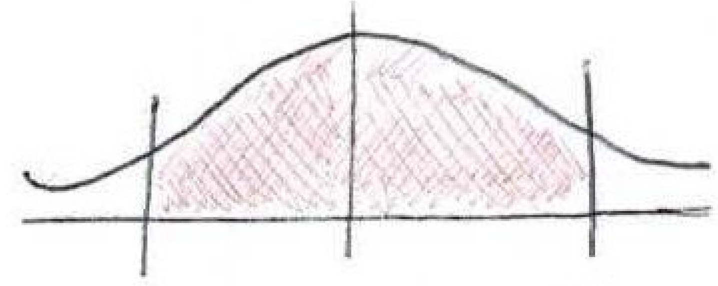
8.44 a) Find $t_{0.025}$ when $v = 14 \rightarrow \underline{2.145}$ $\left\{ \begin{array}{l} \text{Table A.4, page 737} \\ \text{look up } \alpha = 0.025 \\ \quad v = 14 \end{array} \right.$

b) Find $t_{-0.10}$ when $v = 10 \rightarrow \underline{-1.372}$ (distribution is symmetrical)

c) Find $t_{0.995}$ when $v = 7 \rightarrow \underline{-3.499}$

8.47 Random sample of size 24 from normal distribution, find K such that
$\llcorner v = 24 - 1 = 23$

a) $P(-2.069 < T < K) = 0.965$



$t_{-0.025} = -2.069$ K

$0.975 - \alpha = 0.965$

$\alpha = 0.01$ (K)

$t_{0.01} = K = \underline{2.500}$

b) $P(K < T < \underbrace{2.807}_{t_{0.005}}) = 0.095$ $v = 23$

$\alpha - 0.005 = 0.095$
$\alpha = 0.1$

$K = t_{0.1} = \underline{1.319}$

c) $P(-K < T < K) = 0.90$

$\dfrac{1 - 0.90}{2} = 0.05$ $\quad t_{0.05} = \underline{1.714}$

8.51 For an F-distribution

a) $f_{0.05}$ with $v_1 = 7$ $v_2 = 15$ $= 2.71$ (Table A.6, p 741)

b) $f_{0.05}$ with $v_1 = 15$ $v_2 = 7$ $= 3.51$

c) $f_{0.01}$ with $v_1 = 24$ $v_2 = 19$ $= 2.92$

d) $f_{0.95}$ with $v_1 = 19$ $v_2 = 24 = \dfrac{1}{2.11} = 0.47$

(Theorem 8.7)

$f_{1-\alpha}(v_1, v_2) = \dfrac{1}{f_\alpha(v_2, v_1)}$

e) $f_{0.99}$ with $v_1 = 28$ $v_2 = 12 = \dfrac{1}{2.90} = 0.34$

8.59 If S_1^2 & S_2^2 represent variances of independent random samples of size $n = 8$ & $n = 12$ taken from normal populations with equal variances find $P(S_1^2/S_2^2 < 4.89)$

Using Theorem 8.8 for F-Distribution

→ Equal variances $\sigma_1^2 = \sigma_2^2$

$F = 4.89 \quad v_1 = 8-1 = 7$

$v_2 = 12-1 = 11$

$P(F<4.89) = \underline{0.99}$ (TABLE A.6 $f_{0.01}(v_1, v_2)$)

8.69 Two distinct solid fuel propellants

Type A

$n = 20$

$\bar{x}_A = 20.5$

Type B

$n = 20$ (samples)

$\bar{x}_B = 24.5$ cm/sec

$$\underbrace{\sigma_1^2 = \sigma_2^2}_{\text{approximately normal → use CLT}} = \sigma^2 = 5 \text{ cm/sec}$$

a) If $\mu_A = \mu_B$

$P(\hat{X}_B - \hat{X}_A \geq 4.0)$

Theorem 8.3

$$Z = \frac{(\hat{X}_1 - \hat{X}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}} = \frac{4}{\sqrt{5^2/20 + 5^2/20}}$$

$P(Z > 2.53) = \underline{0.0057}$

b) Given the data observed, unlikely that $\mu_A = \mu_B$

8.43 (Optional)

Show that the variance of S^2 for random samples of size n from a normal population decreases as n becomes large.

Hint→ find variance of $\frac{(n-1)S^2}{\sigma^2}$ } Theorem 8.4 Has a χ^2 distribution with $v = n-1$ degrees of freedom

$$\underbrace{Var\left[\frac{(n-1)S^2}{\sigma^2}\right]}_{\chi^2,\, n-1 = v} = \frac{(n-1)^2}{\sigma^4} Var(S^2) = 2\underbrace{(n-1)}_{v}$$

$$Var(S^2) = \frac{2\sigma^4}{n-1}$$

From Theorem 6.5

→ variance of χ^2 distribution is $\underline{2v}$

→ $Var(cX) = c^2 Var(X)$

This value decreases as n increases

Download:

http://cran.utstat.utoronto.ca/

or – link from U of T Library with R Guide

https://mdl.library.utoronto.ca/technology/tutorials/r-guide-and-download

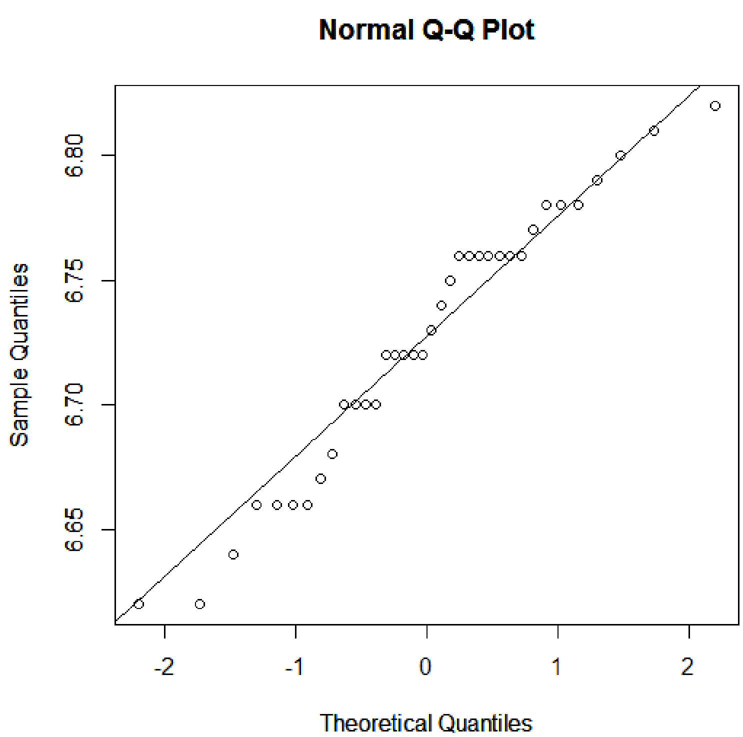
8.55 (in R)

>
X=c(6.72,6.77,6.82,6.70,6.78,6.70,6.62,6.75,6.66,6.66,6.64,6.76,6.73,6.80,6.72,6.76,6.76,6.68,6.66,6.62,6.72,6.76,6.70,6.78,6.76,6.67,6.70,6.72,6.74,6.81,6.79,6.78,6.66,6.76,6.76,6.72)

> qqnorm(X)

>qqline(X)

Out:



The data appear to be normally distributed.