2.19

(c) $T \to aT$

$$\Rightarrow G(f) = \frac{|a|AT}{2}\left[\mathrm{sinc}\left(aTf - \tfrac{1}{2}\right) + \mathrm{sinc}\left(aTf + \tfrac{1}{2}\right)\right] e^{-j\pi f aT}$$

(d) let $a = -1$, and multiply the result by (-1)

$$G(f) = -\frac{AT}{2}\left[\mathrm{sinc}\left(fT + \tfrac{1}{2}\right) + \mathrm{sinc}\left(fT - \tfrac{1}{2}\right)\right] e^{j\pi f T}$$

(e) consider as a superposition of (b) & (c) $\Rightarrow$

$$G(f) = \frac{AT}{2}\left[\mathrm{sinc}\left(fT - \tfrac{1}{2}\right) + \mathrm{sinc}\left(fT + \tfrac{1}{2}\right)\right]\left[e^{-j\pi f T} - e^{j\pi f T}\right]$$

$$= -jAT\sin(\pi f T)\left[\mathrm{sinc}\left(fT - \tfrac{1}{2}\right) + \mathrm{sinc}\left(fT + \tfrac{1}{2}\right)\right]$$

2.22

(b) $$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

take $\left(\frac{d}{df}\right)$ n times $\Rightarrow$ $$\frac{d^n G(f)}{df^n} = (-j2\pi)^n \int_{-\infty}^{\infty} t^n e^{-j2\pi f t}\, dt \quad (*)$$

$$\Rightarrow t^n g(t) \rightleftharpoons \left(\frac{j}{2\pi}\right)^n \frac{d^n G(f)}{df^n}$$

(d) let $f = 0$ in (b)(*)

$$\int_{-\infty}^{\infty} t^n g(t)\, dt = \left(\frac{j}{2\pi}\right)^n G^n(0)$$

(d) $g_2^*(t) \overset{F}{\Longleftrightarrow} G_2^*(-f)$

$$g_1(t)\, g_2^*(t) \Rightarrow \int_{-\infty}^{\infty} G_1(\tau)\, G_2^*(\tau - f)\, d\tau$$

multiplication in time → convolution in freq.

take the fourier of the left hand side

$$F[g_1(t)\, g_2^*(t)] = \int_{-\infty}^{\infty} g_1(t)\, g_2^*(t)\, e^{-j2\pi t f}\, dt$$

$$= \int_{-\infty}^{\infty} G_1(\tau)\, G_2^*(\tau - f)\, d\tau$$

let $f = 0$

$$\Rightarrow \int_{-\infty}^{\infty} g_1(t)\, g_2^*(t) = \int_{-\infty}^{\infty} G_1(\tau)\, G_2^*(\tau)\, d\tau$$

$\square$

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