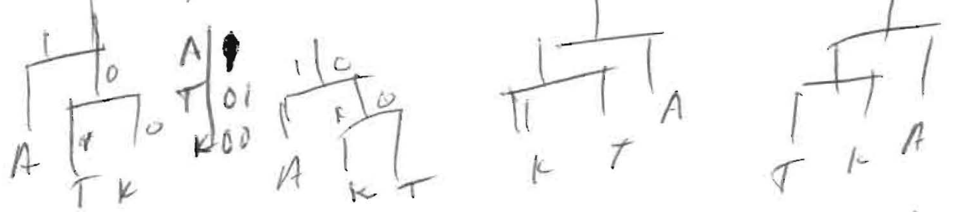
$$\begin{bmatrix} 2.19 & -1.941 \\ -1.941 & 2.19 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1.74 \\ -1.94 \end{bmatrix} \Rightarrow \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{\begin{bmatrix} 2.19 & 1.941 \\ 1.94 & 2.19 \end{bmatrix}}{2.19^2 - 1.94^2} \begin{bmatrix} -1.94 \\ +1.74 \end{bmatrix}$$

$\Rightarrow a_1 = -0.96, \quad a_2 = -0.074$

and $\hat{x}(8) = -0.96(-1.1) - 0.074 \cdot 1.5 = \dots = 1.167$

(5) A T K

prob table: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$



4 diffe codes

rate for all codes $= 1.5$ Entropy $= 1.5 = \frac{1}{4}\log_2 4 + 2\frac{1}{2}\log_2 2 =$

$= \frac{2}{4} + 1 = \frac{1}{2} + 1 = 1.5$

most efficient codes

(6) Assuming ideal conditions $QSNR = 6.02B + 10.8 = 6.02 \cdot 8 + 10.8 = 58.96\,dB$

$PSNR = \frac{2^8}{D} = \frac{2^8}{\frac{\Delta^2}{12}} = \frac{2^8}{\frac{4/2^{2B}}{12}} = \frac{2^8 \cdot 2^{16}}{4 \cdot 12} = \frac{256 \cdot 514}{4 \cdot 12} = 27$

PSNR in dB $= 10\log_{10} 27 = 3.44\ dB$

(7) $Y^T = T x^T \Rightarrow \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = T \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.75 \\ 0.2 \\ -0.75 \\ -0.4 \end{bmatrix}$

Then $\mathrm{Round}\left[\frac{Y}{Q}\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ RL = (9,0)

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Aid sheet (Useful relations)

- 4-DCT Matrix

$$T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.4 & 0.2 & -0.2 & -0.4 \\ 0.25 & -0.25 & -0.25 & 0.25 \\ 0.2 & -0.4 & 0.4 & -0.2 \end{pmatrix},$$

- 2-DCT matrix

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- MSE prediction optimization : $\begin{pmatrix} R(0) & R(1) & R(2) \\ R(1) & R(0) & R(1) \\ R(2) & R(1) & R(0) \end{pmatrix} \begin{pmatrix} a1 \\ a2 \\ a3 \end{pmatrix} = \begin{pmatrix} R(1) \\ R(2) \\ R(3) \end{pmatrix}$

- Distortion $D \sim \sigma^2 2^{-2R}$

- Max-Lloyd relations:

$$b_i = \frac{\hat{x}_{i-1} + \hat{x}_i}{2},$$

$$\hat{x}_i = \frac{\int_{b_i}^{b_{i+1}} x f_X(x) dx}{\int_{b_i}^{b_{i+1}} f_X(x) dx}$$