2.10.

Since the system is time-invariant, we have $x_3(n+1) \xleftrightarrow{T} y_3(n+1)$.
Thus we have $x_3(n+1) = \{\underset{\uparrow}{0}, 0, 1\} \xleftrightarrow{T} y_3(n+1) = \{1, 2, \underset{\uparrow}{1}\}$.

Since $x_2(n) = 3x_3(n+1)$, we have that if the system is time-invariant, it has to satisfy that $x_2(n) = 3x_3(n+1) \xleftrightarrow{T} 3y_3(n+1) = \{3, 6, \underset{\uparrow}{3}\}$.
However, $y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\} \neq \{3, 6, \underset{\uparrow}{3}\}$.

Therefore, this system is nonlinear.
Since the system is time-invariant, we can also have
$x_3(n+3) = \{\underset{\uparrow}{1}\} = \delta(n) \xleftrightarrow{T} y_3(n+3) = \{1, 2, 1, 0, \underset{\uparrow}{0}\}$

Therefore, the impulse response of the system is $\{1, 2, 1, 0, \underset{\uparrow}{0}\}$.

2.11

Since the system is linear, we have that $\delta(n) = x_1(n) + x_2(n) \xleftrightarrow{T} y_1(n) + y_2(n)$

Thus the impulse response of the system is $h(n) = y_1(n) + y_2(n)$

$$= \{0, \underset{\uparrow}{3}, -1, 2, 1\}.$$

We can get $x_3(n) = \delta(n) + \delta(n+1)$.

If the system is time-invariant, we have $\delta(n+1) \xleftrightarrow{T} h(n+1) = \{0, 3, \underset{\uparrow}{-1}, 2, 1\}$

Thus we should be able to get as follows:

$x_3(n) = \delta(n) + \delta(n+1) \xleftrightarrow{T} h(n) + h(n+1) = \{\underset{\uparrow}{3}, 2, 1, 3, 1\}$.

However, $y_3(n) = \{\underset{\uparrow}{1}, 2, 1\} \neq \{\underset{\uparrow}{3}, 2, 1, 3, 1\}$

Therefore, the system is time-varying.

2.23.

$$y(n) = h(n) * x(n)$$
$$= h(n) * \delta(n) * x(n)$$
$$= h(n) * [u(n) - u(n-1)] * x(n)$$
$$= [h(n) * u(n) - h(n) * u(n-1)] * x(n) \quad ①$$

Let $s(n) = h(n) * u(n)$.
Since the system is time-invariant, we have:
$$h(n) * u(n-1) = s(n-1).$$

Therefore $y(n) = ① = [s(n) - s(n-1)] * x(n)$
$$= s(n) * x(n) - s(n-1) * x(n)$$

2.29 In this problem, we consider $M \geqslant N > 0$. This is reasonable.

$$h(n) = h_1(n) * h_2(n)$$
$$= \{a^n[u(n)-u(n-N)]\} * [u(n)-u(n-M)]$$
$$= \sum_{k=-\infty}^{\infty} a^k [u(k)-u(k-N)][u(n-k)-u(n-k-M)]$$
$$= \sum_{k=-\infty}^{\infty} a^k u(k)u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k)u(n-k-M)$$
$$- \sum_{k=-\infty}^{\infty} a^k u(k-N)u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N)u(n-k-M) \quad ①$$

(1) If $n<0$

$$① = 0-0-0+0 = 0$$

(2) If $0 \leqslant n < N$

$$① = \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$$

(3) If $N \leq n < M$

$$① = \sum_{k=0}^{n} a^k - \sum_{k=N}^{n} a^k = \sum_{k=0}^{N-1} a^k = \frac{1-a^N}{1-a}$$

(4) If $M \leq n < M+N$

$$① = \sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k - \sum_{k=N}^{n} a^k = \frac{1-a^N}{1-a} - \frac{1-a^{n-M+1}}{1-a} = \frac{a^{n-M+1}-a^N}{1-a}$$

(5) If $n \geqslant M+N$

$$① = \sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k - \sum_{k=N}^{n} a^k + \sum_{k=N}^{n-M} a^k$$
$$= \sum_{k=n-M+1}^{n} a^k - \sum_{k=n-M+1}^{n} a^k$$
$$= 0$$

2-31

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

Since the particular solution $y_p = 0$ when the excitation is an impulse, we have the impulse response $h(n) = y_h(n)$ for $n \geq 0$.

By assuming $y_h(n) = \lambda^n$, we obtain the characteristic equation as follows: $\lambda^2 - 3\lambda - 4 = 0$. Therefore, $\lambda = -1, 4$ and $y_h(n) = C_1(-1)^n + C_2(4)^n$ ①

Since the system must be relaxed, we have $y(-1) = 0$ and $y(-2) = 0$.

Thus for $n = 0, 1$, $x(n) = \delta(n)$, we have

$$\begin{cases} y(0) = 1 \text{ ②} \\ y(1) - 3y(0) = 2 \overset{②}{\Rightarrow} y(1) = 5 \text{ ③} \end{cases}$$

By using Eqs. ① ② and ③ we have

$$\begin{cases} C_1 + C_2 = 1 \\ 4C_2 - C_1 = 5 \end{cases} \Rightarrow \begin{cases} C_1 = -\frac{1}{5} \\ C_2 = \frac{6}{5} \end{cases}$$

Therefore $h(n) = [\frac{6}{5} 4^n - \frac{1}{5}(-1)^n] u(n)$.

2.35

(a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$

(b) $h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$h_3(n) * h_4(n) = (n+1)u(n) * \delta(n-2) \underset{\uparrow}{=} (n-1)u(n-2)$

shifting property

$h_2(n) - h_3(n) * h_4(n) = (n+1)u(n) - (n-1)u(n-2)$

$= (n-1)[u(n) - u(n-2)] + 2u(n)$

$= (n-1)[\delta(n) + \delta(n-1)] + 2u(n)$

$= (n-1)\delta(n) + (n-1)\delta(n-1) + 2u(n)$

$= -\delta(n) + 0 + 2u(n)$

$= 2u(n) - \delta(n)$

$h(n) = [\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)] * [2u(n) - \delta(n)]$

$= u(n) + \frac{1}{2}u(n-1) + u(n-2) - \frac{1}{2}\delta(n) - \frac{1}{4}\delta(n-1) - \frac{1}{2}\delta(n-2)$

Identity and shifting property

$= [u(n) - \delta(n)] + \frac{1}{2}\delta(n) + \frac{1}{2}[u(n-1) - \delta(n-1)] + \frac{1}{4}\delta(n-1) + [u(n-2) - \delta(n-2)] + \frac{1}{2}\delta(n-2)$

$= u(n-1) + \frac{1}{2}u(n-2) + u(n-3) + \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$= [u(n-1) - \delta(n-1)] + \frac{5}{4}\delta(n-1) + \frac{1}{2}[u(n-2) - \delta(n-2)] + \delta(n-2) + u(n-3) + \frac{1}{2}\delta(n)$

$= u(n-2) + \frac{1}{2}u(n-3) + u(n-3) + \frac{5}{4}\delta(n-1) + \delta(n-2) + \frac{1}{2}\delta(n)$

$= [u(n-2) - \delta(n-2)] + 2\delta(n-2) + \frac{3}{2}u(n-3) + \frac{5}{4}\delta(n-1) + \frac{1}{2}\delta(n)$

$= \frac{5}{2}u(n-3) + \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2)$

(c) $y(n) = x(n) * h(n) = h(n+2) + 3h(n-1) - 4h(n-3)$

$= \{\frac{1}{2}, \frac{5}{4}, \underset{\uparrow}{2}, 4, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \cdots\}$

2.37

$$y(n) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-k)$$

$$= \frac{1}{M}\sum_{k=-\infty}^{\infty}[u(k) - u(k-M)]\, x(n-k)$$

$$= \frac{1}{M}[u(n) - u(n-M)] * x(n)$$

Thus the impulse response is $h(n) = \frac{1}{M}[u(n) - u(n-M)]$.

Therefore the step response is as follows:

$$s(n) = h(x) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, u(n-k)$$

$$= \sum_{k=0}^{n} h(k) = \frac{1}{M}\left[\sum_{k=0}^{n} u(k) - \sum_{k=0}^{n} u(k-M)\right]$$

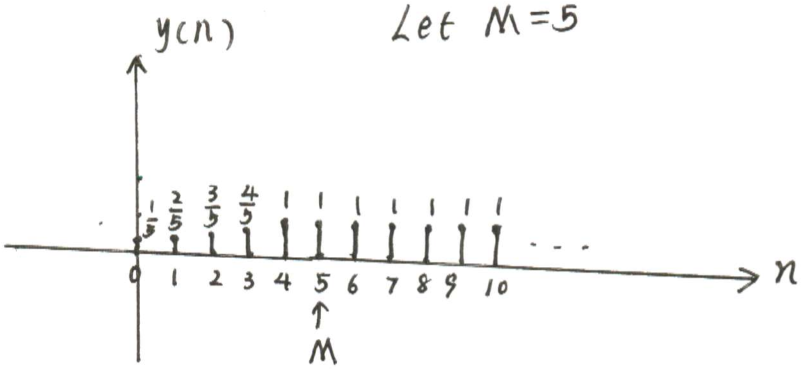
If $0 \le n < M$, $\sum_{k=0}^{n} h(k) = \frac{1}{M}\sum_{k=0}^{n} u(k) = \frac{n+1}{M}$

If $n \geq M$ $\quad \sum_{k=0}^{n} h(k) = \frac{1}{M}\left[\sum_{k=0}^{n} u(k) - \sum_{k=M}^{n} u(k-M)\right]$

$$= \frac{1}{M}[(n+1) - (n-M+1)]$$

$$= 1$$

Let $M=5$



2-38

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,\, even}^{\infty} |a|^n$$

$$= \sum_{k=0}^{\infty} |a|^{2k}$$

$$= \lim_{k \to \infty} \frac{1-|a|^{2k+2}}{1-|a|^2}, \text{ where } k = \tfrac{1}{2}n$$

Therefore in order to ensure $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$, a needs to satisf

$|a| < 1$.

2.41.

$$h(n) = (\tfrac{1}{2})^n u(n)$$

(a) $x(n) = 2^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} (\tfrac{1}{2})^k u(k)\, 2^{n-k} u(n-k)$$

$$= \begin{cases} \sum_{k=0}^{n} (\tfrac{1}{2})^k 2^{n-k} & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

$$= \tfrac{1}{3} u(n) (2^{n+2} - 2^{-n})$$

(b) $x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} (\tfrac{1}{2})^k u(k)\, u(k-n)$$

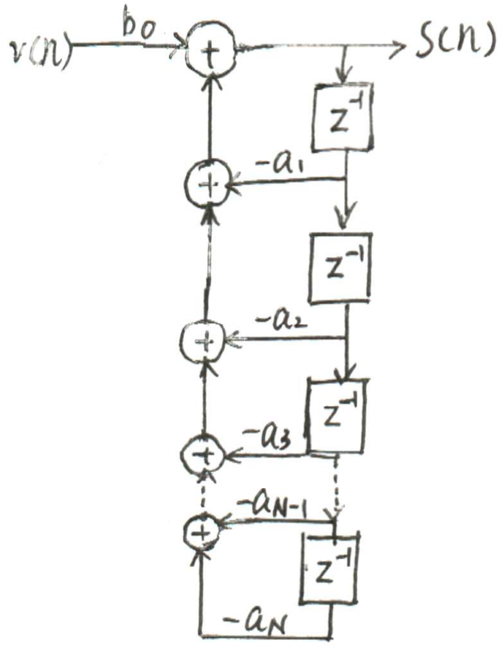
If $n<0$, $y(n) = \sum_{k=0}^{\infty} (\tfrac{1}{2})^k = \frac{1}{1-\frac{1}{2}} = 2$

If $n \geq 0$ $y(n) = \sum_{k=n}^{\infty} (\tfrac{1}{2})^k = \frac{(\frac{1}{2})^n}{1-\frac{1}{2}} = 2^{(1-n)}$

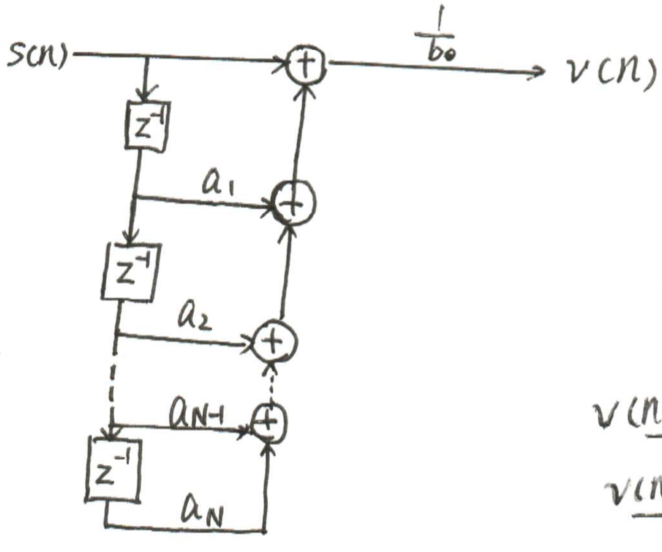
2.44

$$s(n) + a_1 s(n-1) + \cdots + a_N s(n-N) = b_0 v(n)$$

(a) $s(n) = -a_1 s(n-1) - a_2 s(n-2) - \cdots - a_N s(n-N) + b_0 v(n)$




(b) $v(n) = \frac{1}{b_0}[s(n) + a_1 s(n-1) + \cdots + a_N s(n-N)]$



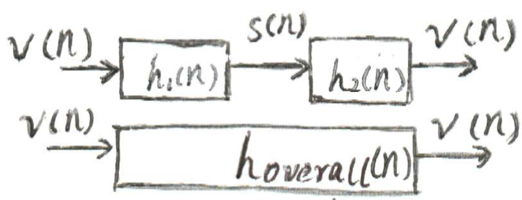



Fig. 1

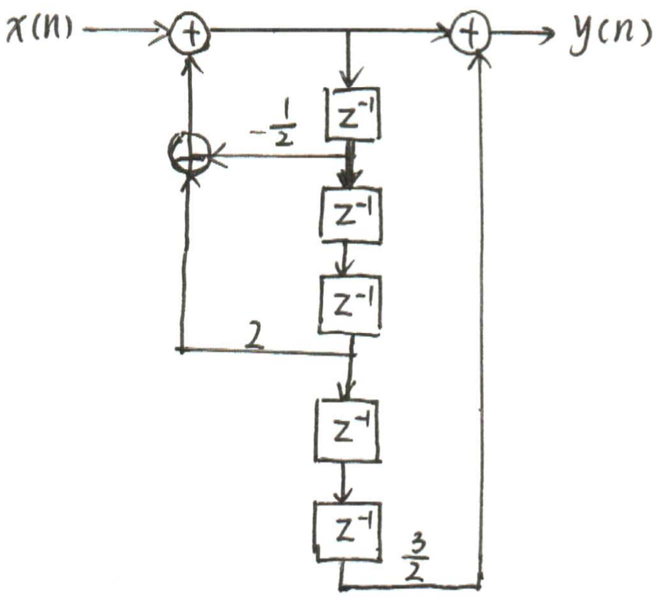
(C) The system can be described as $v(n) = h(n) * v(n)$, this is an identity system. Therefore the impulse response is $h(n) = \delta(n)$

Note: this procedure can be illustrated in Fig. 1.

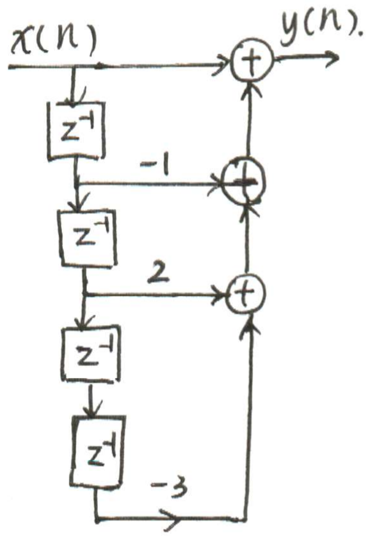
2.46.

(a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$

$$y(n) = -\frac{1}{2}y(n-1) + 2y(n-3) + \frac{1}{2}x(n) + \frac{3}{2}x(n-5)$$

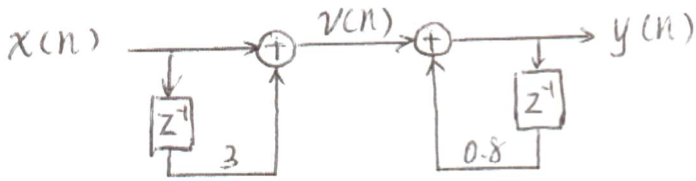



(b) $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$



2.49 $y(n) = 0.8\, y(n-1) + 3x(n-1) + 2x(n)$, $y(n) - 0.8\, y(n-1) = 2x(n) + 3x(n-1)$

(a)



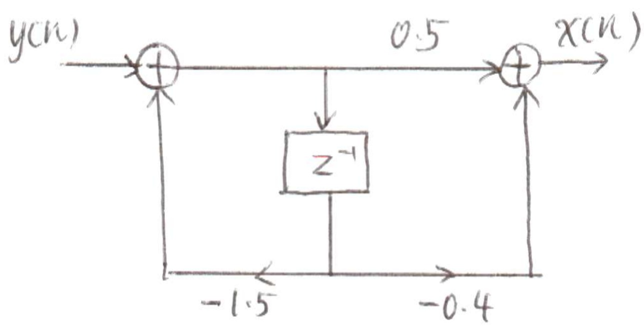

$$\begin{cases} v(n) = 2x(n) + 3x(n-1) \Rightarrow h_1(n) = 2\delta(n) + 3\delta(n-1) \\ y(n) = v(n) + 0.8\, y(n-1) \end{cases}$$

| $n$ | $v(n) = \delta(n)$ | $y(n-1)$ | $y(n)$ |
|---|---|---|---|
| $-2$ | 0 | 0 | 0 |
| $-1$ | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0.8 |
| 2 | 0 | 0.8 | $(0.8)^2$ |
| 3 | 0 | $(0.8)^2$ | $(0.8)^3$ |
| ⋮ | ⋮ | ⋮ | ⋮ |

$$h_2(n) = (0.8)^n u(n)$$

$$h_{overall}(n) = h_1(n) * h_2(n) = 3(0.8)^{n-1} u(n-1) + 2(0.8)^n u(n).$$

(b) $x(n) = -1.5\, x(n-1) + 0.5\, y(n) - 0.4\, y(n-1)$



2.52

(a) $h_1(n) = C_0\delta(n) + C_1\delta(n-1) + C_2\delta(n-2)$

$h_2(n) = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2)$

$h_3(n) = a_0\delta(n) + (a_1 + a_0a_2)\delta(n-1) + a_1a_2\delta(n-2)$

(b) In order to ensure $h_1(n) = h_2(n)$, we need to assign

$$\begin{cases} b_2 = C_0 \\ b_1 = C_1 \\ b_0 = C_2 \end{cases}$$ , and this is easily to be realized.

In order to ensure $h_1(n) = h_3(n)$, we need to assign

$$\begin{cases} a_0 = C_0 \\ a_1 + a_0a_2 = C_1 \\ a_1a_2 = C_2 \end{cases}$$

Therefore $\begin{cases} a_1 + C_0a_2 = C_1 \\ a_1a_2 = C_2 \end{cases}$

we can further get $C_0a_2^2 - C_1a_2 + C_2 = 0$.

Thus if $C_0 \neq 0$, if and only if $\Delta = C_1^2 - 4C_0C_2 \geqslant 0$, it can be satisfied.