ECE 316 CHAPTER 6 PROBLEM SET SOLUTIONS

1. Drill problem 6.2.

Show that for positive frequencies, the area under the normalized raised-cosine curve of $P(f)/(\sqrt{E}/2B_0)$ versus f/B_0 is equal to unity for all values of the roll-off factor in the range $0 \leq \alpha \leq 1$.

$$P(f) = \begin{cases} \sqrt{E}/2B_0 & 0 \leq |f| < f_1 \\ \dfrac{\sqrt{E}}{4B_0}\left\{1 + \cos\left[\dfrac{\pi/2\,(|f| - f_1)}{B_0 - f_1}\right]\right\} & f_1 \leq |f| < 2B_0 - f_1 \\ 0 & \text{o.w.} \end{cases}$$

$$\alpha = 1 - \frac{f_1}{B_0}$$

rewrite $P(f)/(\sqrt{E}/2B_0)$ for positive frequencies:

$$P(f)/(\sqrt{E}/2B_0) = \begin{cases} 1 & 0 \leq f/B_0 < f_1/B_0 \\ \dfrac{1}{2}\left\{1 + \cos\left[\dfrac{\pi/2\,(f/B_0 - f_1/B_0)}{1 - f_1/B_0}\right]\right\} & f_1/B_0 \leq f/B_0 < 2 - f_1/B_0 \\ 0 & \text{o.w.} \end{cases}$$

$\Rightarrow$ $f_1/B_0 = 1-\alpha$. , $f/B_0 = x$ (new variable), $2 - f_1/B_0 = 1 + (1 - f_1/B_0) = 1+\alpha$.

The area under the curve for positive frequencies is :

$$\underbrace{\int_{x=0}^{1-\alpha} dx}_{①} + \underbrace{\int_{x=1-\alpha}^{1+\alpha} \frac{1}{2}\left(1 + \cos\left(\frac{\pi/2\,(x - (1-\alpha))}{\alpha}\right)\right) dx}_{②} =$$

$① = 1-\alpha$

$$② = \frac{1}{2}x\Big]_{1-\alpha}^{1+\alpha} + \frac{1}{2}\frac{\alpha}{\pi/2}\sin\left(\frac{\pi/2\,(x-(1-\alpha))}{\alpha}\right)\Big]_{1-\alpha}^{1+\alpha} =$$

$$\frac{1}{2}(1+\alpha-1+\alpha) + \frac{\alpha}{\pi}\left[\sin\left(\frac{\pi/2\,(1+\alpha-1+\alpha)}{\alpha}\right) - \sin\left(\frac{\pi/2\,(1-\alpha-1+\alpha)}{\alpha}\right)\right]$$

$$= \alpha + \frac{\alpha}{\pi}\left[\underbrace{\sin\pi}_{0} - \underbrace{\sin(0)}_{0}\right] = \alpha$$

$\Rightarrow$ ① + ② = Area under the curve = $1-\alpha+\alpha = \underline{\underline{1}}$.

regardless of the value of α.

Drill problem.

6.3 $\quad P(f)$ is an even function. Rewriting the normalized curve versus f/B_0 (only for simplicity), we have:

$$\frac{P(f)}{\sqrt{E}/2B_0} = \begin{cases} 1 & 0 \le f/B_0 < f_1/B_0 \\ \frac{1}{2}\left[1+\cos\left[\frac{\pi}{2}\frac{(f/B_0 - f_1/B_0)}{1-f_1/B_0}\right]\right] & f_1/B_0 \le f/B_0 < 2-f_1/B_0 \\ 0 & 2-f_1/B_0 \le f/B_0 \end{cases}$$

(only for positive frequencies)

changing $f/B_0 = x$ and $\alpha = 1 - f_1/B_0 \Rightarrow f_1/B_0 = 1-\alpha$.

we have:

$$\frac{P(x)}{\sqrt{E}/2B_0} = \begin{cases} 1 & 0 \le x < 1-\alpha \\ \frac{1}{2}\left[1+\cos\left[\frac{\pi/2\,(x-(1-\alpha))}{\alpha}\right]\right] & 1-\alpha \le x \le 1+\alpha \\ 0 & 1+\alpha \le x \end{cases} \quad (A)$$

using Ⓐ : for 3 values of $\alpha = 0, 1/2, 1$

1 $\alpha = 0.$ $\Rightarrow \dfrac{P(x)}{\sqrt{E}/2B_0} = \begin{cases} 1 & 0 \le x < 1 \\ 0 & x \geq 1 \end{cases}$ (positive frequencies) $x \geq 0.$

$\Rightarrow$ the curve is discontinuous. $\Rightarrow$ $k = 0.$

(0-th derivative is the curve itself). $\Rightarrow$ using the theorem, the pulse should decrease with time as

$\dfrac{1}{t^{k+1}} = \underbrace{\dfrac{1}{t}}.$

* Now, double checking with the equation of the pulse as given in (6.19):

$$p(t) = \sqrt{E}\ \text{Sinc}\ (2B_0 t) \left(\frac{\cos(2\pi\alpha B_0 t)}{1 - 16\alpha^2 B_0^2 t^2} \right)$$

when $\alpha = 0$ $\Rightarrow p(t)\Big|_{\alpha=0} = \sqrt{E}\ \text{Sinc}\ (2B_0 t) = \sqrt{E}\ \dfrac{\sin 2\pi B_0 t}{2\pi B_0 t}$

Since $|\sin\alpha| \le 1$ $\Rightarrow$ it is clear the $p(t)$ decrease as $\underbrace{1/t}.$ ✓

2. $\alpha = 1/2$. $\Rightarrow$ $\dfrac{p(x)}{\sqrt{E}/2B_0} = \begin{cases} 1 & 0 \le x < 1/2 \\ 1/2\left[1 + \cos\left(\dfrac{\pi/2\ (x - 1/2)}{1/2}\right)\right] & 1/2 \le x \le 3/2 \\ 0 & 3/2 \le x \end{cases}$

$\dfrac{P(x)}{\sqrt{E}/2B_0} = g(x)$

$\Rightarrow dg(x)/dx = \begin{cases} 0 & 0 \le x < 1/2 \\ -1/2\,\pi \sin[(x - 1/2)\pi] & 1/2 \le x < 3/2 \\ 0 & x \geq 3/2. \end{cases}$

at $x = 1/2$ $\quad \sin\left[\frac{(x-1/2)\pi/2}{1/2}\right] = 0$

at $x = 3/2$ $\quad \sin\left[(x-1/2)\pi\right] = 0$

$\Rightarrow \frac{dg(x)}{dx}$ is continuous.

$$\frac{d^2 g(x)}{dx^2} = \begin{cases} 0 & 0 \le x < 1/2 \\ -1/2\,\pi^2 \cos\pi(x-1/2) & 1/2 \le x < 3/2 \\ 0 & x \geq 3/2 \end{cases} \quad (2)$$

$\Rightarrow$ at $x = 1/2$ $\quad \frac{d^2 g(x)}{dx^2}$ according to (2) is: $-1/2\,\pi^2 \neq 0$

$\Rightarrow$ the 2nd derivative is discontinuous. Therefore, $k = 2$.

Using the theorem, $p(t)$ decreases with time as $1/t^{k+1} = 1/t^3$.

* Double checking with equation of the pulse given in (6.19):

$$p(t)\Big|_{\alpha=1/2} = \sqrt{E}\,\mathrm{Sinc}(2B_0 t)\left[\frac{\cos(\pi B_0 t)}{1-4B_0^2 t^2}\right]$$

$$= \sqrt{E}\,\underbrace{\frac{\sin(2\pi B_0 t)}{2\pi B_0 t}}\,\underbrace{\left[\frac{\cos \pi B_0 t}{1-4B_0^2 t^2}\right]} = \frac{\sqrt{E}\,\sin(2\pi B_0 t)\cos(\pi B_0 t)}{2\pi B_0\,(t - 4B_0^2 t^3)}$$

decreases as $1/t^3$.

3. $\alpha = 1$.

$$\frac{p(x)}{\sqrt{E}/2B_0} = \begin{cases} 1/2\left[1 + \cos(\pi/2\, x)\right] & 0 \le x < 1+1 \\ 0 & x \geq 2 \end{cases}$$

$$\frac{p(x)}{\sqrt{E}/2B_0} = g(x)$$

$$\frac{dg(x)}{dx} = \begin{cases} -\pi/4\,\sin(\pi/2\, x) & 0 \le x < 2 \\ 0 & x \geq 2 \end{cases}$$

$$\left.\frac{dg(x)}{dx}\right|_{at\ x=2} = 0 \Rightarrow \text{continuous.}$$

$$d^2g(x)/dx^2 = \begin{cases} -\pi^2/8 \cos(\pi/2\, x) & 0 \leq x < 2 \\ 0 & x \geq 2 \end{cases}$$

$$\Rightarrow \left. d^2g(x)/dx^2 \right|_{at\ x=2} = -\pi^2/8 \cos(\pi) \neq 0 \Rightarrow \text{discontinuous.}$$

$\Rightarrow \underline{k=2}$ Using the theorem, the pulse decreases with time as

$\underline{1/t^{k+1} = 1/t^3}$

3 Drill problem 6.4

(a) A linear phase characteristics is represented by:

$e^{-j2\pi f(\tau)}$ Multiplied by the pulse spectrum $P(f)$.

$\Rightarrow$ The inverse Fourier Transform $\left(P(f)e^{-j2\pi f\tau}\right)$ is the delayed function: $\underline{p(t-\tau)}$. For $\tau \geq 0$, it shifts the pulse response in Fig. 6.3 to the right by τ seconds.

(b) The impulse response of a causal system has the property of: $\boxed{h(t) = 0 \quad t \leq 0}$ $\Rightarrow$ by shifting $p(t)$ to the right, we can make $p(t)$ essentially causal, when τ is enough. The value of τ should be large enough, but decreases with increasing α.

$\alpha = 0 \rightarrow \tau \approx 5T_b.$

$\alpha = 1/2 \rightarrow \tau \approx 3T_b.$

$\alpha = 1 \rightarrow \tau \approx 2.5T_b.$

Drill problem 6.5

in (6.24) rewriting $P(f)$ for positive frequencies, (same approach applies for negative frequencies $f<0$).

$|f| = f$ since $f \geq 0$

$$P_v(f) = \begin{cases} 0 & 0 \leq f \leq f_1 \\ \sqrt{E}/4B_0 \left\{ 1 - \cos\left[\frac{\pi}{2} \frac{(f-f_1)}{(B_0 - f_1)} \right] \right\} , & f_1 \leq f \leq B_0 \\ -\sqrt{E}/4B_0 \left\{ 1 + \cos\left[\pi/2 \frac{(f-f_1)}{B_0 - f_1} \right] \right\} , & B_0 \leq f \leq 2B_0 - f_1 \\ 0 & 2B_0 - f_1 \leq f \leq 2B_0 \end{cases}$$

$\Rightarrow$ if $f' = f - B_0$

$$P_v(f') = \begin{cases} 0 & -B_0 \leq f' \leq f_1 - B_0 \\ ① \ \sqrt{E}/4B_0 \left\{ 1 - \cos\left[\pi/2 \frac{f'}{(B_0 - f_1)} + \pi/2 \right] \right\} & f_1 - B_0 \leq f' \leq 0 \\ ② \ -\sqrt{E}/4B_0 \left\{ 1 + \cos\left[\frac{\pi f'}{2(B_0 - f_1)} + \pi/2 \right] \right\} & 0 \leq f' \leq -(f_1 - B_0) \\ 0 & B_0 \geq f' \geq B_0 - f_1 \end{cases}$$

we show that $P_v(f')$ above is an odd function of f'

Note: ① $\sqrt{E}/4B_0 \left[1 - \cos\left(\frac{\pi f'}{2(B_0 - f_1)} + \pi/2 \right) \right] = \sqrt{E}/4B_0 \left[1 + \sin\left(\pi f'/2(B_0 - f_1) \right) \right]$

and ② $-\sqrt{E}/4B_0 \left[1 + \cos\left(\frac{\pi f'}{2(B_0 - f_1)} + \pi/2 \right) \right] = -\sqrt{E}/4B_0 \left[1 - \sin\left(\frac{\pi f'}{2(B_0 - f_1)} \right) \right]$

if ① $= g_1(f')$ $\Rightarrow$ $g_1(f') = \sqrt{E}/4B_0 \left[1 + (-1)\sin\left(\frac{\pi(-f')}{2(B_0 - f_1)} \right) \right] = -g_2(-f')$

② $= g_2(f')$ and $g_2(f') = -g_1(-f')$.

since same approach applies to $f \geq 0 \Rightarrow$

$$\underline{P_v(-f') = -P_v(f')}.$$

problem $\underline{6.7}$

$p(f)$ is an even function and real $\Rightarrow$

$$p(t) = \int_{-\infty}^{+\infty} p(f) e^{j2\pi ft} df = \int_{-\infty}^{+\infty} p(f)\left[\cos 2\pi ft + j \sin 2\pi ft\right] df$$

$$= \int_{-\infty}^{+\infty} p(f) \cos 2\pi ft \, df + \underbrace{j \int_{-\infty}^{+\infty} p(f) \sin(2\pi ft)\, df}_{0}$$

$$= \int_{-\infty}^{+\infty} p(f) \cos 2\pi ft \, df$$ since $p(f)$ is even $\Rightarrow p(f) \sin 2\pi ft$ is odd.

Now: $$\int_{-\infty}^{+\infty} p(f) \cos 2\pi ft \, df = 2 \int_{0}^{\infty} p(f) \cos(2\pi ft)\, df.$$

Ignoring $\sqrt{E}$ factor for simplicity and rewriting the integral in terms of $\alpha = \frac{(B_0 - f_1)}{B_0}$, we have:

$$p(t) = \frac{1}{B_0} \int_0^{f_1} \cos(2\pi ft)\, df + \frac{1}{2B_0} \int_{f_1}^{2B_0 - f_1} \left[1 + \cos\left(\frac{\pi(f - f_1)}{2B_0\alpha}\right)\right] \cos 2\pi ft \, df$$

using: $\cos(\alpha)\cos(\beta) = 1/2\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$.

The integral is written as:

$$\overbrace{\left[\frac{\sin(2\pi f t)}{2\pi B_0 t}\right]_0^{f_1}}^{1} + \overbrace{\left[\frac{\sin(2\pi f t)}{4\pi B_0 t}\right]_{f_1}^{2B_0 - f_1}}^{2} + \overbrace{\cancel{\tfrac{1}{4}}B_0\left[\frac{\sin\left(2\pi f t + \frac{\pi(f-f_1)}{2B_0\alpha}\right)}{2\pi t + \pi/2B_0\alpha}\right]_{f_1}^{2B_0-f_1}}^{3} +$$

$$\underbrace{\frac{1}{4B_0}\left[\frac{\sin\left[2\pi f t - \frac{\pi(f-f_1)}{2B_0\alpha}\right]}{2\pi t + \pi/2B_0\alpha}\right]_{f_1}^{2B_0-f_1}}_{4}$$

1 : $\dfrac{\sin(2\pi f_1 t)}{2\pi B_0 t}$

2 : $\dfrac{\sin(2\pi(2B_0 - f_1)t)}{4\pi B_0 t} - \dfrac{\sin(2\pi f_1 t)}{4\pi B_0 t}$

3 : $$\cancel{\tfrac{1}{4}}B_0\left[\frac{\sin\left[2\pi(2B_0 - f_1)t + \frac{\pi(2B_0 - 2f_1)}{2B_0\alpha}\right]}{2\pi t + \frac{\pi}{2B_0\alpha}}\right]$$

$$-\frac{1}{4B_0}\left[\frac{\sin\left[2\pi f_1 t + 0\right]}{2\pi t + \frac{\pi}{2B_0\alpha}}\right]$$

4 : $$\frac{1}{4B_0}\left[\frac{\sin\left[2\pi(2B_0 - f_1)t - \frac{\pi(2B_0 - 2f_1)}{2B_0\alpha}\right]}{2\pi t + \frac{\pi}{2B_0\alpha}}\right]$$

$$-\frac{1}{4B_0}\left[\frac{\sin 2\pi f_1 t}{2\pi t + \frac{\pi}{2B_0\alpha}}\right].$$

1 + 2 + 3 + 4 = $p(t)$

problem 6.8.

Starting with (6.19) we have:

$$p(t) = \sqrt{E}\ \text{Sinc}(2B_0t)\left(\frac{\cos(2\pi\alpha B_0 t)}{1-16\alpha^2 B_0^2 t^2}\right) \quad \text{when } \alpha = 1 \Rightarrow$$

$$p(t)_{\alpha=1} = \sqrt{E}\ \frac{\sin(2\pi B_0 t)}{2\pi B_0 t}\ \frac{\cos(2\pi B_0 t)}{1-16 B_0^2 t^2}$$

using : $\sin(a)\cos(b) = 1/2 \sin(2a)$
when $a=b$

$$\Rightarrow p(t) = \frac{\sqrt{E}}{2}\cdot\frac{1}{2\pi B_0 t}\sin(4\pi B_0 t)\frac{1}{1-16B_0^2t^2} = \frac{\sqrt{E}\ \text{Sinc}(4B_0t)}{1-16B_0^2t^2}$$

problem 6.9.

$R_b = 56k \Rightarrow \frac{1}{T_b} = R_b = 56k$

$B_0 = \frac{1}{2T_b} = 1/2\ 56k = 28k$

$B_T = B_0(1+\alpha)$ using (6.21)

$\Rightarrow$ (a) $\alpha = 0.25 \rightarrow B_T = 1.25 \times 28k = 35kHz$

(b) $\alpha = 0.5 \rightarrow B_T = 1.5 \times 28k = 42kHz$

(c) $\alpha = 0.75 \rightarrow B_T = 1.75 \times 28k = 49kHz$

(d) $\alpha = 1.0 \rightarrow B_T = 2. \times 28k = 56kHz$.

problem 6.10.

$B_T = 75\ kHz$.

$T_b = 10\mu s \Rightarrow B_0 = \frac{1}{2T_b} = 50kHz$.

$B_T = (1+\alpha)B_0 \Rightarrow \alpha = \frac{B_T}{B_0} - 1$

$\Rightarrow \alpha = \frac{75k}{50k} - 1 = \boxed{0.5}$

$\alpha = 1 - \frac{f_1}{B_0} \Rightarrow \underset{\sim}{f_1} = (1-\alpha) B_0 = 25\,kHz.$

$\begin{cases} f_1 = 25\,kHz \\ \alpha = 0.5 \end{cases}$ are the design parameters.

problem 6.11

B_T : channel bandwidth $= 3k$

$B_T = (1+\alpha)B_0 = B_0 + \alpha B_0$ $\quad f_v = \alpha B_0$ and $B_0 = \frac{1}{2T_b} = \frac{1}{2} R_b$

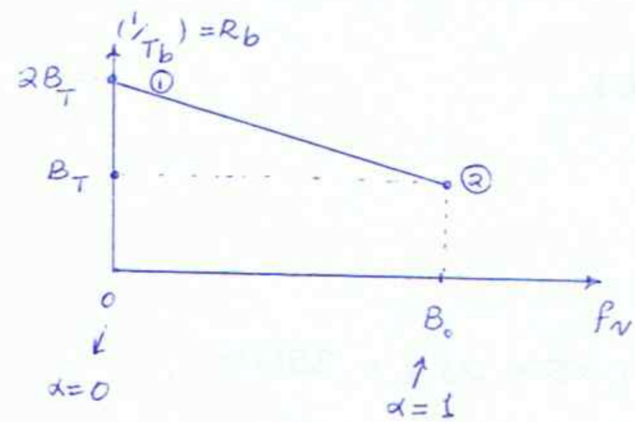
where $R_b = \frac{1}{T_b}$ bit rate.

$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{T_b}\right) = B_T + (-f_v)$

$\boxed{\frac{1}{T_b} = 2B_T - 2f_v}$ relationship between $\left(\frac{1}{T_b}\right)$ and f_v.

when α changes from 0 to 1, f_v changes from 0 to B_0.

at $\alpha = 1$ $\quad B_T = (1+\alpha)B_0 = 2B_0 \Rightarrow \underline{B_0 = 1.5k}$



$\Rightarrow$ ① at zero excess BW $\alpha = 0$ $\quad f_v = 0$ $\quad B_T = 3k \Rightarrow \underline{R_b = 6\,kbps}$

② at $\alpha = 1$ $\quad f_v = B_0 = 1.5\,kHz$ $\quad B_T = 3\,kHz \Rightarrow \underline{R_b = 2B_0 = 3\,kbps}.$

6.13

The pulse shaping criterion for zero intersymbol interference is (6.30) : $\sum_{-\infty}^{+\infty} P(f-\frac{m}{T}) = \text{const.}$ where $\frac{1}{T}$ is signalling rate.

(a) The pulse-shaping spectrum of Fig. 6.13(a) is given by:

$$P(f) = \begin{cases} \sqrt{E}/2B_0 & f=0. \\ \frac{\sqrt{E}}{2B_0}(1-\frac{f}{B_0}) \quad 0<f<B_0 \ , & \frac{\sqrt{E}}{2B_0}(\frac{B_0+f}{B_0}) \quad -B_0<f<0 \\ 0 & \text{for } f=B_0, -B_0 \text{ and } |f|>B_0. \end{cases}$$

substituting $P(f)$ in (6.30) gives the condition:

$\frac{1}{T} = B_0/2$ or $B_0 = \frac{2}{T}$

(b) Similarly: pulse-shaping spectrum of Fig 6.13(b) is:

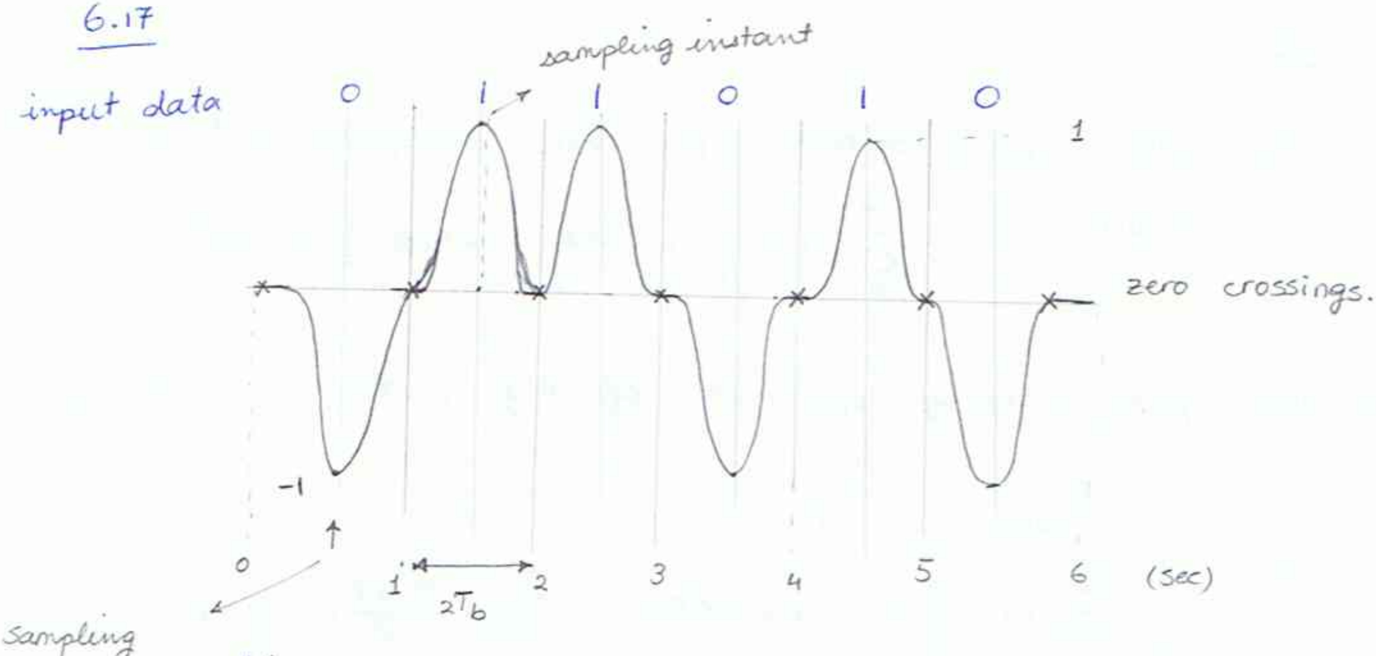
$$P(f) = \begin{cases} \sqrt{E}/2B_0 & 0<|f|<f_1 \\ \sqrt{E}/2B_0\left[1-\frac{|f|-f_1}{B_0-f_1}\right] & f_1<f<B_0 \\ 0 & |f|>B_0 \end{cases}$$

$\Rightarrow$ (6.30) gives the condition: $\frac{1}{T} = \frac{1}{2}(f_1+B_0)$.

or $B_0 = \frac{2}{T} - f_1$

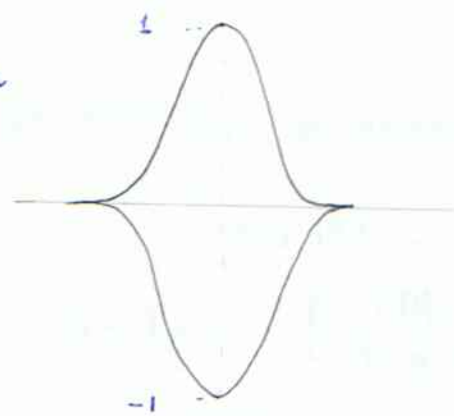
(c) Among the pulse-shaping spectra, the raised-cosine function corresponding to roll-off factor $\alpha=1$ and $\alpha=1/2$ are preferred due to 1 improved signalling rate and 2 mathematical simplicity and ease of implementation.

6.17



if $1/T_b = 1$ bps (scaled) $\Rightarrow B_0 = \frac{1}{2T_b} = 0.5$ Hz.

Eye diagram



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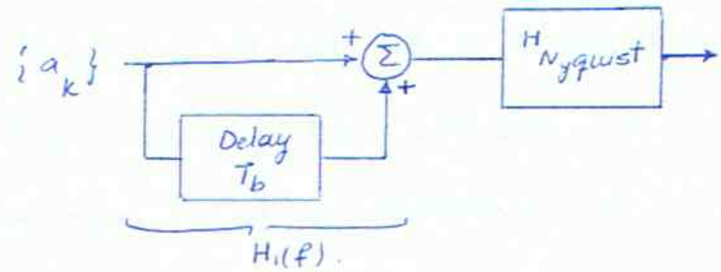
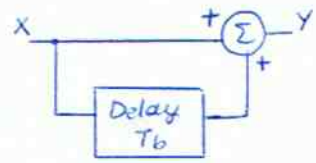
6.20 6.21 6.22 6.23.

6.20

(a) Duobinary Conversion Filter



$$H(f) = H_1(f)\cdot H_{Nyquist}(f)$$



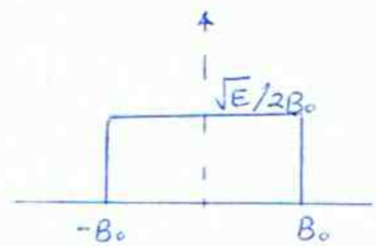
for $H_1(f)$: $\quad Y(f) = X(f) + X(f)e^{-j2\pi f T_b}$

$$\Rightarrow H_1(f) = \frac{Y(f)}{X(f)} = 1 + e^{-j2\pi f T_b}.$$

$$\Rightarrow \underline{H(f) = H_{Nyquist}(f)\left[1 + e^{-j2\pi f T_b}\right].}$$

where :

$H_{Nyquist} =$



For ideal Nyquist channel:

$$B_0 = \frac{1}{2T_b}$$

(b) $h(t) = F^{-1}(H(f))$

↳ Inverse Fourier transform

$$= F^{-1}\left[H_{Nyquist}(f) + H_{Nyquist}(f)\, e^{-j2\pi f T_b}\right].$$

$$= \sqrt{E}\ \mathrm{Sinc}(2B_0 t) + \sqrt{E}\ \mathrm{Sinc}(2B_0(t - T_b)) =$$

$$= \sqrt{E}\ \frac{\sin(2\pi \frac{1}{2T_b} t)}{2\pi \frac{1}{2T_b} t} + \sqrt{E}\ \frac{\sin(\overrightarrow{2B_0\pi(t - T_b)})}{2B_0\pi(t - T_b)}^{1/2T_b} =$$

using : $\underline{\sin(\theta - \pi) = -\sin(\theta)}$

$$\boxed{\sqrt{E}\ \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)}}$$

(c) The original sequence is detected using decision feedback by: $\hat{a}_k = c_k - \hat{a}_{k-1}$.

A major drawback is the for the current detection $\hat{a}_k$ to be correct, the previous detection has to be correct. If $\hat{a}_{k-1}$ is not detected corrected, we have error propagation.

6.21

$d_k = b_k \oplus d_{k-1}$ symbol $\oplus$ denotes modulo-two addition (EXCLUSIVE OR).

$$d_k = \begin{cases} 1 & \text{if either } d_{k-1} \text{ or } b_k \text{ is } 1. \\ 0 & \text{otherwise.} \end{cases}$$

d_{k-1} \ b_k	0	1
0	0	1
1	1	0

truth table for d_k.

Also:

$$a_k = \begin{cases} 1 & \text{if } d_k \text{ is } 1 \\ -1 & \text{if } d_k \text{ is } 0 \end{cases}$$

And:

$$c_k = a_k + a_{k-1} = \begin{cases} -2 & \text{if both } a_k \text{ \& } a_{k-1} \text{ are } -1 \\ 0 & \text{if } a_k = 1 \;\; a_k = -1 \text{ or } a_k = -1 \;\; a_k = 1. \\ 2 & \text{if both are } 1. \end{cases}$$

Decision Rule:

if $|c_k| < 1 \Rightarrow \quad b_k = 1$

if $|c_k| > 1 \Rightarrow \quad b_k = 0$

Solutions to parts (a)-(c) is given below for sequence $\{b_k\}$ = 0010110.

b_k		0	0	1	0	1	1	0	Mathes the sequence $\{b_k\}$.
d_k	1	1	1	0	0	1	0	0	
a_k	1	1	1	-1	-1	1	-1	-1	
c_k		2	2	0	-2	0	0	-2	
Applying the decision rule $\to \{\hat{b}_k\}$		0	0	1	0	1	1	0	

* chosen arbitrarily. can be chosen 0.

6.22

(a) Similar to 6.20 part (a), the overall transfer function of the modified duobinary Conversion Filter is given by:

$$H(f) = H_{Nyquist}(f)\left[1 - e^{-j4\pi f T_b}\right] \qquad e^{-j2\pi f(2T_b)} = e^{-4\pi f T_b j}$$

$$\Rightarrow \quad h(t) = F^{-1}(H(f)). \qquad \text{Again: } B_0 = \frac{1}{2T_b}$$

(b) $$h(t) = F^{-1}(H_{Nyquist}(f)) + F^{-1}\left(H_{Nyquist}(f)e^{-j4\pi f T_b}\right)$$

$$= \sqrt{E}\frac{\sin(\pi t/T_b)}{\pi t/T_b} + (-1)\sqrt{E}\frac{\sin(\pi(t-2T_b)/T_b)}{\pi(t-2T_b)/T_b} \rightarrow \text{Simplify}$$

$$= \sqrt{E}\,\frac{\sin(\pi t/T_b)}{\pi t/T_b} - \sqrt{E}\,\frac{\sin(\pi t/T_b - 2\pi)}{\pi(t-2T_b)/T_b} = \frac{\sin(\pi t/T_b)}{1}\left[\frac{1}{\pi t/T_b} - \frac{1}{\pi(t-2T_b)/T_b}\right]$$

using: $\sin(\theta - 2\pi) = \sin\theta$

$$= \frac{2T_b^2 \sin(\pi t/T_b)}{\pi t(2T_b - 1)} \cdot \sqrt{E}$$

(c) $d_{k-2} \oplus b_k = d_k$ where b_k is the incoming binary sequence.

Note: in Fig. (6.15), the delay element in the precoder should read $2T_b$.

Given the same sequence as b_k: 0010110

b_k:			0	0	1	0	1	1	0
d_k:	0	0	0	0	1	0	0	1	0
	chosen arbitrarily								
a_k:	-1	-1	-1	-1	1	-1	-1	1	-1
c_k:			0	0	2	0	-2	2	0
$(c_k = a_k - a_{k-2})$									
Decision Rule $\{\hat{b}_k\}$			0	0	1	0	1	1	0

$$\begin{cases} |c_k| > 1 \rightarrow b_k = 1 \\ |c_k| < 1 \rightarrow b_k = 0 \end{cases}$$

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(d) In the absence of channel noise, the detected binary sequence $\{b_k\}$ is exactly the same as the original sequence. Also, the performance does not allow for error propagation and is independent of the two initial bits chosen for the pre-coder. $\{0,0\}$ in our example.

6.23

The transfer function of Modified duobinary Conversion Filter in $\boxed{(6.22)}$ is :

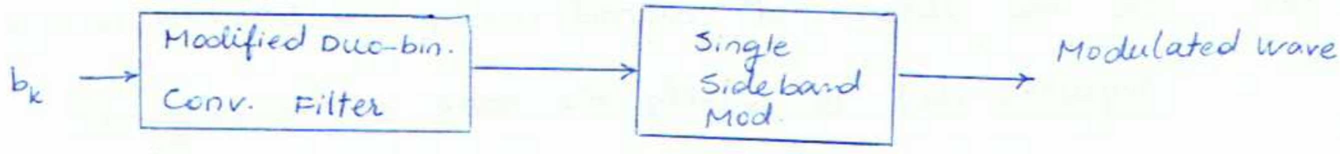
$$H(f) = H_{Nyquist}(f)\left[1 - e^{-j4\pi f T_b}\right]$$

$$\underbrace{H(f)\Big|_{at\ f=0}}\ = 0.$$

where as: $H(f)_{\text{as in } \boxed{6.20}} = H_{Nyquist}(f)\left[1 + e^{-j2\pi f T_b}\right]$

$$\Rightarrow \underbrace{H(f)\Big|_{at\ f=0}} \neq 0.$$

$\Rightarrow$ The transfer function in 6.22 makes it easier to design a pass-band filter required for a single-side band modulation. More specifically, the transmitter consists of two functional blocks:



transforms the incoming binary data into a new format whose spectrum has low-frequency content around origin.

up converts the transformed data to the desired band, transmitting the lower or the upper sideband of the modulated wave.

Receiver: