Homework #4: Discrete-Time Filters II

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Questions

Please print this out and answer the following questions in the space provided below. Please add additional sheets if necessary or use the backs of sheets. For full points, please provide explanations and reasoning in your solutions.

1. Consider the following LTI filter with impulse response:

$$h(n) = \frac{1}{2}\delta(n) + \frac{1}{2}\delta(n-1)$$

a. Please determine the system function $H(z)$.

b. Is the system implementable? Please explain your reasoning.

c. Find an input-output expression for this system. That is, compute

$$y(n) = x(n) * h(n)$$

and obtain $y(n)$ in terms of $x(n)$ and its shifted versions. Explain in your own words what the system does.

2. *Linear Phase FIR Filters*. The impulse responses of a few FIR filters are given below. In each case determine whether the filter is linear phase and show your reasoning.

a. $h(n) = 0.5\delta(n) + 0.25\delta(n-1) + \delta(n-2) + 0.25\delta(n-3) + 0.5\delta(n-4)$

b. $h(n) = 0.5\delta(n) + 0.25\delta(n-1) - \delta(n-2) + 0.25\delta(n-3) + 0.5\delta(n-4)$

c. $h(n) = 0.5\delta(n) + 0.25\delta(n-1) - \delta(n-2) - 0.25\delta(n-3) + 0.5\delta(n-4)$

d. $h(n) = 0.5\delta(n) - 0.25\delta(n-1) - \delta(n-2) - 0.25\delta(n-3) + 0.5\delta(n-4)$

e. $h(n) = 0.5\delta(n) + 0.25\delta(n-1) - \delta(n-2) - 0.25\delta(n-3) - 0.5\delta(n-4)$

3. *Linear Phase FIR Filters*. Consider the following linear phase FIR filter with impulse response:

$$h(n) = \delta(n) - \delta(n-3).$$

a. Find the frequency response of the filter.

b. Graph the magnitude of the frequency response. Would you classify this filter as lowpass, bandpass, bandstop, highpass filter or none of these?

c. Graph the phase of the frequency response to verify that the filter is linear phase. Is it exactly a line for all frequencies? If not, please explain why the phase is still considered to be linear.

d. Determine the group delay of the filter.

e. In the lectures we considered a linear phase FIR filter with impulse response:

$$h(n) = \delta(n) - \delta(n-1)$$

with a group delay of 0.5 time units. The filter is similar to that in this question except the delta functions are closer together in time. Based on your computation in this question and your relative observations from this similar example in class, how will the group delay change (i.e., will it increase or decrease?) for the following filter with impulse response:

$$h(n) = \delta(n) - \delta(n-k)$$

as k, a positive integer constant, increases? Does this make sense? Please explain your answer intuitively.

4. *Design of Linear Phase FIR Filters using Windows*. Design an FIR linear phase filter approximating the ideal frequency response:

$$H_d(\omega) = \begin{cases} 1 & |\omega| \le \dfrac{\pi}{6} \\ 0 & \dfrac{\pi}{6} < |\omega| \le \pi \end{cases}$$

a. Determine the coefficients of a 25-tap filter based on the window method using a rectangular window as discussed in the lectures. You should leave your answer as a fairly simplified expression as done in the lectures. *Hint*: Do not forget to appropriately delay the frequency response the appropriate amount in anticipation of forcing the filter to be of length 25.

b. Using MATLAB/Simulink to help, sketch the magnitude and phase response of the filter.

c. Repeat parts (a) and (b) of this question assuming a Bartlett (triangular) window is used instead of a rectangular. A length-M Bartlett window $w(n)$ is given by the following expression:

$$w(n) = 1 - \frac{2\left| n - \frac{M-1}{2} \right|}{M-1}$$

for $0 \le n \le M\text{-}1$ and $w(n) = 0$ otherwise.

5. *Design of IIR Filters using Bilinear Transformation*. Use the bilinear transformation to convert the analog filter with system function:

$$H(s) = \frac{s+1}{s^2 - s - 2}.$$

into a digital IIR filter. Select $T = 0.1$.

a. Find the system function $H(z)$ of the digital IIR filter. Leave your expression in rational form (one polynomial of z^{-1} over another).

b. Is the analog filter stable? Please explain.

c. Is the digital filter stable? Please explain.

d. Draw the Direct Form II Realization of $H(z)$. Please use the back of this sheet for your computations and drawing.