3.4 Costas Receiver

Coherent Demodulation Output

\[ v_0(t) = \frac{1}{2} A_c A'_c \cos(\phi) m(t) \]

- To maximize \( v_0(t) \), would like \( \phi \approx 0 \).

- For \( v_0(t) \) to be proportional to \( m(t) \), \( \phi \) should be constant.

Thus, we wish to synchronize the local oscillator. Let \( A'_c = 1 \) for simplicity.
3.4 Costas Receiver

**Costas Receiver**

Coherent Demodulation

\[
\begin{align*}
\text{DSB-SC wave} & \quad A, \cos(2\pi f_c t + \phi) \\
\text{Product Modulator} & \quad A, \cos(2\pi f_c t + \phi) \\
\text{Low-pass filter} & \quad \text{Voltage-controlled Oscillator} \\
\text{Phase Shifter} & \quad -90^\circ \\
\sin(2\pi f_c t + \phi) & \quad \cos(2\pi f_c t + \phi) \\
\text{Product Modulator} & \quad \text{Low-pass filter} \\
\text{Low-pass Filter} & \quad \text{Phase Discriminator} \\
\text{Demodulated Signal} & \quad \text{Local oscillator output}
\end{align*}
\]

Goals: (1) Coherent demodulation of DSB-SC input signal.
(2) Tweak the local oscillator phase such that \( \phi = 0 \).

**Costas Receiver: Phase Lock Circuit**

\[
\begin{align*}
\text{DSB-SC wave} & \quad A, \cos(2\pi f_c t + \phi) \\
\text{Product Modulator} & \quad \text{Voltage-controlled Oscillator} \\
\text{Low-pass filter} & \quad \text{Phase Discriminator} \\
-90^\circ & \quad \sin(2\pi f_c t + \phi) \\
\text{Product Modulator} & \quad \text{Low-pass filter} \\
\text{Low-pass Filter} & \quad \text{Phase Discriminator}
\end{align*}
\]

Goals: (1) Coherent demodulation of DSB-SC input signal.
(2) Tweak the local oscillator phase such that \( \phi = 0 \).
3.4 Costas Receiver

Costas Receiver: Phase Lock Circuit

$\phi < 0$: Freq of local oscillator needs to temporarily increase

local oscillator phase lags behind carrier and must increase its frequency to catch up.
Costas Receiver: In-Phase Coherent Detector

\[ V_I(f) = \frac{e^{j\phi}}{2} S(f - f_c) + \frac{e^{-j\phi}}{2} S(f + f_c) \]

\[ = \frac{\cos(\phi) + j \sin(\phi)}{2} S(f - f_c) + \frac{\cos(-\phi) + j \sin(-\phi)}{2} S(f + f_c) \]

\[ = \frac{\cos(\phi) + j \sin(\phi)}{2} S(f - f_c) + \frac{\cos(-\phi) - j \sin(\phi)}{2} S(f + f_c) \]

\[ = \frac{\cos(\phi) + j \sin(\phi)}{2} [S(f - f_c) + S(f + f_c)] + \frac{\cos(\phi) - j \sin(\phi)}{2} [S(f - f_c) - S(f + f_c)] \]

\[ \approx \frac{1}{2} \left[ S(f - f_c) + S(f + f_c) \right] + j \sin(\phi) \frac{1}{2} \left[ S(f - f_c) - S(f + f_c) \right] \]

for \( \phi \ll 1 \).

The closer \( \phi \) is to zero, the more significant the baseband term of \( V_I(t) \) and vice versa.
Costas Receiver: In-Phase Coherent Detector

\[ s(t) = A_c \cos(2\pi f_c t) m(t) \]

\[ v_Q(t) = s(t) \cdot \sin(2\pi f_c t + \phi) \]

Recall

\[ \sin(2\pi f_c t + \phi) = \frac{e^{i\phi}}{2j} e^{2\pi f_c t} - \frac{e^{-j\phi}}{2j} e^{-2\pi f_c t} = \frac{e^{i\phi}}{2j} \delta(f - f_c) - \frac{e^{-j\phi}}{2j} \delta(f + f_c) \]

\[ V_Q(f) = S(f) \ast \left[ \frac{e^{i\phi}}{2j} \delta(f - f_c) - \frac{e^{-j\phi}}{2j} \delta(f + f_c) \right] \]

\[ = \frac{e^{i\phi}}{2j} S(f) \ast \delta(f - f_c) - \frac{e^{-j\phi}}{2j} S(f) \ast \delta(f + f_c) \]

\[ = \frac{e^{i\phi}}{2j} S(f - f_c) - \frac{e^{-j\phi}}{2j} S(f + f_c) \]

\[ \approx \frac{1}{2} \mathrm{f}_{\text{small}} \left[ S(f - f_c) + S(f + f_c) \right] \]

For \( \phi \) small.

Costas Receiver: Quadrature-Phase Detector

\[ V_Q(f) = \frac{e^{i\phi}}{2j} S(f - f_c) - \frac{e^{-j\phi}}{2j} S(f + f_c) \]

\[ = \frac{\cos(\phi) + j \sin(\phi)}{2j} S(f - f_c) - \frac{\cos(-\phi) + j \sin(-\phi)}{2j} S(f + f_c) \]

\[ = \frac{\cos(\phi) + j \sin(\phi)}{2j} S(f - f_c) - \frac{\cos(\phi) - j \sin(\phi)}{2j} S(f + f_c) \]

\[ = \cos(\phi) \left[ S(f - f_c) - S(f + f_c) \right] + j \sin(\phi) \left[ S(f - f_c) + S(f + f_c) \right] \]

for \( \phi \ll 1 \).
Costas Receiver: Quadrature-Phase Detector

\[ V_Q(f) = \frac{e^{j\phi}}{2j} S(f - f_c) - \frac{e^{-j\phi}}{2j} S(f + f_c) \]
\[ = \frac{\cos(\phi) + j \sin(\phi)}{2j} S(f - f_c) - \frac{\cos(\phi) - j \sin(\phi)}{2j} S(f + f_c) \]
\[ = \frac{\cos(\phi)}{2j} \left[ S(f - f_c) - S(f + f_c) \right] + \frac{\sin(\phi)}{2j} \left[ S(f - f_c) + S(f + f_c) \right] \]

For \( \phi \ll 1 \), the closer \( \phi \) is to zero, the more negligible the baseband term of \( v_Q(t) \) and vice versa.

For \( \phi \) small.

\[ v_I(t) \quad \text{and} \quad v_Q(t) \]

\[ v_I(t) = \frac{A_c}{2} \cos(2\pi f_c t + \phi) \]
\[ v_Q(t) = \frac{A_c}{2} \sin(2\pi f_c t + \phi) \]
3.4 Costas Receiver

**Costas Receiver: \(v_I(t)\) and \(v_Q(t)\)**

\[
A_c \cos(2\pi f_c t + \phi) m(t) = \frac{A_c}{2} \cos(\phi m(t)) + \frac{A_c}{2} \cos(\phi m(t))
\]

- **Product Modulator**
- **Low-pass Filter**
- **Phase Shifter**
- **Voltage-controlled Oscillator**
- **Phase Discriminator**

**DSB-SC wave**

\[
A_c \cos(2\pi f_c t) m(t)
\]

**Demodulated Signal**

**Spectrum**

**Costas Receiver: Low-Pass Filter**

\[
A_c \cos(2\pi f_c t + \phi) m(t) = \frac{A_c}{2} \cos(\phi m(t)) + \frac{A_c}{2} \cos(\phi m(t))
\]

- **Product Modulator**
- **Low-pass Filter**
- **Voltage-controlled Oscillator**
- **Phase Shifter**
- **Phase Discriminator**

**DSB-SC wave**

\[
A_c \cos(2\pi f_c t) m(t)
\]

**Demodulated Signal**

**Spectrum**

**Costas Receiver: Local Oscillator Control**

\[
A_c \cos(2\pi f_c t + \phi) m(t) = \frac{A_c}{2} \cos(\phi m(t)) + \frac{A_c}{2} \cos(\phi m(t))
\]

- **Product Modulator**
- **Low-pass Filter**
- **Voltage-controlled Oscillator**
- **Phase Shifter**
- **Phase Discriminator**

**DSB-SC wave**

\[
A_c \cos(2\pi f_c t) m(t)
\]

**Demodulated Signal**

**Spectrum**
Costas Receiver: Phase Discriminator

Two components in sequence:

(1) multiplier

\[ v_0(t) \cdot v'_0(t) = \frac{A_c}{2} \cos(\phi) m(t) \cdot \frac{A_c}{2} \sin(\phi) m(t) \]

\[ = \frac{A_c^2}{4} \cos(\phi) \sin(\phi) m^2(t) \approx \frac{A_c^2}{4} \phi m^2(t) \]

for \( \phi \ll 1 \).

Costas Receiver: Voltage-controlled Oscillator

▶ If \( g(t) > 0 \) (or \( \phi > 0 \)), then the local oscillator will decrease from \( f_c \) proportional to the value of \( g(t) \) (or \( \phi \)).

▶ If \( g(t) < 0 \) (or \( \phi < 0 \)), then the local oscillator will increase from \( f_c \) proportional to the value of \( g(t) \) (or \( \phi \)).
### 3.4 Costas Receiver

**Costas Receiver: Voltage-controlled Oscillator**

- $\phi > 0$: Freq of local oscillator needs to temporarily decrease
- $\phi < 0$: Freq of local oscillator needs to temporarily increase

![Diagram of Costas Receiver](image)

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### 3.5 Quadrature-Carrier Multiplexing

**Multiplexing and QAM**

**Multiplexing**: to send multiple message simultaneously

**Quadrature Amplitude Multiplexing (QAM)**: (a.k.a quadrature-carrier multiplexing) amplitude modulation scheme that enables two DSB-SC waves with independent message signals to occupy the same channel bandwidth (i.e., same frequency channel) yet still be separated at the receiver.
3.5 Quadrature-Carrier Multiplexing

QAM: Transmitter

Message signal \( m_1(t) \)

Product Modulator

\[ s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t) \]

-90 degree Phase Shifter

Product Modulator

Message signal \( m_2(t) \)

QAM: Receiver

Product Modulator

Low-pass filter

\[ \frac{1}{2} A_c A'_c m_1(t) \]

-90 degree Phase Shifter

Product Modulator

Low-pass filter

\[ \frac{1}{2} A_c A'_c m_2(t) \]

Costas receiver may be used to synchronize the local oscillator for demodulation.