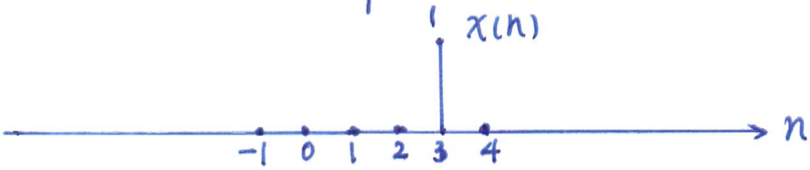
2.2

(e) $x(n-1)\delta(n-3)$

$$\delta(n-3)=\{\cdots \underset{\uparrow}{0}, 0, 0, 1, 0 \cdots\}$$

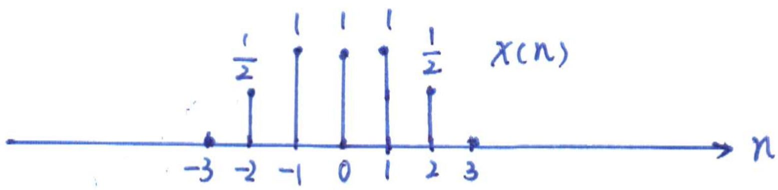
$$x(n-1)=\{\cdots 0, 0, \underset{\uparrow}{1}, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0 \cdots\}$$

$$x(n-1)\delta(n-3)=\{\cdots \underset{\uparrow}{0}, 0, 0, 1, 0, \cdots\}$$



(f) $x(n^2)=\{\cdots 0, x(4), x(1), x(0), x(1), x(4), 0, \cdots\}$

$$=\{\cdots 0, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, 0 \cdots\}$$



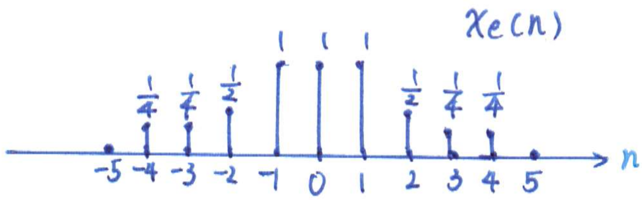
(g)(h) $x_e(n)=\frac{1}{2}[x(n)+x(-n)]$

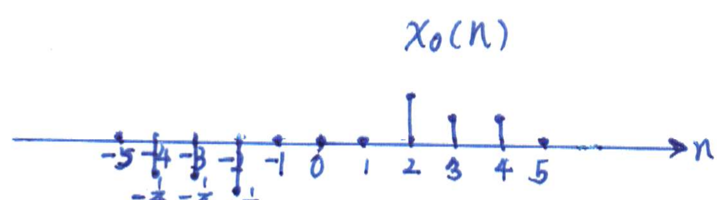
$$x_o(n)=\frac{1}{2}[x(n)-x(-n)]$$

$$x(-n)=\{\cdots 0, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0, \cdots\}$$

So $x_e(n)=\{\cdots 0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0, \cdots\}$

$$x_o(n)=\{\cdots 0, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{2}, 0, \underset{\uparrow}{0}, 0, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0, \cdots\}$$





2.3

(a) $u(n) \equiv \begin{cases} 1, & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases} = \begin{cases} 1, & \text{for } n > 0 \\ 1, & \text{for } n = 0 \\ 0, & \text{for } n < 0 \end{cases}$

Thus $u(n-1) = \begin{cases} 1, & \text{for } n \geq 1 \\ 0, & \text{for } n < 1 \end{cases} = \begin{cases} 1, & \text{for } n > 0 \\ 0, & \text{for } n = 0 \\ 0, & \text{for } n < 0 \end{cases}$

Thus $u(n) - u(n-1) = \begin{cases} 0, & \text{for } n > 0 \\ 1, & \text{for } n = 0 \\ 0, & \text{for } n < 0 \end{cases} = \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases} = \delta(n)$

(b) $\delta(n) \equiv \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases}$

Thus $\sum_{k=-\infty}^{n} \delta(k) = \begin{cases} 0, & \text{for } n < 0 \\ 1, & \text{for } n \geq 0 \end{cases} = u(n)$

Let $m = n-k$, we can get $\sum_{k=0}^{\infty} \delta(n-k) = \sum_{m=-\infty}^{n} \delta(m)$.

Based on the result we get above, we have $\sum_{m=-\infty}^{n} \delta(m) = u(n)$.

Therefore $\sum_{k=0}^{\infty} \delta(n-k) = u(n)$.

2.4 (1) Let $x(n)$ be any signal, then we have:

$$x(n) = \frac{1}{2}[x(n)+x(-n)] + \frac{1}{2}[x(n)-x(-n)] = x_1(n)+x_2(n),$$

where $x_1(n) = \frac{1}{2}[x(n)+x(-n)]$, and $x_2(n)=\frac{1}{2}[x(n)-x(-n)]$.

$x_1(-n)= \frac{1}{2}[x(-n)+x(n)] = x_1(n)$, therefore $x_1(n)$ is even.

$x_2(-n) = \frac{1}{2}[x(-n)-x(n)] = -\frac{1}{2}[x(n)-x(-n)]$, therefore $x_2(n)$ is odd.

Thus any signal can be decomposed into an even and an odd component.

(2) The decomposition is unique. The proof is as follows:

Assume that there exits another different decomposition $x(n)=x_3(n)+x_4(n)$ such that $x_3(n)$ is even and $x_4(n)$ is odd.

Defining $a(n)= x_3(n)-x_1(n)$, we have $x_4(n)-x_2(n)=-a(n)$, and $a(n)$ is not a zero function.

Since $x_1(n)$ and $x_3(n)$ are both even, we have as follows:

$a(-n) = x_3(-n) - x_1(-n) = x_3(n) - x_1(n) = a(n)$, thus $a(n)$ is even.

Similarly, we can get
$$\begin{aligned} a(-n) &= x_4(-n)-x_2(-n) \\ &= [-x_4(n)]-[-x_2(n)] \\ &= x_2(n)-x_4(n) \\ &= -[x_4(n)-x_2(n)] \\ &= -a(n). \end{aligned}$$
Thus $a(n)$ is odd.

Since $a(n)$ is not a zero function, it is impossible to be both even and odd. Therefore, there is a contradiction and we have proved that the decomposition is unique.

(3). $x[n] = [2,3,\underset{\uparrow}{4},5,6]$

$x[-n] = [6,5,\underset{\uparrow}{4},3,2]$

Thus the even component $x_1[n]=\frac{1}{2}[x(n)+x(-n)]=[4,4,\underset{\uparrow}{4},4,4]$,

and the odd component $x_2(n)=\frac{1}{2}[x(n)-x(-n)] = [-2,-1,\underset{\uparrow}{0},1,2]$.

2-7. P_7

(a) $y(n) = \cos[x(n)]$

(1) Since $y(n)$ depends only on $x(n)$, it is static.

(2) Since $\cos[a_1 x_1(n)] \neq a_1 \cos[x(n)]$ where $a_1 \in \mathbb{R}$, we can get $y(n)$ is not Homogeneity, and thus it is nonlinear.

(3) $\cos[x(n-k)] = y(n-k)$. Thus it is time-invariant.

(4) Since $y(n)$ only depends on present input, it is causal.

(5) Since $|\cos[x(n)]| \leq 1$, $y(n)$ is always bounded. Thus $y(n)$ is stable.

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

(1) Since $y(n)$ does not only depend on $x(n)$, it is dynamic.

(2)

$$
\begin{aligned}
T[a_1x_1(n)+a_2x_2(n)] &= \sum_{k=-\infty}^{n+1}[a_1x_1(k)+a_2x_2(k)] \\
&= \sum_{k=-\infty}^{n+1}[a_1x_1(k)] + \sum_{k=-\infty}^{n+1}[a_2x_2(k)] \\
&= a_1\sum_{k=-\infty}^{n+1}[x_1(k)] + a_2\sum_{k=-\infty}^{n+1}[x_2(k)] \\
&= a_1y_1(n) + a_2y_2(n) \\
&= a_1T[x_1(n)] + a_2T[x_2(n)]
\end{aligned}
$$

Thus $y(n)$ is linear.

(3) $T[x(n-m)] = \sum_{k=-\infty}^{n+1} x(k-m) = y(n-m)$. Thus $y(n)$ is time-invariant.

(4) Since $y(n)$ also depends on the future input $x(n+1)$, it is noncasual.

(5) $y(n)$ is unstable.

Counter-example: $x(n) = u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n<0. \end{cases}$ is bounded.

$y(n) = \sum_{k=-\infty}^{n+1} x(k) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0, & \text{for } n<-1 \\ n+2, & \text{for } n \geq -1. \end{cases}$ is unbounded.

(c) $y(n) = x(n)\cos(\omega_0 n)$

(1) Static

(2)

$$
\begin{aligned}
T[a_1x_1(n)+a_2x_2(n)] &= [a_1x_1(n)+a_2x_2(n)]\cos(\omega_0 n) \\
&= a_1[x_1(n)\cos(\omega_0 n)] + a_2[x_2(n)\cos(\omega_0 n)] \\
&= a_1T[x_1(n)] + a_2T[x_2(n)]
\end{aligned}
$$

Thus $y(n)$ is linear.

2.7 (c) $y(n) = x(n)\cos(w_0 n)$

(3) $T[x(n-k)] = x(n-k)\cos(w_0 n) \neq y(n-k)$

Thus $y(n)$ is time-varying.

(4) Since $y(n)$ only depends on the present input, it is causal.

(5) Let $x(n)$ is bounded by A, where $0 \leq A < \infty$, i.e. $|x(n)| \leq A$.

Thus $|y(n)| = |x(n)\cos(w_0 n)|$

$= |x(n)|\,|\cos(w_0 n)|$

$\leq A\,|\cos(w_0 n)|$

Since $|\cos(w_0 n)| \leq 1$, we have $|y(n)| \leq A$.

Since $y(n)$ is also bounded, it is stable.

(d) $y(n) = x(-n+2)$

(1) Dynamic. (2) Linear

(3) $T[x(n-k)] = x[-n+2-k] \neq y(n-k)$, thus $y(n)$ is time-varying

(4) noncausal, counter example: for $n=0$, $y(0)=x(2)$, thus $y(0)$ depends on the future input $x(2)$.

(5) Stable.

(e) $y(n) = \mathrm{Trun}[x(n)]$

(1) Static. (3) Time-invariant (4) Causal.

(2) nonlinear. Counter-example: $x_1(n)\equiv 1.6$, $x_2(n)\equiv 0.1$, $a_1=2$, $a_2=1$.

Thus $T[a_1x_1(n)+a_2x_2(n)] = \mathrm{Trun}[2\times 1.6 + 1\times 0.1] = \mathrm{Trun}[3.4] = 3$

$a_1T[x_1(n)] + a_2T[x_2(n)] = 2\cdot\mathrm{Trun}[1.6] + 1\cdot\mathrm{Trun}[0.1]$

$= 2\times 1 + 1\times 0 = 2$

Thus $T[a_1x_1(n)+a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$

(3) Let $x(n)$ is bounded by $0 \leq A < \infty$, we have $|x(n)| \leq A$

Thus $y(n) = \mathrm{Trun}[x(n)] \leq \mathrm{Trun}[A] \leq A$, we can get $y(n)$ is stable.

2.7

(f) $y(n) = \text{Round}[x(n)]$

(1) Static (3) time-invariant (4) Causal.

(2) Nonlinear, counter-example: $x_1(n) \equiv 1.6$, $x_2(n) \equiv 0.1$, $a_1 = 2$, $a_2 = 1$

$$T[a_1 x_1(n) + a_2 x_2(n)] = \text{Round}[a_1 x_1(n) + a_2 x_2(n)]$$
$$= \text{Round}[2 \times 1.6 + 0.1] = \text{Round}[3.3] = 3$$

$$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 \text{Round}[x_1(n)] + a_2 \text{Round}[x_2(n)]$$
$$= 2\,\text{Round}[1.6] + 1 \cdot \text{Round}[0.1]$$
$$= 2 \times 2 + 1 \times 0 = 4$$

Thus $T[a_1 x_1(n) + a_2 x_2(n)] \neq a_1 T[x_1(n)] + a_2 T[x_2(n)]$.

(5) Let $|x(n)| \leq A$, where $0 \leq A < \infty$

$|y(n)| = |\text{Round}[x(n)]| \leq |\text{Round}[A]| \leq A+1$

Thus $y(n)$ is stable.

(g) $y(n) = |x(n)|$

(1) Static (3) time-invariant (4) Causal (5) Stable

(2) Nonlinear. Counter-example: $x_1(n) \equiv -2$, $x_2(n) \equiv 3$, $a_1 = 5$, $a_2 = 3$

Then $T[a_1 x_1(n) + a_2 x_2(n)] = |a_1 x_1(n) + a_2 x_2(n)| = |5 \times (-2) + (3) \times 3|$
$= 1$

$$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 |x_1(n)| + a_2 |x_2(n)|$$
$$= 5 \times |-2| + 3 \times |3|$$
$$= 19$$

Thus $T[a_1 x_1(n) + a_2 x_2(n)] \neq a_1 T[x_1(n)] + a_2 T[x_2(n)]$

(h) $y(n) = x(n)\,u(n) = \begin{cases} 0, & \text{for } n < 0 \\ x(n), & \text{for } n \geq 0 \end{cases}$

(1) Static (2) Linear (3) time-invariant (4) Causal (5) Stable.

2.7 (i) $y(n) = x(n) + n x(n+1)$

(1) Dynamic (2) Linear (4) noncausal

(3) $T[x(n-k)] = x(n-k) + n x(n+1-k)$

$$= [x(n-k) + (n-k) x(n+1-k)] + k x(n+1-k)$$
$$= y(n-k) + k x(n+1-k)$$

Thus $T[x(n-k)] \neq y(n-k)$, $y(n)$ is time-variant.

(5) Unstable. Counter-example. $x(n) = u(n)$ is bounded

$$y(n) = x(n) + n x(n+1) = u(n) + n u(n+1) = \begin{cases} n+1, & \text{for } n \geq 0 \\ n, & \text{for } n = -1 \\ 0, & \text{for } n < -1 \end{cases}$$

Thus $y(n)$ is unbounded.

(j) $y(n) = x(2n)$

(1) Dynamic (2) Linear (4) noncausal (5) Stable

(3) $T[x(n-k)] = x(2n-k)$

$y(n-k) = x[2(n-k)] = x(2n-2k) \neq T[x(n-k)]$

Thus $y(n)$ is time variant.

(k) $y(n) = \begin{cases} x(n) & \text{if } x(n) \geq 0 \\ 0 & \text{if } x(n) < 0 \end{cases}$

(1) Static (3) Time-invariant (4) Causal (5) Stable.

(2) Nonlinear. Counter-example: $x_1(n) \equiv 6$, $x_2(n) \equiv -2$, $a_1 = 1$, $a_2 = 2$

Thus $T[a_1 x_1(n) + a_2 x_2(n)] = T[6 \times 1 + (-2) \times 2] = T[2] = 2$

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = 1 \times T[6] + 2 \times T[-2] = 6$.

$T[a_1 x_1(n) + a_2 x_2(n)] \neq a_1 T[x_1(n)] + a_2 T[x_2(n)]$

2.7 (l) $y(n)=x(-n)$

(1) Dynamic, (2) Linear (3) Time-invariant (5) Stable

(4) Noncausal, counter-example: Let $n=-1$, then $y(-1)=x(1)$
$y(-1)$ depends on the future input $x(1)$

(m) $y(n)=\text{sign}[x(n)] = \begin{cases} -1 & \text{if } x(n)<0 \\ 0 & \text{if } x(n)=0 \\ 1 & \text{if } x(n)>0 \end{cases}$

(1) Static (3) Time-invariant

(4) Causal (5) Since $|y(n)|\le 1$, $y(n)$ is stable.

(2) Nonlinear. Counter example: $x_1(n)=u(n)$, $x_2(n)=u(n)$, $a_1=2$, $a_2=3$.

Thus $\mathcal{T}[a_1x_1(n)+a_2x_2(n)]=\text{sign}[5u(n)] = \begin{cases} 0 & \text{if } n<0 \\ 1 & \text{if } n\geq 0 \end{cases}$

$a_1\mathcal{T}[x_1(n)]+a_2\mathcal{T}[x_2(n)]=2\text{sign}[u(n)]+3\text{sign}[u(n)]$

$=5\text{sign}[u(n)] = \begin{cases} 0 & \text{if } n<0 \\ 5 & \text{if } n\geq 0 \end{cases}$

Thus $\mathcal{T}[a_1x_1(n)+a_2x_2(n)] \neq a_1\mathcal{T}[x_1(n)]+a_2\mathcal{T}[x_2(n)]$.

(n) $\mathcal{T}[x_a(t)] = x_a(nT)$

(1) Static (2) Linear (3) time-invariant (4) causal (5) Stable

2.10.

Since the system is time-invariant, we have $x_3(n+1) \overset{T}{\longleftrightarrow} y_3(n+1)$.
Thus we have $x_3(n+1) = \{\underset{\uparrow}{0}, 0, 1\} \overset{T}{\longleftrightarrow} y_3(n+1) = \{1, 2, \underset{\uparrow}{1}\}$.

Since $x_2(n) = 3x_3(n+1)$, we have that if the system is time-invariant, it has to satisfy that $x_2(n) = 3x_3(n+1) \overset{T}{\longleftrightarrow} 3y_3(n+1) = \{3, 6, \underset{\uparrow}{3}\}$.
However, $y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\} \neq \{3, 6, \underset{\uparrow}{3}\}$.

Therefore, this system is nonlinear.
Since the system is time-invariant, we can also have
$x_3(n+3) = \{\underset{\uparrow}{1}\} = \delta(n) \overset{T}{\longleftrightarrow} y_3(n+3) = \{1, 2, 1, 0, \underset{\uparrow}{0}\}$

Therefore, the impulse response of the system is $\{1, 2, 1, 0, \underset{\uparrow}{0}\}$.

2.11

Since the system is linear, we have that $\delta(n) = x_1(n) + x_2(n) \overset{T}{\longleftrightarrow} y_1(n) + y_2(n)$

Thus the impulse response of the system is $h(n) = y_1(n) + y_2(n)$

$$= \{0, \underset{\uparrow}{3}, -1, 2, 1\}.$$

We can get $x_3(n) = \delta(n) + \delta(n+1)$.

If the system is time-invariant, we have $\delta(n+1) \overset{T}{\longleftrightarrow} h(n+1) = \{0, 3, \underset{\uparrow}{-1}, 2, 1\}$

Thus we should be able to get as follows:

$x_3(n) = \delta(n) + \delta(n+1) \overset{T}{\longleftrightarrow} h(n) + h(n+1) = \{\underset{\uparrow}{3}, 2, 1, 3, 1\}$.

However, $y_3(n) = \{\underset{\uparrow}{1}, 2, 1\} \neq \{\underset{\uparrow}{3}, 2, 1, 3, 1\}$

Therefore, the system is time-varying.

2.23.

$$y(n) = h(n) * x(n)$$
$$= h(n) * \delta(n) * x(n)$$
$$= h(n) * [u(n) - u(n-1)] * x(n)$$
$$= [h(n) * u(n) - h(n) * u(n-1)] * x(n) \quad ①$$

Let $s(n) = h(n) * u(n)$.
Since the system is time-invariant, we have:
$h(n) * u(n-1) = s(n-1)$.

Therefore $y(n) = ① = [s(n) - s(n-1)] * x(n)$
$= s(n) * x(n) - s(n-1) * x(n)$

2.29 In this problem, we consider $M \geqslant N > 0$. This is reasonable.

$$h(n) = h_1(n) * h_2(n)$$

$$= \{a^n[u(n)-u(n-N)]\} * [u(n)-u(n-M)]$$

$$= \sum_{k=-\infty}^{\infty} a^k [u(k)-u(k-N)][u(n-k)-u(n-k-M)]$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k-M)$$
$$- \sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k-M) \quad ①$$

(1) If $n<0$

$$① = 0-0-0+0 = 0$$

(2) If $0 \leqslant n < N$

$$① = \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$$

(3) If $N \leqslant n < M$

$$① = \sum_{k=0}^{n} a^k - \sum_{k=N}^{n} a^k = \sum_{k=0}^{N-1} a^k = \frac{1-a^N}{1-a}$$

(4) If $M \leqslant n < M+N$

$$① = \sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k - \sum_{k=N}^{n} a^k = \frac{1-a^N}{1-a} - \frac{1-a^{n-M+1}}{1-a} = \frac{a^{n-M+1}-a^N}{1-a}$$

(5) If $n \geqslant M+N$

$$① = \sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k - \sum_{k=N}^{n} a^k + \sum_{k=N}^{n-M} a^k$$

$$= \sum_{k=n-M+1}^{n} a^k - \sum_{k=n-M+1}^{n} a^k$$

$$= 0$$

2.35

(a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$

(b) $h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$h_3(n) * h_4(n) = (n+1)u(n) * \delta(n-2) \underset{\uparrow}{=} (n-1)u(n-2)$

shifting property

$$h_2(n) - h_3(n) * h_4(n) = (n+1)u(n) - (n-1)u(n-2)$$
$$= (n-1)[u(n) - u(n-2)] + 2u(n)$$
$$= (n-1)[\delta(n) + \delta(n-1)] + 2u(n)$$
$$= (n-1)\delta(n) + (n-1)\delta(n-1) + 2u(n)$$
$$= -\delta(n) + 0 + 2u(n)$$
$$= 2u(n) - \delta(n)$$

$$h(n) = [\tfrac{1}{2}\delta(n) + \tfrac{1}{4}\delta(n-1) + \tfrac{1}{2}\delta(n-2)] * [2u(n) - \delta(n)]$$

$$= u(n) + \tfrac{1}{2}u(n-1) + u(n-2) - \tfrac{1}{2}\delta(n) - \tfrac{1}{4}\delta(n-1) - \tfrac{1}{2}\delta(n-2)$$

Identity and shifting property

$$= [u(n) - \delta(n)] + \tfrac{1}{2}\delta(n) + \tfrac{1}{2}[u(n-1) - \delta(n-1)] + \tfrac{1}{4}\delta(n-1) + [u(n-2) - \delta(n-2)] + \tfrac{1}{2}\delta(n-2)$$

$$= u(n-1) + \tfrac{1}{2}u(n-2) + u(n-3) + \tfrac{1}{2}\delta(n) + \tfrac{1}{4}\delta(n-1) + \tfrac{1}{2}\delta(n-2)$$

$$= [u(n-1) - \delta(n-1)] + \tfrac{5}{4}\delta(n-1) + \tfrac{1}{2}[u(n-2) - \delta(n-2)] + \delta(n-2) + u(n-3) + \tfrac{1}{2}\delta(n)$$

$$= u(n-2) + \tfrac{1}{2}u(n-3) + u(n-3) + \tfrac{5}{4}\delta(n-1) + \delta(n-2) + \tfrac{1}{2}\delta(n)$$

$$= [u(n-2) - \delta(n-2)] + 2\delta(n-2) + \tfrac{3}{2}u(n-3) + \tfrac{5}{4}\delta(n-1) + \tfrac{1}{2}\delta(n)$$

$$= \tfrac{5}{2}u(n-3) + \tfrac{1}{2}\delta(n) + \tfrac{5}{4}\delta(n-1) + 2\delta(n-2)$$

(c) $y(n) = x(n) * h(n) = h(n+2) + 3h(n-1) - 4h(n-3)$

$$= \{\tfrac{1}{2}, \tfrac{5}{4}, \underset{\uparrow}{2}, 4, \tfrac{25}{4}, \tfrac{13}{2}, 5, 2, 0, 0, \cdots\}$$

2.37

$$y(n) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-k)$$

$$= \frac{1}{M}\sum_{k=-\infty}^{\infty}[u(k) - u(k-M)]\, x(n-k)$$

$$= \frac{1}{M}[u(n) - u(n-M)] * x(n)$$

Thus the impulse response is $h(n) = \frac{1}{M}[u(n) - u(n-M)]$.

Therefore the step response is as follows:

$$s(n) = h(x) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, u(n-k)$$

$$= \sum_{k=0}^{n} h(k) = \frac{1}{M}\left[\sum_{k=0}^{n} u(k) - \sum_{k=0}^{n} u(k-M)\right]$$

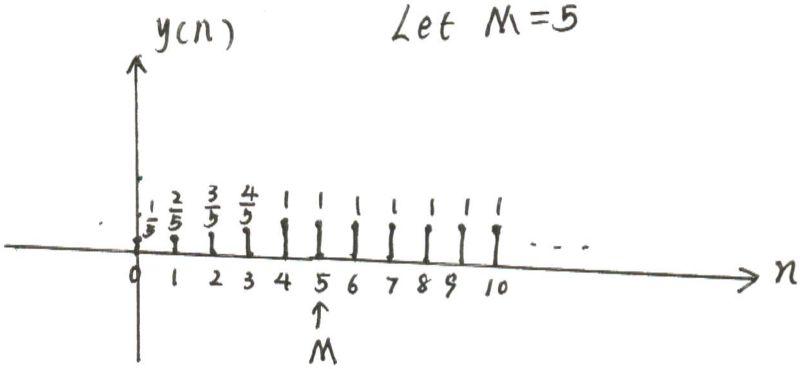
If $0 \le n < M$, $\displaystyle\sum_{k=0}^{n} h(k) = \frac{1}{M}\sum_{k=0}^{n} u(k) = \frac{n+1}{M}$

If $n \geq M$ $\displaystyle\sum_{k=0}^{n} h(k) = \frac{1}{M}\left[\sum_{k=0}^{n} u(k) - \sum_{k=M}^{n} u(k-M)\right]$

$$= \frac{1}{M}[(n+1) - (n-M+1)]$$

$$= 1$$

Let $M=5$



2-38

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,\,even}^{\infty} |a|^n$$

$$= \sum_{k=0}^{\infty} |a|^{2k}$$

$$= \lim_{k\to\infty} \frac{1-|a|^{2k+2}}{1-|a|^2}, \text{ where } k=\tfrac{1}{2}n$$

Therefore in order to ensure $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$, a needs to satisf

$|a|<1$.

2.41.

$$h(n) = (\tfrac{1}{2})^n u(n)$$

(a) $x(n) = 2^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} (\tfrac{1}{2})^k u(k)\, 2^{n-k} u(n-k)$$

$$= \begin{cases} \sum_{k=0}^{n} (\tfrac{1}{2})^k 2^{n-k} & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

$$= \tfrac{1}{3} u(n) (2^{n+2} - 2^{-n})$$

(b) $x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} (\tfrac{1}{2})^k u(k)\, u(k-n)$$

If $n < 0$, $y(n) = \sum_{k=0}^{\infty} (\tfrac{1}{2})^k = \frac{1}{1-\frac{1}{2}} = 2$

If $n \geq 0$ $y(n) = \sum_{k=n}^{\infty} (\tfrac{1}{2})^k = \frac{(\frac{1}{2})^n}{1-\frac{1}{2}} = 2^{(1-n)}$