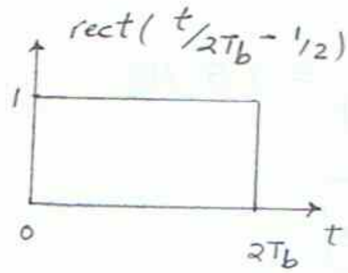
problem 7.5

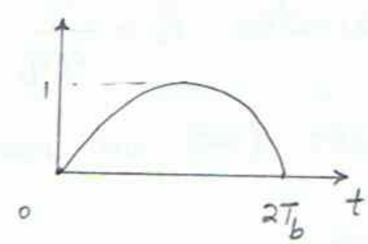
$$p(t) = \sin\left(\frac{\pi t}{2T_b}\right) \text{rect}\left(\frac{t}{2T_b} - 1/2\right)$$

In MSK, each in-phase and quadrature component is weighted by a half-cycle of a cosinusoid as shown in Fig. (7.14) and problem 7.18. The frequency of the cosinusoid is $f_0 = \frac{1}{4T_b}$. Since each in-phase or quadrature component lasts for a duration of $T = 2T_b$ sec, such process is equivalent to multiplying the components by:




x

$\sin\left(\frac{2\pi t}{4T_b}\right)$
$= \sin\left(\frac{\pi t}{2T_b}\right)$

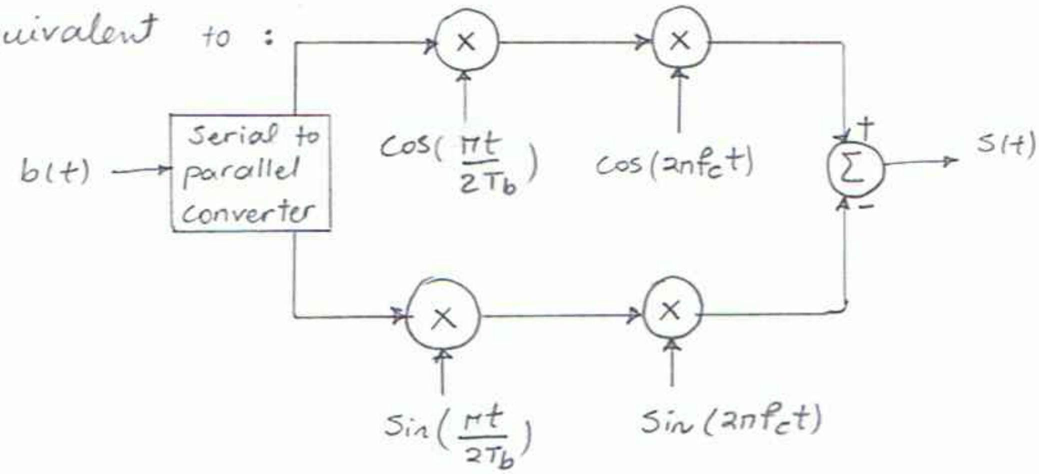



$$\Rightarrow p(t) = \sin\left(\frac{\pi t}{2T_b}\right) \text{rect}\left(\frac{t}{2T_b} - 1/2\right).$$

## 7.7

MSK signal $s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\theta(t)) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin(\theta(t)) \sin(2\pi f_c t)$

equivalent to :




This system satifies the principle of superposition $\Rightarrow$ it is linear.

7.10 Justify Eqs. (7.47) and (7.49)

Eq.(7.46): $s_1 = \int_0^{T_b} \varphi(t)\, s_1(t)\, dt = \int_0^{T_b} \frac{2}{T_b}\sqrt{E_b}\cos^2(2\pi f_c t)\, dt$

using: $\cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}$

$$\Rightarrow S_1 = \int_0^{T_b} \frac{2\sqrt{E_b}}{2T_b}\left(1+\cos(4\pi f_c t)\right) dt = \frac{\sqrt{E_b}\, T_b}{T_b} + \int_0^{T_b} \frac{\sqrt{E}}{T_b}\cos(4\pi f_c t)\, dt$$

under band pass assumption $\Rightarrow \int_0^{T_b} \cos(4\pi f_c t)\, dt = 0$
if $f_c \gg \frac{1}{T_b}$

Therefore $\boxed{S_1 = \sqrt{E}}$ (7.47)

Similarly

Eq(7.48): $s_2 = \int_0^{T_b} s_2(t)\cdot \varphi_1(t)\, dt = -\frac{2}{T_b}\sqrt{E_b}\int_0^{T_b} \cos^2(2\pi f_c t)\, dt$

using same approach as above $\boxed{S_2 = -\sqrt{E_b}}$ (7.49)

6. problem 7.11.

ASK modulator.

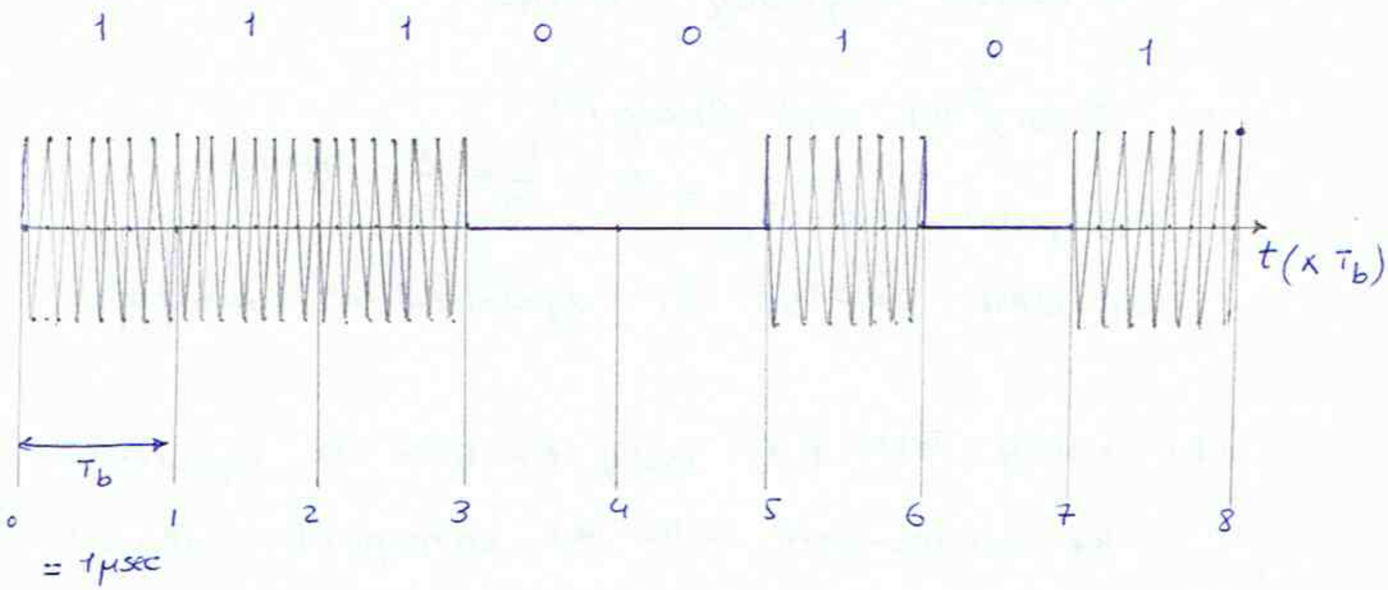
$T_b$: bit duration = 1 $\mu s$

$f_c$: carrier frequency = 7 MHz.

(a) $B_T$ = transmission bandwidth

$$= \frac{2}{T_b} \text{ (for ASK) } = \frac{2}{1\mu s} = 2 \times 10^6 \text{ Hz} : 2\text{MHz}.$$

(b) in ASK : symbol 1 is represented by carrier wave for a duration of $T_b$ seconds.




since $f_c$ = 7MHz $\Rightarrow T_c = \frac{1}{f_c} = \frac{1}{7\text{MHz}} \rightarrow$ in each $T_b$ seconds,

7 complete periods of the carrier wave is transmitted.

7. Problem 7.12.

If the line encoder and the carrier-wave generator operate independently, the transmitted signal might not be continuous in time when a succession of 1's is transmitted.

8. problem 7.14.

1 1 1 0 0 1 0 1 is applied to QPSK modulator.

$T_b$ = bit duration = 1 μs.

$f_c$ = carrier frequency = 6 MHz.

(a) $B_T = \frac{2}{T}$

T: symbol duration $\Rightarrow B_T = \frac{2}{2T_b} = \frac{1}{T_b} = 1\,MHz$.

in QPSK $T = 2T_b$ (T: symbol (dibit) duration).

(b) using Table 7.1, every two bits are represented by the carrier-wave with the corresponding phase.

so : $\underbrace{11}_{7\pi/4}\ \underbrace{10}_{\pi/4}\ \underbrace{01}_{5\pi/4}\ \underbrace{01}_{5\pi/4}$

phase shift: $7\pi/4$ $\pi/4$ $5\pi/4$ $5\pi/4$.

$\Rightarrow$ The signal transmitted for each pair of bits is:

$$\sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \theta(t))$$

↳ phase shift found from Table 7.1.

To clarify:

For $\underline{1}\,\underline{1}$ : the transmitted signal is

$$s(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 7\pi/4)$$

This signal is transmitted for the duration of $T = 2T_b$ seconds.

Similarly:

$\underline{1}\,\underline{0}$ : $s(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \pi/4)$

$\underline{0}\,\underline{1}$ : $s(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 5\pi/4)$.

Since $T_b = 1\,\mu s$ $\quad T = 2T_b = 2\,\mu s$.

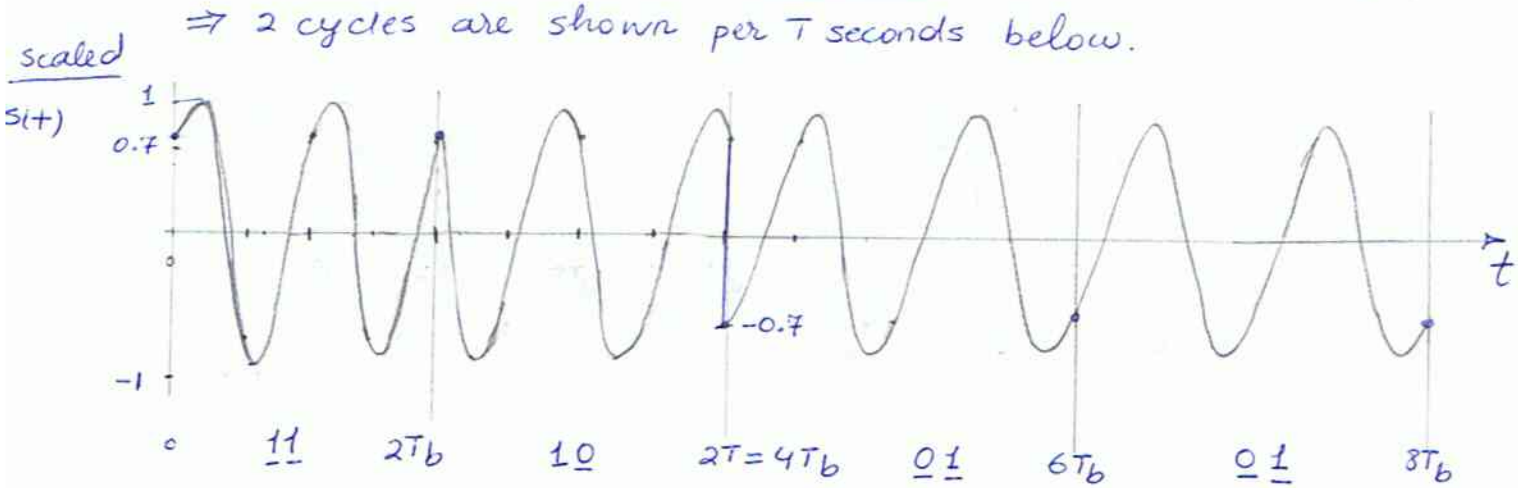
and $f_c = 6\text{ MHZ} \Rightarrow \frac{1}{f_c} = \frac{1}{6}\,\mu s$. $\quad \Rightarrow \frac{T}{T_c} = \frac{2\,\mu s}{1/6\,\mu s} = 12$.

$T_c = \frac{1}{f_c} = \frac{1}{6}\,\mu s$

Therefore: in each $T$ seconds, 12 complete cycles of the signal with the corresponding phase shift derived from Table 7.1 is transmitted.

[For ease of representation] below: the carrier is <u>assumed</u> to be 1 MHZ. $\Rightarrow T_c = 1\,\mu s$

$\Rightarrow$ 2 cycles are shown per $T$ seconds below.



7.15 Repeat problem 7.14 for OQPSK.

The binary sequence is : 1 1 1 0 0 1 0 1

Grouping the input binary sequence into dibits :

$\underbrace{11}$ $\underbrace{10}$ $\underbrace{01}$ $\underbrace{01}$ The first bit of each dibit is $b_1$
" second " " " " is $b_2$.

$\Rightarrow$

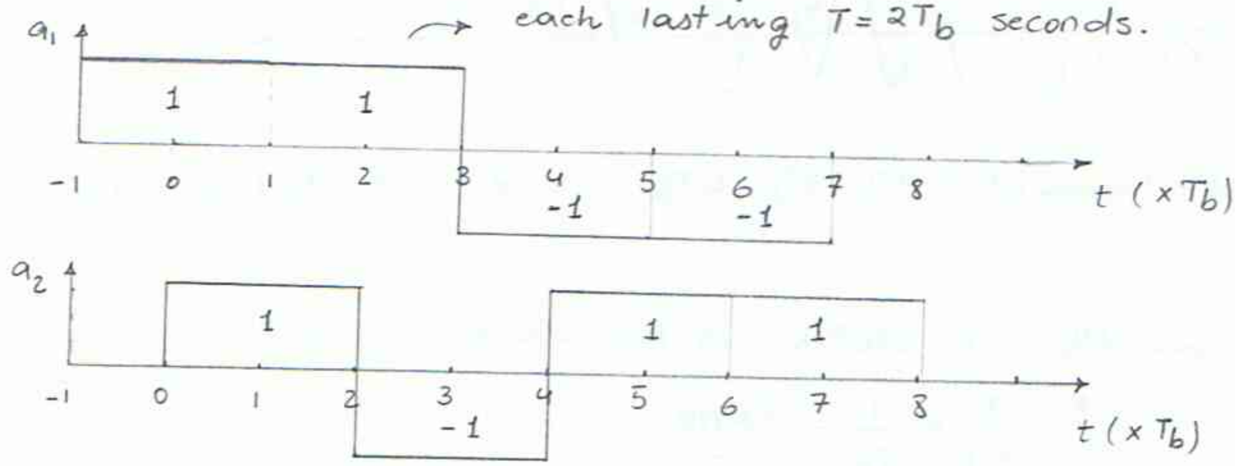
$b_1$ : 1 1 0 0
$b_2$ : 1 0 1 1

Using NRZ encoder :

$a_1$ : 1 1 -1 -1
$a_2$ : 1 -1 1 1

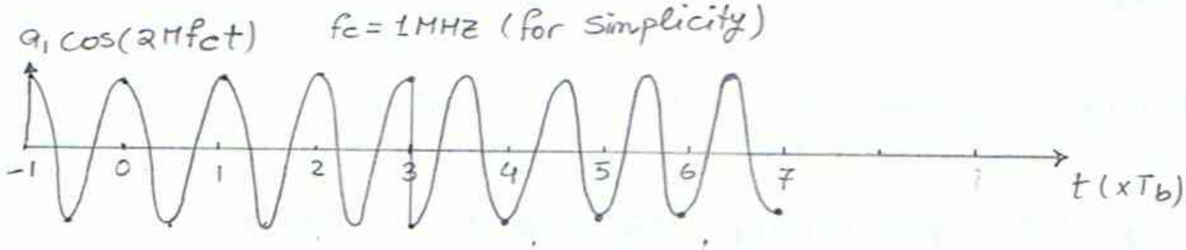
$a_2$ is delayed by $T_b$ seconds ; in other words $a_1$ starts $T_b$ seconds earlier $\Rightarrow$ ( $T_b$ is bit duration)

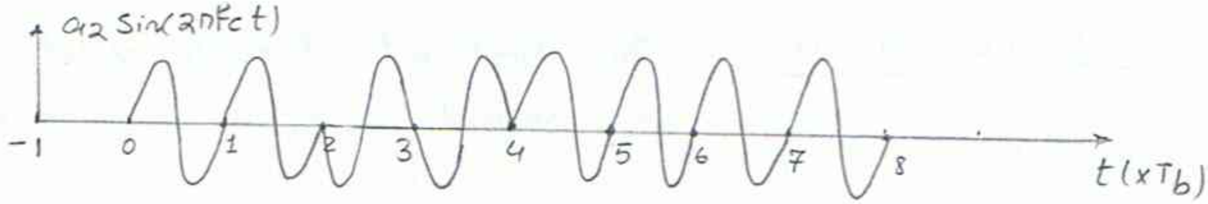
each lasting $T = 2T_b$ seconds.




Multiply $a_1$ by $\cos(2\pi f_c t)$ and $a_2$ by $\sin(2\pi f_c t)$.

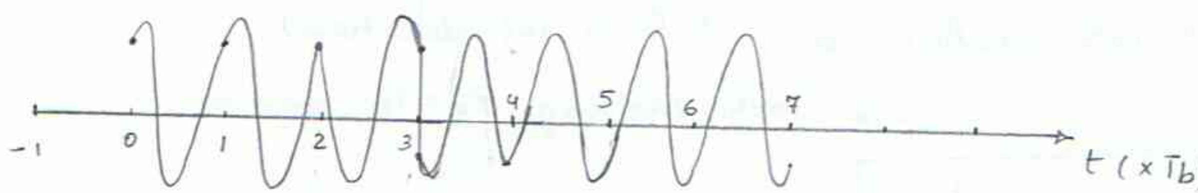
Here : $f_c = 6$ MHz $\Rightarrow$ in $T_b = 1\,\mu s$ there are 6 complete cycles of the carrier wave. For simplicity of representation, $f_c = 1$ MHz is chosen, hence 1 complete cycle per $T_b$ sec.




$a_1 \cos(2\pi f_c t)$ starts $T_b$ seconds earlier. (at $-T_b$)

$a_2 \sin(2\pi f_c t)$ starts $T_b$ seconds after $a_1 \cos(2\pi f_c t) \Rightarrow$ at 0.




phase shifts happen at: $2T_b$, $3T_b$, $4T_b$. and is limited to $\pm \pi/2$.

The bandwidth of OQPSK is the same as QPSK.

$$\Rightarrow BW = \frac{2}{T} = \frac{2}{2T_b} = \frac{1}{T_b} = \underline{1\,MHZ}.$$

7.18

The binary sequence of 1 1 1 0 0 1 0 1 is applied to MSK modulator. The bit duration is $1\,\mu s$.

$f_1$ : carrier frequency representing symbol 0 = 2.5 MHZ

$f_2$ : " " " " 1 = 3.5 MHZ.

$$f_c = \frac{f_1 + f_2}{2} = \left(\frac{2.5 + 3.5}{2}\right)M = 3 \text{ MHZ. carrier frequency.}$$

$$f_0 = \frac{1}{4T_b} = \frac{1}{4(1\mu s)} = 250 \text{ kHZ.}$$

as in 7.15 : group the input binary sequence into dibits.

⇒ $b_1$ : 1 1 0 0 using NRZ encoding: $a_1$ : 1 1 −1 −1

$b_2$ : 1 0 1 1 $a_2$ : 1 −1 1 1

as in OQPSK : $a_1$ starts $T_b$ seconds earlier than $a_2$, or $a_2$ is delayed by $T_b$ seconds.

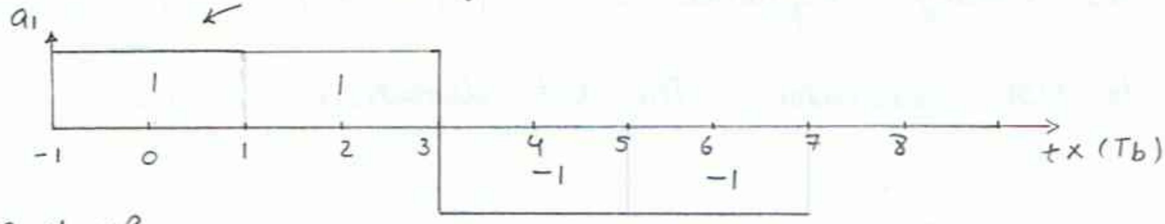
Also : in $T_b$ seconds $\frac{T_b}{1/f_c} = \frac{1M}{(\frac{1}{3})M} = 3$ complete cycles are drawn.

$a_1$ is multiplied by half cycle $\cos(2\pi f_0 t)$ where as

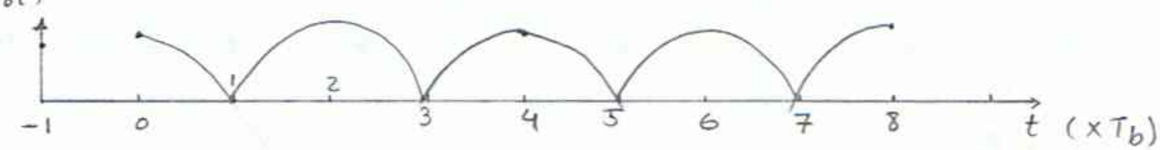
$a_2$ is multiplied by half cycle $\sin(2\pi f_0 t)$.

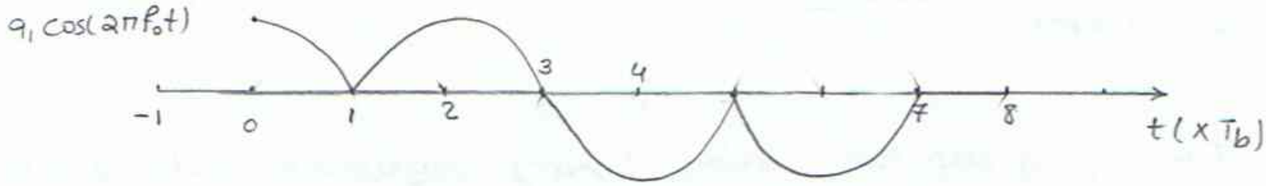
as shown in the figure.

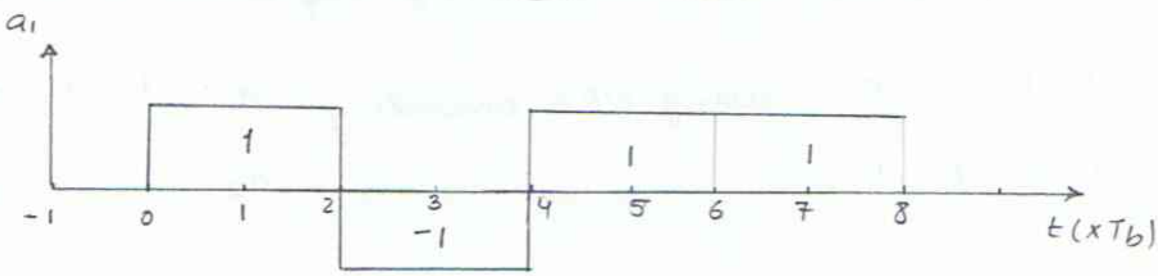
each lasting for $2T_b$ seconds.



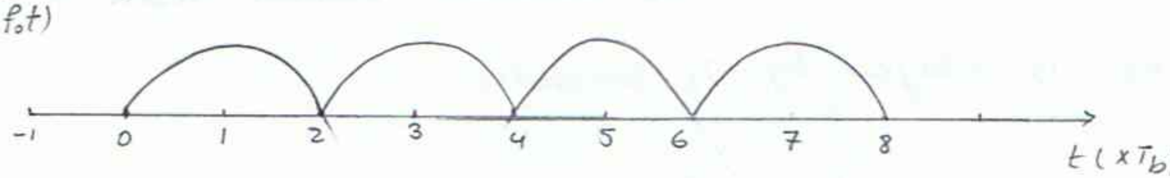

Half cycle of $\cos(2\pi f_0 t)$

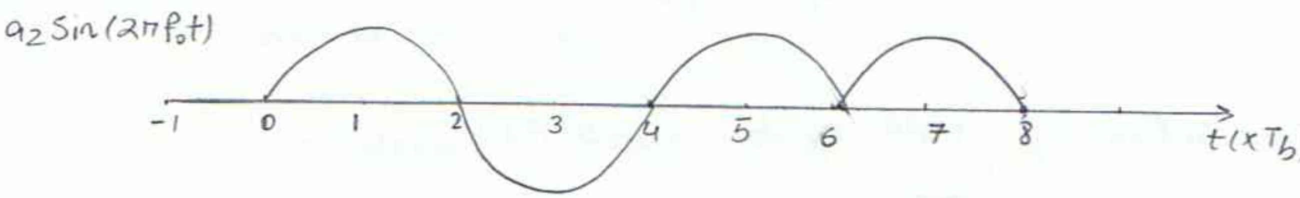


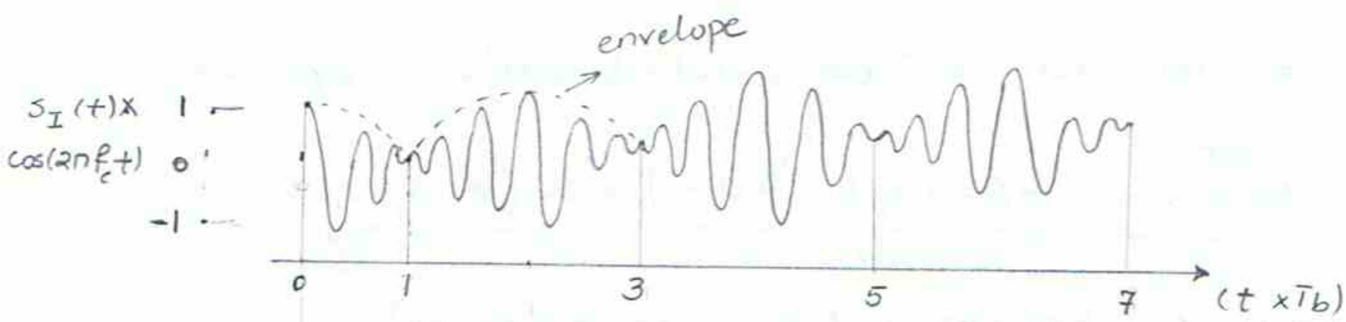



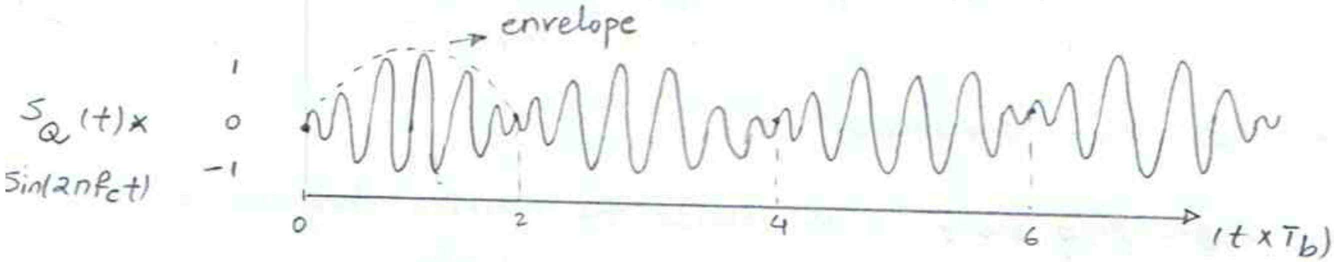




Half cycle of $\sin(2\pi f_0 t)$

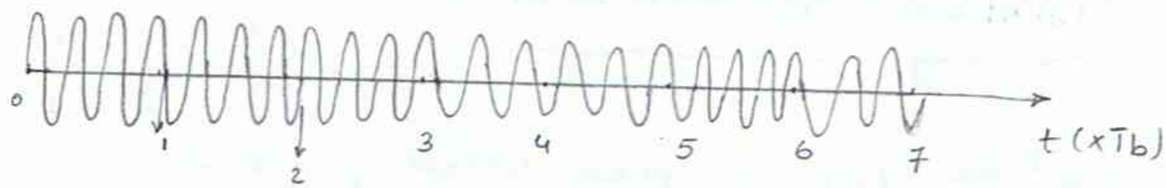


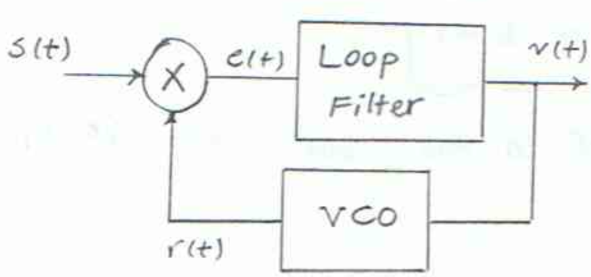



$$s(t) = a_1 \cos(2\pi f_0 t)\cos(2\pi f_c t) - a_2 \sin(2\pi f_0 t)\sin(2\pi f_c t).$$




**7.24** The block diagram of PLL is as follows.




binary input $\begin{cases} +1 \\ -1 \end{cases}$

where: $s(t) = A_c \cos\left[2\pi f_c t + k_p b(t)\right]$

phase sensitivity

$r(t) = A_c \sin\left[2\pi f_c t + \theta(t)\right]$

$$e(t) = s(t)\cdot r(t) = A_c^2 \cos\left[2\pi f_c t + k_p b(t)\right]\cdot \sin\left[2\pi f_c t + \theta(t)\right]$$

$$= \tfrac{1}{2} A_c^2 \left[ \underbrace{\sin\left(4\pi f_c t + k_p b(t) + \theta(t)\right)}_{1} + \underbrace{\sin\left(\theta(t) - k_p b(t)\right)}_{2} \right]$$

using : $\sin(\alpha)\cos(\beta) = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$.

$\Rightarrow$ $e(t)$ has two components :

1 : high frequency with twice the carrier frequency $(2f_c)$

Hence : 1 is filtered.

$v(t)$ = Output of Loop filter will only have component 2.

$$\Rightarrow \underline{v(t) = \tfrac{1}{2} A_c^2 \sin\left(\theta(t) - k_p b(t)\right)}$$

(b) when the Loop is phase locked $\Rightarrow \theta(t) = 0$

$$\Rightarrow v(t) = -\tfrac{1}{2} A_c^2 \sin\left(k_p b(t)\right) = \begin{cases} -\tfrac{1}{2} A_c^2 \sin(k_p)\cdot 1 & b(t) = 1 \\ -\tfrac{1}{2} A_c^2 \sin(k_p)(-1) & b(t) = -1 \end{cases}$$

$$= \boxed{b(t)\cdot\left(-\tfrac{1}{2} A_c^2 \sin(k_p)\right) \propto b(t)}$$

Hence, it is proportional to the data signal $b(t)$.

7.25

$$s_1(t) = A_c \cos\left[2\pi(f_c + \Delta f/2)t\right] \qquad 0 \le t \le T_b$$

$$s_2(t) = A_c \cos\left[2\pi(f_c - \Delta f/2)t\right] \qquad 0 \le t \le T_b$$

$$\rho = \frac{\int_0^{T_b} s_1(t)s_2(t)\,dt}{\int_0^{T_b} s_1^2(t)\,dt}.$$

$$s_1(t)s_2(t) = A_c^2 \cos\left[2\pi(f_c + \Delta f/2)t\right]\cos\left[2\pi(f_c - \Delta f/2)t\right].$$

using: $\cos(\alpha)\cos(\beta) = 1/2\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

$$s_1(t)s_2(t) = 1/2\, A_c^2\left[\cos(4\pi f_c t) + \cos(2\pi\Delta f t)\right].$$

$$\Rightarrow \int_0^{T_b} s_1(t)s_2(t)\,dt = 1/2\, A_c^2 \int_0^{T_b} \left(\cos(4\pi f_c t) + \cos(2\pi \Delta f t)\right) dt$$

assuming $f_c \gg \frac{1}{T_b} \Rightarrow \int_0^{T_b} \cos(4\pi f_c t)\,dt = 0$

$$\int_0^{T_b} s_1(t)s_2(t)\,dt = \frac{1}{2} A_c^2 \frac{1}{2\pi\Delta f} \sin(2\pi\Delta f t)\Big]_0^{T_b} = \underbrace{\frac{A_c^2}{4\pi\Delta f}\sin(2\pi\Delta f T_b)}_{\text{Numerator}}$$

$$\int_0^{T_b} s_1^2(t)\,dt = \int_0^{T_b} A_c^2 \cos^2(2\pi(f_c + \Delta f)t)\,dt =$$

$$\int_0^{T_b} 1/2\, A_c^2 \left(1 + \cos(4\pi(f_c + \Delta f)t)\right)dt = \int_0^{T_b} 1/2\, A_c^2\, dt +$$

using: $\cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2}$

$$\int_0^{T_b} \frac{A_c^2}{4}\cos(4\pi(f_c + \Delta f)t)\,dt$$

if $f_c \gg \Delta f \Rightarrow \int_0^{T_b} \frac{1}{2} A_c^2 \cos(4\pi(f_c+\Delta f)t)\, dt \simeq 0.$

$$\Rightarrow \int_0^{T_b} s_1^2(t)\, dt \simeq \int_0^{T_b} \frac{1}{2} A_c^2 dt = \underbrace{\frac{1}{2} A_c^2 T_b}_{\text{Denominator.}}$$

$$\Rightarrow \rho \simeq \frac{\frac{A_c^2}{4\pi \Delta f} \sin(2\pi \Delta f T_b)}{\frac{1}{2} A_c^2 T_b} = \frac{\sin(2\pi\Delta f T_b)}{2\pi \Delta f T_b} = \boxed{\operatorname{Sinc}(2\Delta f T_b)}$$

using: $\boxed{\frac{\sin(\pi x)}{\pi x} = \operatorname{Sinc}(x)}$

(b) $s_1(t)$ and $s_2(t)$ are orthogonal when $\rho = 0$

$\operatorname{Sinc}(2\Delta f T_b) = 0$ when $\sin(2\pi \Delta f T_b) = 0$

$$\Rightarrow 2\pi \Delta f T_b = k\pi \quad \Delta f T_b = \frac{k}{2} \Rightarrow \boxed{\Delta f = \frac{k}{2T_b}} \quad k = 1, 2, \cdots$$

The minimum value of frequency shift $\Delta f$ is when $k=1$

$$\Rightarrow \boxed{\Delta f = \frac{1}{2T_b}}$$

## 7.27

Eq. (7.23) states : $s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta(t))$

where $\theta(t) = \frac{\pi t}{2T_b}$ if symbol 1 is transmitted.

$\theta(t) = -\frac{\pi t}{2T_b}$ " " 0 " " .

Hence : $\theta(T_b) = \begin{cases} \pi/2 & \text{if } 1 \\ -\pi/2 & \text{if } 0. \end{cases}$

- The transmission of symbol 1 <u>increases</u> the phase of MSK by $\pi/2$ radians.
- The transmission of symbol 0 <u>decreases</u> the phase of MSK by $-\pi/2$ radians.

Examining Table 7.4 :

if $\theta(0) = 0$ after transmitting 0, $\theta(T_b) = -\pi/2$ ✓.

if $\theta(0) = \pi$ after transmitting 1, $\theta(T_b) = \pi + \pi/2 = 3\pi/2$

$3\pi/2$ in modulo-2 is equivalent to $-\pi/2$ ✓

$(3\pi/2 - 2\pi = -\pi/2)$

if $\theta(0) = \pi$ after transmitting 0, $\theta(T_b) = \pi - \pi/2 = \pi/2$ ✓

if $\theta(0) = 0$ after transmitting 1, $\theta(T_b) = 0 + \pi/2 = \pi/2$ ✓

$\Rightarrow$ <u>All 4 rows have been verified.</u>