

3.22  
a)

$$x(t) = \frac{d}{dt} e^{-2|t|}$$

HW 10

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$$\frac{d}{dt} y(t) \xleftrightarrow{FT} j\omega Y(j\omega)$$

$$y(t) = e^{-2|t|}$$

$$Y(j\omega) = \frac{4}{4 + \omega^2}$$

$$X(j\omega) = \frac{4j\omega}{4 + \omega^2}$$

b)  $x(t) = \frac{d}{dt} (2 + e^{-2t} u(t))$

$$y(t) = 2 + e^{-2t} u(t)$$

$$Y(j\omega) = \frac{2}{(2 + j\omega)^2}$$

$$X(j\omega) = \frac{2j\omega}{(2 + j\omega)^2}$$

3.23

$$X(j\omega) = \begin{cases} j\omega & |\omega| < 1 \\ 0 & |\omega| > 1 \end{cases}$$

$$Y(j\omega) = \begin{cases} 1 & |\omega| < 1 \\ 0 & |\omega| > 1 \end{cases}$$

$$X(j\omega) = j\omega Y(j\omega)$$

$$y(t) = \frac{1}{\pi t} \sin(t)$$

$$x(t) = \frac{d}{dt} y(t)$$

$$x(t) = \frac{1}{\pi t} \cos(t) - \frac{1}{\pi t^2} \sin(t)$$

3.25

$$x(t) = t e^{-at} u(t)$$

$$-j t y(t) \xleftrightarrow{\text{FT}} \frac{d}{d\omega} X(j\omega)$$

$$x(t) = \frac{-j}{-j} t e^{-at} u(t) = \left( \frac{-1}{j} \right) -j t e^{-at} u(t)$$

$$y(t) = e^{-at} u(t)$$

$$Y(j\omega) = \frac{1}{j\omega + a}$$

$$X(j\omega) = \left( \frac{-1}{j} \right) \cdot \frac{d}{d\omega} \left( \frac{1}{j\omega + a} \right)$$

$$= \left( \frac{-1}{j} \right) \cdot (-1) \cdot j \cdot \frac{1}{(j\omega + a)^2}$$

$$= \frac{1}{(j\omega + a)^2}$$

3.26

$$y(t) = \frac{d}{dt} \{ t e^{-3t} u(t) * e^{-2t} u(t) \}$$

$$\text{let } f(t) = t e^{-3t} u(t)$$

$$g(t) = e^{-2t} u(t)$$

$$Y(j\omega) = j\omega \cdot F(j\omega) \cdot G(j\omega)$$

$$= j\omega \cdot \frac{1}{(j\omega + 3)^2} \cdot \frac{1}{(j\omega + 2)}$$

$$= \frac{j\omega}{(j\omega + 3)^2 (j\omega + 2)}$$

3.29

$$X(j\omega) = \frac{1}{j\omega(j\omega+1)} + \pi \delta(\omega)$$

$$\int_{-\infty}^t y(\tau) d\tau \leftrightarrow \frac{1}{j\omega} Y(j\omega) + \pi Y(j0) \delta(\omega)$$

$$Y(j\omega) = \frac{1}{j\omega+1}$$

$$y(t) = e^{-t} u(t)$$

$$x(t) = \int_{-\infty}^t e^{-\tau} u(\tau) d\tau$$

$$= u(t) \int_0^t e^{-\tau} d\tau = u(t) \cdot (-1)(e^{-t} - 1)$$

$$= u(t) (e^{-t} - 1)$$

3.34

b)  $X(j\omega) =$ 

$$\frac{1}{2 + j(\omega - 3)} + \frac{1}{2 + j(\omega + 3)}$$

$$\begin{aligned}
 x(t) &= e^{j3t} e^{-2t} u(t) + e^{-j3t} e^{-2t} u(t) \\
 &= e^{-2t} u(t) (e^{-j3t} + e^{j3t}) \\
 &= e^{-2t} u(t) 2 \cos(3t)
 \end{aligned}$$

3.40

b)

$$X(j\omega) = \frac{2 \sin(\omega - 2)}{(\omega - 2)} * \frac{e^{-j2\omega} \sin(2\omega)}{\omega}$$

$$= \frac{1}{2\pi} \cdot 2\pi \cdot \frac{2 \sin(\omega - 2)}{\omega - 2} * \frac{e^{-j2\omega} \sin(2\omega)}{\omega}$$

$$Y(j\omega) = \frac{4\pi \sin(\omega - 2)}{\omega - 2}$$

$$y(t) = 2\pi e^{j2t} (u(t+1) - u(t-1))$$

$$z(j\omega) = \frac{e^{-j2\omega} \sin(2\omega)}{\omega}$$

$$z(t) = \frac{1}{2} (u((t-2)+2) - u((t+2)-2))$$

$$x(t) = y(t) \cdot z(t) = \pi e^{j2t} (u(t) - u(t-1))$$

3.41

$$x(t) \xrightarrow{FT} X(j\omega) = e^{-j\omega} |\omega| e^{-2|\omega|}$$

$$y(t) = x(-2t)$$

~~$$x(t) = y(t) = x(-2t) \xrightarrow{F} \frac{1}{|2|} X(j\omega/2)$$~~

$$Y(j\omega) = \frac{1}{2} \cdot e^{\frac{j\omega}{2}} \left| \frac{\omega}{2} \right| e^{-2|\omega/2|}$$

$$= \frac{1}{2} \cdot e^{\frac{j\omega}{2}} \left| \frac{\omega}{2} \right| e^{-|\omega|}$$



3.43

a)

$$X_1 = \int_{-\infty}^{\infty} \frac{z}{|j\omega + 2|^2} d\omega = 2 \int_{-\infty}^{\infty} \left| \frac{1}{j\omega + 2} \right|^2 d\omega$$

$$X(j\omega) = \frac{1}{2 + j\omega}$$

$$x(t) = e^{-2t} u(t)$$

$$X_1 = 2 \cdot 2\pi \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{1}{j\omega + 2} \right|^2 d\omega$$

$$= 4\pi \cdot \int_{-\infty}^{\infty} e^{-4t} u(t) dt$$

$$= 4\pi \cdot \left(-\frac{1}{4}\right) \cdot \left[ e^{-4t} \right]_0^{\infty}$$

$$= 4\pi \cdot \left(-\frac{1}{4}\right) \cdot (-1) = \pi$$

3.44

$$X(j\omega) = u(\omega)$$

first

$$f(t) = u(t) \xleftrightarrow{F} F(j\omega) = \frac{1}{j\omega} + \pi \delta(\omega)$$

$$x(t) = \frac{-1}{2\pi jt} + \frac{\delta(t)}{2}$$

3.58

a)  $x(t) = \sin(2\pi t) e^{-t} u(t)$

Solve by

1) breaking down the sin function into complex exponential form

2) use frequency shift property

b)  $x(t) = t e^{3|t-1|}$

1) Find FF of  $e^{3|t-1|}$  by time shift property

2) Take derivative of step 1

e)  $x(t) = \int_{-\infty}^t \frac{\sin(2\pi\tau)}{\pi\tau} d\tau$

1) Notice integral of sinc

2) use integration property

3.58

$$g) \quad x(t) = \left( \frac{\sin(t)}{\pi t} \right) * \frac{d}{dt} \left( \frac{\sin(2t)}{\pi t} \right)$$

1) use derivative property

2) use convolution  $\xleftrightarrow{F}$  multiplication property

3.59

$$a) X(j\omega) = \frac{j\omega}{(1+j\omega)^2}$$

1) Notice  $X(j\omega) = j\omega \cdot \frac{1}{(1+j\omega)^2}$

2) Use time differentiation property

$$c) X(j\omega) = \frac{1}{j\omega(j\omega+2)} - \pi \delta(\omega)$$

1) Use time integration property

$$e) X(j\omega) = \frac{2 \sin(\omega)}{\omega(j\omega+2)}$$

1) Notice  $X(j\omega) = \frac{2 \sin(\omega)}{\omega} \cdot \frac{1}{j\omega+2}$

2) Use convolution  $\xleftrightarrow{F}$  multiplication property