(7.1) The first five points of the eight-point DFT of a real-valued sequence are $\{0.25, 0.125-j0.3018, 0, 0.125-j0.0518, 0\}$. Determine the remaining three points.

⇒ Real signal implies these symmetric

- real part of DFT is even symmetric
- imaginary part of DFT is odd symmetric.

Hence, the remaining points are $\{0.125+j0.0518, 0, 0.125+j0.3018\}$.

(7.2) Compute the eight-points circular convolution for the following sequences.

(a). $x_1(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$x_2(n) = \sin\frac{3\pi}{8}n, \quad 0 \le n \le 7$

$$\rightarrow \quad x_1(n) ⑧ x_2(n) = \sum_{n=0}^{7} x_2(n)\, x_1((m-n))_7$$

for $m=0$, $x_1(n) Ⓝ x_2(n) = \sum_{n=0}^{7} x_2(n)\, x_1((-n))_7$

$$= \sin(\tfrac{3\pi}{8}\cdot 0) + \sin(\tfrac{3\pi}{8}\cdot 5) + \sin(\tfrac{3\pi}{8}\cdot 6) + \sin(\tfrac{3\pi}{8}\cdot 7)$$

$$= 1.25$$

for $m = 1, 2, 3 \ldots 7$ - - -

$\Rightarrow x_1(n) ⑧ x_2(n) = \{1.25, 2.55, 2.55, 1.25, 0.25, -1.06, -1.06, 0.25\}$

(b). $x_1(n) = (\frac{1}{4})^n, \quad 0 \le n \le 7$

$x_2(n) = \cos\frac{3\pi}{8}n, \quad 0 \le n \le 7$

$$\rightarrow \quad x_1(n) ⑧ x_2(n) = \sum_{n=0}^{7} x_1(n)\, x_2((m-n))_7$$

$$= \{0.96, 0.62, -0.55, -1.06, -0.26, -0.86, 0.92, -0.15\}$$

(c) Compute the DFT of the two circular convolution sequences using DFTs of $x_1(n)$ and $x_2(n)$.

$$\rightarrow \text{for (a)}, \quad X_1(k) = \sum_{n=0}^{7} x_1(n)\, e^{-j\frac{2\pi kn}{N}} = \sum_{n=0}^{7} x_1(n)\, e^{-j\frac{2\pi kn}{8}}$$

$$= \sum_{n=0}^{7} x_1(n)\, e^{-j\frac{\pi}{4}kn} = \sum_{n=0}^{4} e^{-j\frac{\pi}{4}kn}$$

$$= \{4, 1-j2.4142, 0, 1-j0.4142, 0, 1+j0.4142, 0, 1+j2.4142\}.$$

Similar,

$X_2(k) = \{1.4966, 2.8478, -2.4142, -0.8478, -0.6682, -0.8478, -2.4142, 2.8478\}$

DFT of $x_1(n) \circledast x_2(n) = X_1(k)\, X_2(k)$

$$= \{5.9864, 2.8478 - j6.8751, 0, -0.8478 + j0.3512, 0, -0.8478 - j0.3512, 0, 2.8478 + j6.8751\}.$$

⇒ for (b), $X_1(k) = \{1.3333, 1.1612 - j0.2493, 0.9412 - j0.2353, 0.8310 - j0.1248, 0.8, 0.8310 + j0.1248, 0.9412 + j0.2353, 1.1612 + j0.2493\}$.

$X_2(k) = \{1, 1 + j2.1796, 1 - j2.6131, 1 - j0.6488, 1, 1 + j0.6488, 1 + j2.6131, 1 - j2.1796\}$

$$\text{DFT}\{x_1(n) \circledast_8 x_2(n)\} = X_1(k) X_2(k)$$

$$= \{1.3333, 1.7046 + j2.2815, 0.3263 - j2.6947, 0.75 - j0.664, 0.8, 0.75 + j0.664, 0.3263 + j2.6947, 1.7046 - j2.2815\}.$$

7.3. Let $X(k)$, $0 \le k \le N-1$, be the N-point DFT of the sequence $x(n)$, $0 \le n \le N-1$. We define
$$\hat{X}(k) = \begin{cases} X(k), & 0 \le k \le k_c,\ N-k_c \le k \le N-1 \\ 0, & k_c < k < N-k_c \end{cases}$$
and we compute the inverse N-point DFT of $\hat{X}(k)$, $0 \le k \le N-1$. What is the effect of this process on the sequence $x(n)$? Explain.

⇒ Consider $\hat{X}(k)$ as the output of the DFT sequence $X(k)$ times
$$F(k) = \begin{cases} 1, & 0 \le k \le k_c,\ N-k_c \le k \le N-1 \\ 0, & k_c < k < N-k_c \end{cases}$$
$F(k)$ acting as an ideal filter removing frequency components between $(k_c+1)\frac{2\pi}{N}$ to $(N-k_c-1)\frac{2\pi}{N}$. Thus, $\hat{x}(n)$ is a filtered version of $x(n)$.

7.7 If $X(k)$ is the DFT of the sequence $x(n)$, determine the N-point DFTs of the sequences $x_c(n) = x(n)\cos\frac{2\pi k_0 n}{N}$ and $x_s(n) = x(n)\sin\frac{2\pi k_0 n}{N}$, $0 \le n \le N-1$ in terms of $X(k)$.

$$\Rightarrow X_c(k) = \sum_{n=0}^{N-1} x(n)\cdot\frac{1}{2}\left[e^{j\frac{2\pi k_0 n}{N}} + e^{-j\frac{2\pi k_0 n}{N}}\right]e^{-j\frac{2\pi k n}{N}}$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi(k-k_0)n}{N}} + \frac{1}{2}\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi(k+k_0)n}{N}}$$

$$= \frac{1}{2}X((k-k_0))_N + \frac{1}{2}X((k+k_0))_N$$

$$X_s(k) = \sum_{n=0}^{N-1} x(n)\frac{1}{2j}\left[e^{j\frac{2\pi k_0 n}{N}} - e^{-j\frac{2\pi k_0 n}{N}}\right]e^{-j\frac{2\pi k n}{N}}$$

$$= \frac{1}{2j}\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi(k-k_0)n}{N}} - \frac{1}{2j}\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi(k+k_0)n}{N}}$$

$$= \frac{1}{2j}X((k-k_0))_N - \frac{1}{2j}X((k+k_0))_N.$$

7.8 Determine the circular convolution of the sequences

$x_1(n) = \{1, 2, 3, 1\}$, $x_2(n) = \{4, 3, 2, 2\}$.

Using the time domain formula in (7.2.39).

$$\Rightarrow y(n) = x_1(n) ④ x_2(n) = \sum_{m=0}^{3} x_1(m)\, x_2((n-m))_4$$

$$= \{17, 19, 22, 19\}.$$

7.11 Given the eight-point DFT of the sequence $x(n) = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & 4 \le n \le 7 \end{cases}$.

Compute the DFT of the sequences

(a) $x_1(n) = \begin{cases} 1 & n = 0 \\ 0 & 1 \le n \le 4 \\ 1 & 5 \le n \le 7 \end{cases}$

$$\Rightarrow x_1(n) = x((n-5))_8$$

$$X_1(k) = X(k)\, e^{-j\frac{2\pi 5k}{8}} = X(k)\, e^{-j\frac{5\pi k}{4}}$$

(b) $x_2(n) = \begin{cases} 0 & 0 \le n \le 1 \\ 1 & 2 \le n \le 5 \\ 0 & 6 \le n \le 7 \end{cases}$

$$\Rightarrow x_2(n) = x((n-2))_8$$

$$X_2(k) = X(k)\, e^{-j\frac{2\pi 2k}{8}} = X(k)\, e^{-j\frac{\pi k}{2}}$$

(7.13) Let $x_p(n)$ be a periodic sequence with fundamental period N. Consider the following DFTs: $x_p(n) \xleftrightarrow[N]{DFT} X_1(k)$, $x_p(n) \xleftrightarrow[3N]{DFT} X_3(k)$.

(a) What is the relationship between $X_1(k)$ and $X_3(k)$?

$\Rightarrow$ $X_1(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{N}}$

$$X_3(k) = \sum_{n=0}^{3N-1} x(n) e^{-j\frac{2\pi k n}{3N}}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{3N}} + \sum_{n=N}^{2N-1} x(n) e^{-j\frac{2\pi k n}{3N}} + \sum_{n=2N}^{3N-1} x(n) e^{-j\frac{2\pi k n}{3N}}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{3N}} + \sum_{n=0}^{N-1} x(n-N) e^{-j\frac{2\pi k(n+N)}{3N}} + \sum_{n=0}^{N-1} x(n-2N) e^{-j\frac{2\pi k(n+2N)}{3N}}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{3N}} + \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{3N}} e^{-j\frac{2\pi k N}{3N}} + \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{3N}} e^{-j\frac{4\pi k N}{3N}}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{3N}} \left[1 + e^{-j\frac{2\pi k}{3}} + e^{-j\frac{4\pi k}{3}}\right]$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi (\frac{k}{3}) n}{N}} \left[1 + e^{-j\frac{2\pi k}{3}} + e^{-j\frac{4\pi k}{3}}\right]$$

$$= X(k)\left[1 + e^{-j\frac{2\pi k}{3}} + e^{-j\frac{4\pi k}{3}}\right]$$

(7.14) Consider the sequence $x_1(n) = \{0, 1, 2, 3, 4\}$, $x_2(n) = \{0, 1, 0, 0, 0\}$, $s(n) = \{1, 0, 0, 0, 0\}$, and their five-point DFTs.

(a) Determine a sequence $y(n)$ so that $Y(k) = X_1(k) X_2(k)$.

$\Rightarrow$ $Y(k) = X_1(k)X_2(k)$, so $y(n) = x_1(n) ⑤ x_2(n)$

$$= \{4, 0, 1, 2, 3\}.$$

(b) Is there a sequence $x_3(n)$ such that $S(k) = X_1(k) X_3(k)$?

$\Rightarrow$ Let $x_3(n) = \{x_0, x_1, x_2, x_3, x_4\}$, then

$x_1(n) ⑤ x_3(n) = s(n)$.

$$\begin{bmatrix} 0 & 4 & 3 & 2 & 1 \\ 1 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $x_3(n) = \{-0.18, 0.22, 0.02, 0.02, 0.02\}$.

7.23 Compute the N-point DFTs of the signals

(a) $x(n)=\delta(n)$

$$\Rightarrow X(k)=\sum_{n=0}^{N}\delta(n)e^{-j\frac{2\pi kn}{N}}=e^{-j\frac{2\pi k\cdot 0}{N}}=1,\quad 0\le k\le N-1$$

(b) $x(n)=\delta(n-n_0)$, $0<n_0<N$.

$$\Rightarrow X(k)=\sum_{n=0}^{N-1}\delta(n-n_0)e^{-j\frac{2\pi kn}{N}}=e^{-j\frac{2\pi kn_0}{N}},\quad 0\le k\le N-1.$$

(c) $x(n)=a^n$, $0\le n\le N-1$

$$\Rightarrow X(k)=\sum_{n=0}^{N-1}x(n)e^{-j\frac{2\pi kn}{N}}=\sum_{n=0}^{N-1}a^n e^{-j\frac{2\pi kn}{N}}=\sum_{n=0}^{N-1}\left[ae^{-j\frac{2\pi k}{N}}\right]^n$$

$$=\frac{1-\left(ae^{-j\frac{2\pi k}{N}}\right)^N}{1-ae^{-j\frac{2\pi k}{N}}}=\frac{1-a^N}{1-ae^{-j\frac{2\pi k}{N}}}$$

(h) $x(n)=\begin{cases}1, & n \text{ even}\\ 0, & n \text{ odd}\end{cases}$, $0\le n\le N-1$

$$\Rightarrow X(k)=\sum_{n=0}^{N-1}x(n)e^{-j\frac{2\pi kn}{N}}\quad \text{(assume } N \text{ odd)}$$

$$=1+e^{-j\frac{2\pi}{N}\cdot 2k}+e^{-j\frac{2\pi}{N}4k}+\cdots+e^{-j\frac{2\pi}{N}(n-1)k}$$

$$=\frac{1-\left(e^{-j\frac{2\pi}{N}2k}\right)^{\frac{N+1}{2}}}{1-e^{-j\frac{2\pi}{N}2k}}=\frac{1-e^{-j\frac{2\pi k}{N}}}{1-e^{-j\frac{4\pi k}{N}}}=\frac{1}{1-e^{-j\frac{2\pi k}{N}}}.$$

7.25 (a) Determine the Fourier transform $X(\omega)$ of the signal $x(n)=\{1,2,\underset{\uparrow}{3},2,1,0\}$

$$\Rightarrow X(\omega)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\omega n}=e^{2j\omega n}+2e^{j\omega}+3+2e^{-j\omega n}+e^{-2j\omega n}$$

$$=3+4\cos(\omega)+2\cos(2\omega)$$

(b) Compute the six point DFT $V(k)$ of the signal $v(n)=\{3,2,1,0,1,2\}$.

$$\Rightarrow V(k)=\sum_{n=0}^{5}v(n)e^{-j\frac{2\pi nk}{6}}$$

$$=3+2e^{-j\frac{2\pi}{6}k}+e^{-j\frac{2\pi}{6}2k}+0+e^{-j\frac{2\pi}{6}4k}+e^{-j\frac{2\pi}{6}5k}$$

$$=3+4\cos\left(\tfrac{\pi}{3}k\right)+2\cos\left(\tfrac{2\pi}{3}k\right)$$

(c) Is there any relation between $X(\omega)$ and $V(k)$? Explain.

$$\Rightarrow V(k)=X(\omega)\big|_{\omega=\frac{\pi k}{3}}.$$

$v(n)$ is one period $(0\le n\le 7)$ of a periodis sequence obtained by repeating $x(n)$.

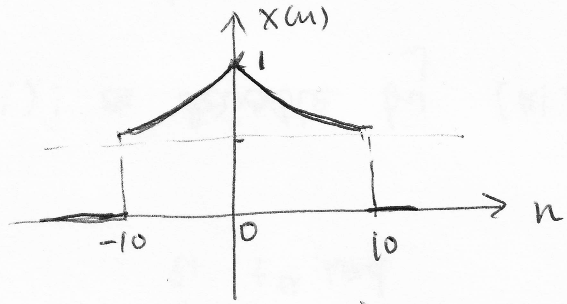
7.28 Frequency-domain sampling. Consider the following discrete-time signal

$$x(n) = \begin{cases} a^{|n|}, & |n| \le L \\ 0, & |n| > L \end{cases} \quad \text{where } a = 0.95 \text{ and } L = 10.$$

(a) Compute and plot the signal $x(n)$.

$$\Rightarrow \quad x(n) = \begin{cases} 0.95^{|n|}, & |n| \le 10 \\ 0, & |n| > L \end{cases}$$

$= \{0.5987,\ 0.6302,\ 0.6634,\ 0.6983,\ 0.7351,\ 0.7738,\ 0.8145,\ 0.8574,$
$0.9025,\ 0.9500,\ 1,\ 0.9500,\ 0.9025,\ 0.8574,\ 0.8145,\ 0.7733,$
$0.7351,\ 0.6983,\ 0.6634,\ 0.6302,\ 0.5987\}$



(b) Show that $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = x(0) + 2\sum_{n=1}^{L} x(n)\cos \omega n$. and plot $X(\omega)$ by computing it at $\omega = \pi k/100$, $k = 0, 1, \cdots 100$.

$$\Rightarrow \quad X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = X(0) + \sum_{n=-L}^{-1} a^{-n} e^{-j\omega n} + \sum_{n=1}^{L} a^{n} e^{-j\omega n}$$

$$= a + \sum_{n=1}^{L} a^n e^{j\omega n} + \sum_{n=1}^{L} a^n e^{-j\omega n}$$

$$= a + \sum_{n=1}^{L} a^n \left(e^{j\omega n} + e^{-j\omega n}\right)$$

$$= a + 2\sum_{n=1}^{L} a^n \cos(\omega n)$$