

3.1

$$(a) \quad x(n) = \{3, 0, 0, 0, 0, 6, 1, -4\}$$

↑

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x(n) z^{-n} \\ &= 3 \cdot z^{-(-5)} + 6 \cdot z^0 + 1 \cdot z^{-1} + (-4) \cdot z^{-2} \\ &= 3z^5 + 6 + z^{-1} - 4z^{-2} \end{aligned}$$

$$\text{ROC: } 0 < |z| < \infty$$

$$(b) \quad x(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 5 \\ 0, & n \leq 4 \end{cases}$$

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x(n) z^{-n} \\ &= \sum_{n=5}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} \\ &= \sum_{n=5}^{\infty} \left(\frac{1}{2z}\right)^n \end{aligned}$$

$$\text{Thus the ROC: } \left|\frac{1}{2z}\right| < 1 \Rightarrow |z| > \frac{1}{2}$$

$$\text{Then } X(z) = \frac{\left(\frac{1}{2z}\right)^5}{1 - \frac{1}{2z}} = \left(\frac{1}{32}\right) \frac{z^{-5}}{1 - \frac{1}{2}z^{-1}}$$

3.3.

P₂

$$(a) x_1(n) = \begin{cases} \left(\frac{1}{3}\right)^n & n \geq 0 \\ \left(\frac{1}{2}\right)^{-n} & n < 0 \end{cases}$$

$$\begin{aligned} X_1(z) &= \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} + \sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^{-n} z^{-n} \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n + \sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^m z^m \quad \Leftarrow \text{Let } m = -n. \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n + \sum_{m=1}^{\infty} \left(\frac{z}{2}\right)^m \end{aligned}$$

Thus ROC: $\left|\frac{1}{3z}\right| < 1$ & $\left|\frac{z}{2}\right| < 1 \Rightarrow \frac{1}{3} < |z| < 2$

$$X_1(z) = \frac{1}{1 - \frac{1}{3z}} + \frac{\frac{z}{2}}{1 - \frac{z}{2}} = \frac{\frac{5}{6}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{2}z)}$$

$$(b) x_2(n) = \begin{cases} \left(\frac{1}{3}\right)^n - 2^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$\begin{aligned} X_2(z) &= \sum_{n=-\infty}^{\infty} x_2(n) z^{-n} \\ &= \sum_{n=0}^{\infty} \left[\left(\frac{1}{3}\right)^n - 2^n\right] z^{-n} \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} - \sum_{n=0}^{\infty} 2^n z^{-n} \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n \end{aligned}$$

Thus ROC: $\left|\frac{1}{3z}\right| < 1$ & $\left|\frac{2}{z}\right| < 1 \Rightarrow 2 < |z|$

$$X_2(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} - \frac{1}{1 - 2z^{-1}} = \frac{-\frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}$$

3.3.

B

$$(c) \quad x_3(n) = x_1(n+4)$$

Thus $X_3(z) = X_1(z)z^4 \Leftarrow$ Time Shifting

Thus ROC of $X_3(z)$ is the same as that of $X_1(z)$, which is $\frac{1}{3} < |z| < 2$.

$$X_3(z) = X_1(z)z^4 = \frac{\frac{5}{6}z^4}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{2}z)}$$

$$(d) \quad x_4(n) = x_1(-n)$$

Thus $X_4(z) = X_1(z^{-1}) \Leftarrow$ Time - Reversal

$$= \frac{\frac{5}{6}}{(1-\frac{1}{3}z)(1-\frac{1}{2}z^{-1})}$$

$$\text{ROC: } \frac{1}{2} < |z| < 3$$

3.5 Let the two-sided sequence $x(n)$ has the z-transform with ROC^{P4} as $r_1 < |z| < r_2$, where $r_1, r_2 \geq 0$

Let the right-sided sequence $x_r(n) = \begin{cases} x(n) & \text{if } n \geq n_0 \\ 0 & \text{otherwise} \end{cases}$

the left-sided sequence $x_l(n) = \begin{cases} x(n) & \text{if } n \leq n_0 \\ 0 & \text{otherwise} \end{cases}$

the finite-duration two-sided sequence $x_f(n) = \begin{cases} x(n) & \text{if } n_0 \leq n \leq n_1 \\ 0 & \text{otherwise} \end{cases}$
where $n_0 < 0 < n_1$.

(a) If $n_0 \geq 0$, then $X_r(z) = \sum_{n=-\infty}^{\infty} x_r(n) z^{-n} = \sum_{n=n_0}^{\infty} x(n) z^{-n}$

Thus ROC is $|z| > r_1$

If $n_0 < \infty$ then $X_r(z) = \sum_{n=-\infty}^{\infty} x_r(n) z^{-n} = \sum_{n=n_0}^{\infty} x(n) z^{-n}$
 $= \underbrace{\sum_{n=0}^{\infty} x(n) z^{-n}}_{\textcircled{1}} + \underbrace{\sum_{n=n_0}^{-1} x(n) z^{-n}}_{\textcircled{2}}$

$\textcircled{1}$ converges for $|z| > r_1$, and $\textcircled{2}$ converges for $|z| < \infty$

Therefore ROC is $r_1 < |z| < \infty$

(b) $X_l(z) = \sum_{n=-\infty}^{\infty} x_l(n) z^{-n} = \sum_{n=-\infty}^{n_0} x(n) z^{-n}$

If $n_0 \leq 0$, then ROC is $|z| < r_2$

If $n_0 > 0$, then $X_l(z) = \underbrace{\sum_{n=-\infty}^0 x(n) z^{-n}}_{\textcircled{1}} + \underbrace{\sum_{n=1}^{n_0} x(n) z^{-n}}_{\textcircled{2}}$

$\textcircled{1}$ converges for $|z| < r_2$, and $\textcircled{2}$ converges for $|z| > 0$.

Thus ROC: $0 < |z| < r_2$.

(c) $X_f(z) = \sum_{n=-\infty}^{\infty} x_f(n) z^{-n} = \sum_{n=n_0}^{n_1} x(n) z^{-n} = \underbrace{\sum_{n=n_0}^0 x(n) z^{-n}}_{\textcircled{1}} + \underbrace{\sum_{n=1}^{n_1} x(n) z^{-n}}_{\textcircled{2}}$

$\textcircled{1}$ converges for $|z| < \infty$, and $\textcircled{2}$ converges for $|z| > 0$

Thus ROC is: $0 < |z| < \infty$.

$$3.6. \quad y(n) = \sum_{k=-\infty}^n x(k) \quad (5)$$

$$\text{Thus } y(n) - y(n-1) = \sum_{k=-\infty}^n x(k) - \sum_{k=-\infty}^{n-1} x(k) = x(n).$$

Based on "time shifting", let $y(n) \xrightarrow{z} Y(z)$, we have $y(n-1) \xrightarrow{z} z^{-1}Y(z)$

$$\text{Thus } y(n) - y(n-1) \xrightarrow{z} Y(z) - z^{-1}Y(z).$$

Since $y(n) - y(n-1) = x(n)$ and $x(n) \xrightarrow{z} X(z)$,

$$\text{we have } Y(z) - z^{-1}Y(z) = X(z).$$

$$\text{Therefore } Y(z) = \frac{X(z)}{1 - z^{-1}}$$