Properties of Angle Modulation

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Angle Modulation

▶ Phase Modulation (PM):

\[ \theta_i(t) = 2\pi f_c t + k_p m(t) \]
\[ f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt} \]
\[ s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)] \]

▶ Frequency Modulation (FM):

\[ \theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau \]
\[ f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = f_c + k_f m(t) \]
\[ s_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right] \]

Reference:

Sections 4.2 and 4.3 of

4.2 Properties of Angle Modulation

**Constancy of Transmitted Power**

- Consider a sinusoid \( g(t) = A_c \cos(2\pi f_0 t + \phi) \) where \( T_0 = \frac{1}{f_0} \) or \( T_0 f_0 = 1 \).
- The power of \( g(t) \) (over a 1 ohm resistor) is defined as:

\[
P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t)^2 \, dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A_c^2 \cos^2(2\pi f_0 t + \phi) \, dt
\]

\[
= \frac{A_c^2}{T_0} \int_{-T_0/2}^{T_0/2} \frac{1}{2} \left[ 1 + \cos(4\pi f_0 t + 2\phi) \right] \, dt
\]

\[
= \frac{A_c^2}{2T_0} \left[ t + \sin(4\pi f_0 T_0/2 + 2\phi) \right]_{-T_0/2}^{T_0/2}
\]

\[
= \frac{A_c^2}{2T_0} \left[ \frac{T_0}{2} - \frac{T_0}{2} \right] + \frac{\sin(4\pi f_0 T_0/2 + 2\phi) - \sin(-4\pi f_0 T_0/2 + 2\phi)}{4\pi f_0}
\]

\[
= \frac{A_c^2}{2T_0} \left[ \frac{T_0}{2} + \frac{\sin(2\pi + 2\phi) - \sin(-2\pi + 2\phi)}{4\pi f_0} \right] = \frac{A_c^2}{2}
\]

- Therefore, the power of a sinusoid is **NOT** dependent on \( f_0 \), just its envelop \( A_c \).
4.2 Properties of Angle Modulation

**Constancy of Transmitted Power: PM**

Therefore, angle modulated signals exhibit constancy of transmitted power.

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**Constancy of Transmitted Power: FM**

Nonlinearity of Angle Modulation

Consider PM (proof also holds for FM).

- Suppose
  \[
  s_1(t) = A_c \cos [2\pi f_c t + k_p m_1(t)] \]
  \[
  s_2(t) = A_c \cos [2\pi f_c t + k_p m_2(t)]
  \]

- Let \( m_3(t) = m_1(t) + m_2(t) \).

\[
 s_3(t) = A_c \cos [2\pi f_c t + k_p (m_1(t) + m_2(t))] 
\neq s_1(t) + s_2(t)
\]

\[
 \therefore \cos(2\pi f_c t + A + B) \neq \cos(2\pi f_c t + A) + \cos(2\pi f_c t + B)
\]
4.2 Properties of Angle Modulation

Nonlinearity of Angle Modulation

Therefore, angle modulation is nonlinear.

Irregularity of Zero-Crossings

- Zero-crossing: instants of time at which waveform changes amplitude from positive to negative or vice versa.
4.2 Properties of Angle Modulation

Zero-Crossings: FM

Irregularity of Zero-Crossings

Therefore angle modulated signals exhibit irregular zero-crossings because they contain information about the message (which is irregular in general).

Visualization Difficulty of Message

 Visualization of a message refers to the ability to glean insights about the shape of \( m(t) \) from the modulated signal \( s(t) \).

Visualization: AM
4.2 Properties of Angle Modulation

Visualization Difficulty of Message

Therefore it is difficult to visualize the message in angle modulated signals due to the nonlinear nature of the modulation process.

Bandwidth vs. Noise Trade-Off

- Noise affects the message signal piggy-backed as amplitude modulation more than it does when piggy-backed as angle modulation.

- The more bandwidth that the angle modulated signal takes, typically the more robust it is to noise.
4.2 Properties of Angle Modulation

carrier
message
amplitude modulation
phase modulation
frequency modulation