

- 1.1 (a) One dimensional, multichannel, discrete time and digital.  $T_1$   
 This signal is generated by multiple sources at certain specific time. Thus it is discrete-time and multichannel signal. Since this discrete-time signal has a set of discrete values, it is digital. Because it is also a function of a single independent variable, it is one-dimensional signal.
- (b) Multi-dimensional, single channel, continuous time and analog.
- (c) One-dimensional, single channel, continuous time and analog.
- (d) One-dimensional, single channel, continuous time and analog.
- (e) One-dimensional, ~~single~~ multichannel, discrete time and digital.

1.3 (a)  $x_a(t) = 3 \cos(5t + \frac{\pi}{6})$

It is periodic, and the period  $T = \frac{1}{F} = \frac{1}{\frac{5}{2\pi}} = \frac{2\pi}{5}$

(b)  $x(n) = 3 \cos(5n + \frac{\pi}{6})$

Since  $f = \frac{5}{2\pi}$  is irrational, it is non-periodic.

(c)  $x(n) = 2 \exp[j(\frac{n}{8} - \pi)]$

Since  $f = \frac{1}{8} = \frac{1}{12\pi}$  is irrational, it is non-periodic.

(d) 
$$\begin{aligned} x(n) &= \cos(\frac{n}{8}) \cos(\frac{\pi n}{8}) \\ &= \frac{1}{2} [\cos(\frac{n}{8} + \frac{\pi n}{8}) + \cos(-\frac{n}{8} + \frac{\pi n}{8})] \\ &= \frac{1}{2} [\cos(\frac{(\pi+1)n}{8}) + \cos(\frac{(\pi-1)n}{8})] \end{aligned}$$

$= x_1(n) + x_2(n)$ , where  $x_1(n) = \frac{1}{2} \cos(\frac{(\pi+1)n}{8})$  and  $x_2(n) = \frac{1}{2} \cos(\frac{(\pi-1)n}{8})$ .

Since the frequency for  $x_1(n)$  is  $f_1 = \frac{(\pi+1)}{8} = \frac{(\pi+1)}{16\pi}$  is irrational,  $x(n)$  is non-periodic. Therefore  $x(n)$  is non-periodic.

(e)  $x(n) = \cos(\frac{\pi n}{2}) - \sin(\frac{\pi n}{8}) + 3 \cos(\frac{\pi n}{4} + \frac{\pi}{3})$

Since  $\cos(\frac{\pi n}{2})$  is periodic with the period  $N_{p_1} = \frac{1}{f_1} = \frac{1}{\frac{\pi/2}{2\pi}} = 4$ ,  $\sin(\frac{\pi n}{8})$  is periodic with period  $N_{p_2} = \frac{1}{f_2} = \frac{1}{\frac{\pi/8}{2\pi}} = 16$ , and  $3 \cos(\frac{\pi n}{4} + \frac{\pi}{3})$  is periodic with period  $N_{p_3} = \frac{1}{f_3} = \frac{1}{\frac{\pi/4}{2\pi}} = 8$ ,  $x(n)$  is periodic with the period  $N_p = 16$ , which is the least common multiple of  $N_{p_1}$ ,  $N_{p_2}$  and  $N_{p_3}$ .

1.4 (a)  $S_k(n) = e^{j2\pi kn/N} = e^{j2\pi k_1 n/N_1}$ , where  $k_1 = \frac{k}{\text{GCD}(k,N)}$  and  $N_1 = \frac{N}{\text{GCD}(k,N)}$ .

$S_k(n+N_p) = e^{j2\pi n(k_1+N_p)/N_1} = e^{j2\pi n N_p/N_1} S_k(n)$

In order to ensure  $S_k(n+N_p) = S_k(n)$ ,  $e^{j2\pi n N_p/N_1} = 1$ .

Therefore the fundamental period  $N_p = N_1 = \frac{N}{\text{GCD}(k,N)}$ .

(b) For  $N=7$ ,  $k=0, 1, 2, 3, 4, 5, 6, 7$ .

Thus  $\text{GCD}(k,N) = 7, 1, 1, 1, 1, 1, 1, 7$ .

Thus  $N_p = \frac{N}{\text{GCD}(k,N)} = 1, 7, 7, 7, 7, 7, 7, 1$ .

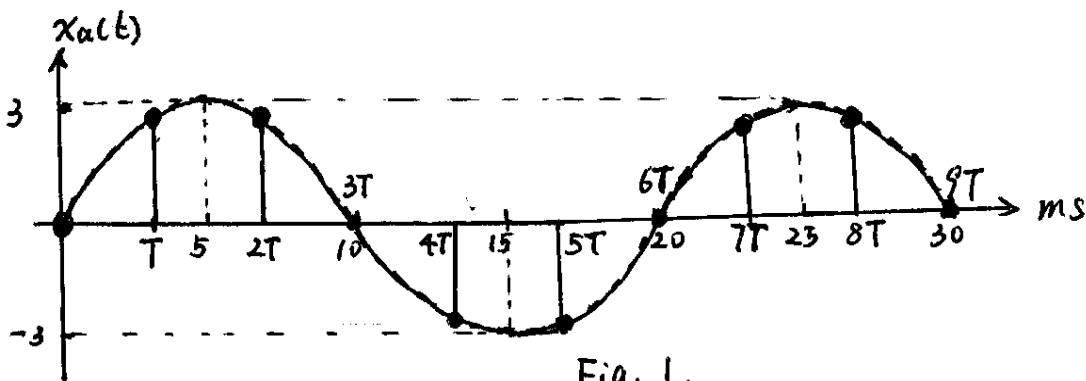
(c) For  $N=16$ ,  $k=0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16$ .

Thus  $\text{GCD}(k,N) = 16, 1, 2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1, 2, 1, 16$ .

Thus  $N_p = \frac{N}{\text{GCD}(k,N)} = 1, 16, 8, 16, 4, 16, 8, 16, 2, 16, 8, 16, 4, 16, 8, 16, 1$ .

1.5 (a)  $T = \frac{1}{F} = \frac{1}{\frac{100\pi}{2\pi}} = \frac{1}{50} \text{ s} = 20 \text{ ms}$ .

We can sketch the signal  $x_a(t)$  for  $0 \leq t \leq 30 \text{ ms}$  as follows:



(b)  $f = \frac{F}{F_s} = \frac{50}{300} = \frac{1}{6}$

$T = \frac{1}{f} = \frac{1}{300} \text{ s}$

Since the frequency  $f = \frac{1}{6}$  is rational,  $x(n)$  is periodic.

(c)  $x(n) = x_a(nT) = 3 \sin(100\pi nT) = 3 \sin\left(\frac{\pi n}{3}\right)$

$T_a = \frac{1}{f_a} = \frac{1}{\frac{\pi/3}{2\pi}} = 6$

$x(0) = 0$ ,  $x(1) = \frac{3\sqrt{3}}{2}$ ,  $x(2) = \frac{3\sqrt{3}}{2}$ ,  $x(3) = 0$ ,  $x(4) = -\frac{3\sqrt{3}}{2}$ ,  $x(5) = -\frac{3\sqrt{3}}{2}$ ,

$x(n)$  is sketched on Fig. 1. with black color. The period of  $x(n)$  is 20 ms.

(d)  $x(n) = x_a(nT) = x_a\left(\frac{n}{F_s}\right) = 3 \sin\left(\frac{100\pi n}{F_s}\right)$  P3

$$3 \sin\left(\frac{100\pi n}{F_s}\right) = 3 \Rightarrow \frac{100\pi n}{F_s} = \frac{\pi}{2} \Rightarrow F_s = 200 n.$$

Therefore the minimum value for  $F_s$  is 200 samples/sec.