Homework 2 ECEN 314 Prof. Dr. Kundur

Dr. Righetti

Solved by
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Problem 1.10

let $x[n] = \begin{cases} n & \text{for } n \text{ odd} \\ 0 & \text{otherwise} \end{cases}$

determine

$$y[n] = x[2n]$$

As noted, $x[n]$ is 1 if n is odd and 0 otherwise.

Taking $x[2n]$ guarantees that the argument of signal x is even. Therefore the signal $y[n] = x[2n] = 0$ for all integers n

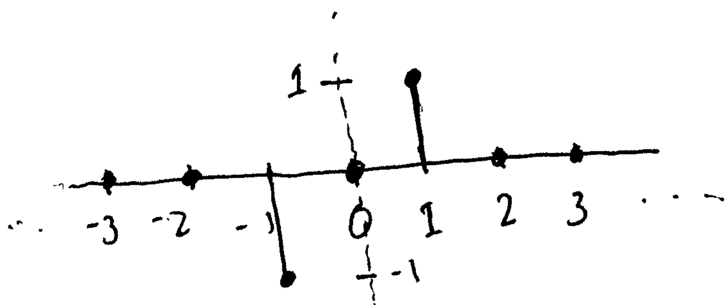
1.11 Discrete signal

$$x[n] = \begin{cases} 1 & n = 1 \\ -1 & n = -1 \\ 0 & n = 0 \text{ and } |n| > 1 \end{cases}$$

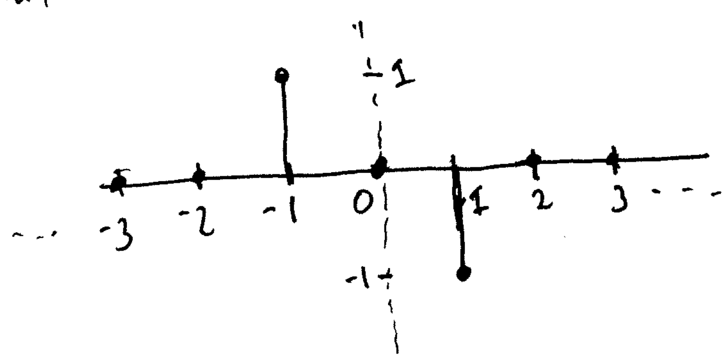
find

$$y[n] = x[n] + x[-n]$$

Let's sketch $x[n]$



$x[-n]$ is the reflection of $x[n]$ about 0.
That looks like this:



Adding $x[n]$ to $x[-n]$ therefore, clearly leads to $y[n] = x[n] + x[-n] = 0$ for all integers n

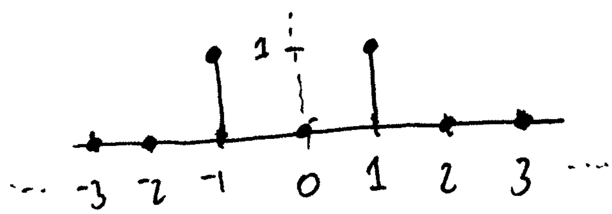
1.12

discrete signal

$$x[n] = \begin{cases} 1, & n = -1 \text{ and } n = 1 \\ 0 & n = 0 \text{ and } |n| > 1 \end{cases}$$

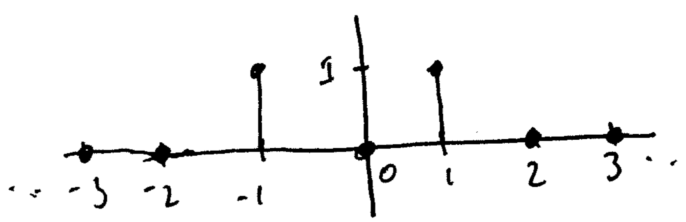
find $y[n] = x[n] + x[-n]$

Again, let's sketch $x[n]$

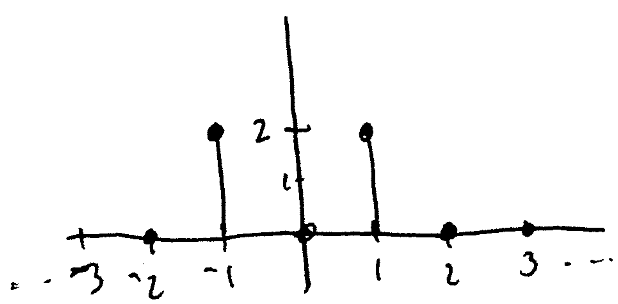


As $x[-n]$ is the reflection of $x[n]$ about 0

~~sho~~ the sketch of $x[-n]$ is:



Adding these two yields:



that is:

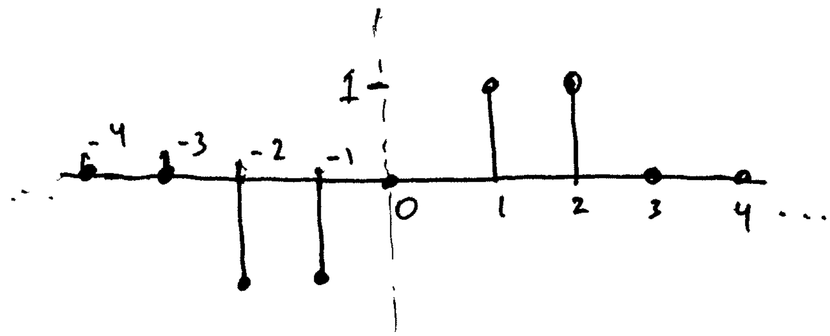
$$y[n] = x[n] + x[-n] = \begin{cases} 2 & n = -1,\ n = 1 \\ 0 & n = 0 \text{ and } |n| > 1 \end{cases}$$

1.13

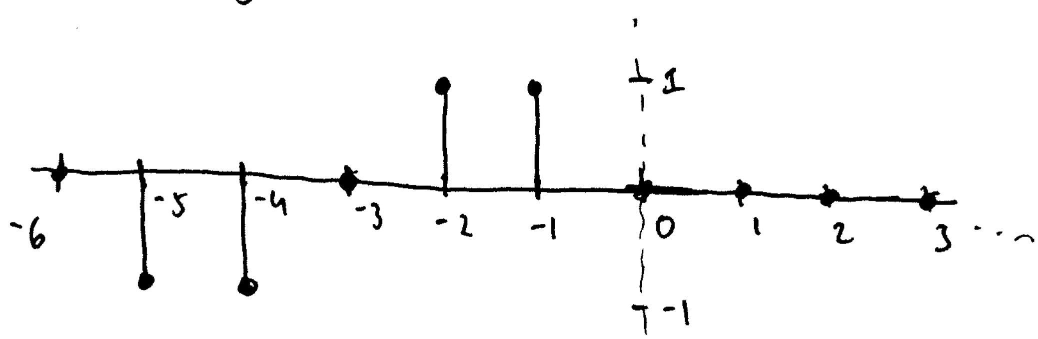
$$x[n] = \begin{cases} 1 & n = 1,\ 2 \\ -1 & n = -1,\ 2 \\ 0 & n = 0 \text{ and } |n| > 2 \end{cases}$$

find $y[n] = x[n+3]$

First, sketch $x[n]$



if we take $x[n+3]$, we have to shift everything left by 3 samples, that looks like:



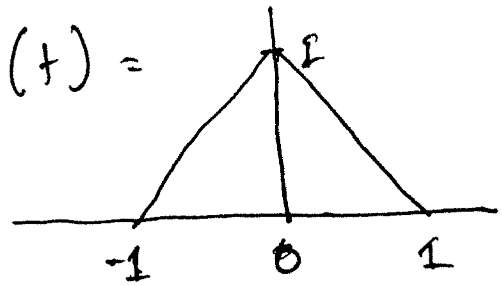
So

$$y[n] = x[n+3] = \begin{cases} 1 & n = -2,\ n = -1 \\ -1 & n = -4,\ n = -5 \\ 0 & \text{otherwise} \end{cases}$$

1.14 a)

sketch $x(3t)$

$x(t) =$

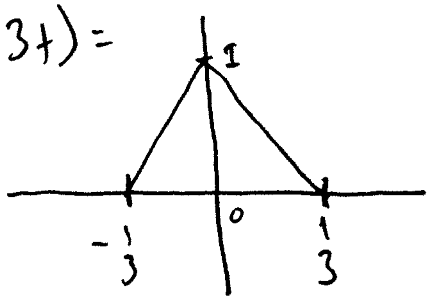


So if we solve the argument for -1 and 1, we know the vertices of the triangle

$$\cancel{3t} \quad 3t = -1 \Rightarrow t = \frac{-1}{3}$$

$$3t = 1 \Rightarrow t = \frac{1}{3}$$

$x(3t) =$



b)

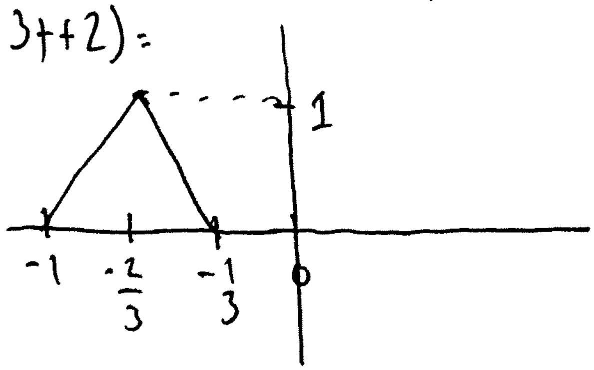
Sketch $x(3t+2)$

$$3t+2 = -1 \Rightarrow 3t = -3 \Rightarrow t = \frac{-3}{3} = -1$$

$$3t+2 = 1 \Rightarrow 3t = -1 \Rightarrow t = \frac{-1}{3}$$

$$3t+2 = 0 \Rightarrow 3t = -2 \Rightarrow t = -\frac{2}{3}$$

$x(3t+2) =$

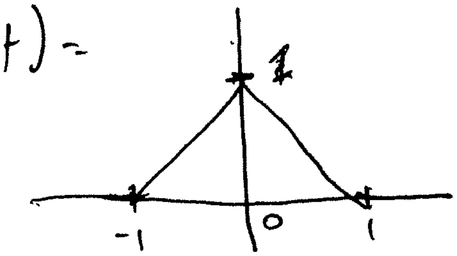


1.14 c)

sketch

$x(-2t-1)$

$x(t) =$

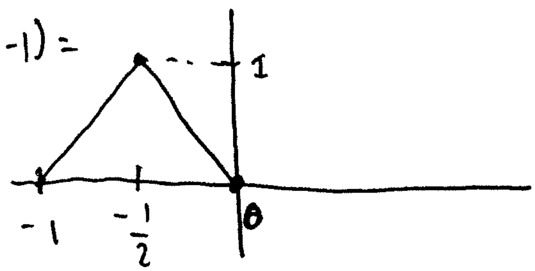


$-2t - 1 = -1 \Rightarrow -2t = 0 \Rightarrow t = 0$

$-2t - 1 = 0 \Rightarrow -2t = 1 \Rightarrow t = -\frac{1}{2}$

$-2t - 1 = 1 \Rightarrow -2t = 2 \Rightarrow t = -1$

$x(-2t-1) =$



d) sketch

$x(2(t+2))$

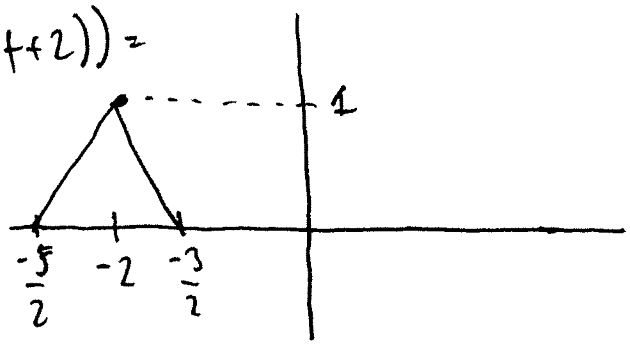
$2(t+2) = 2t+4$

$2t+4 = -1 \Rightarrow 2t = -5 \Rightarrow t = -\frac{5}{2}$

$2t+4 = 0 \Rightarrow 2t = -4 \Rightarrow t = -2$

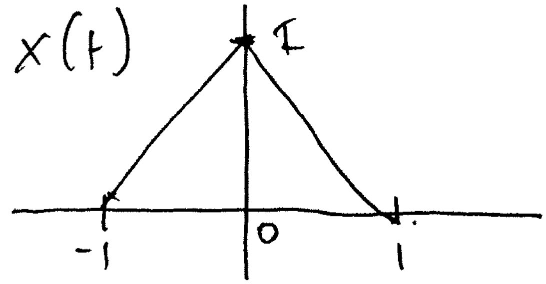
$2t+4 = 1 \Rightarrow 2t = -3 \Rightarrow t = -\frac{3}{2}$

$x(2(t+2)) =$



1.14 sketch

e) $x(2(t-2))$



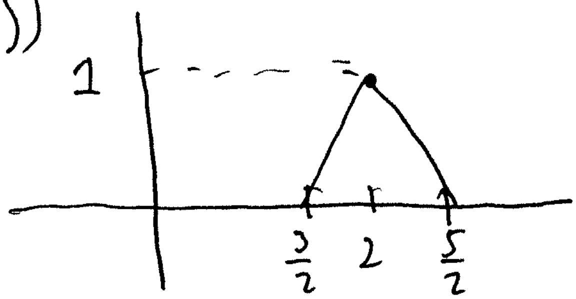
$$2(t-2) = 2t - 4$$

$$2t-4 = -1 \Rightarrow 2t = \cancel{1}3 \Rightarrow t = \cancel{2}\ \tfrac{3}{2}$$

$$2t-4 = 0 \Rightarrow 2t = 4 \Rightarrow t = 2$$

$$2t-4 = 1 \Rightarrow 2t = 5 \Rightarrow t = \tfrac{5}{2}$$

$x(2(t-2))$

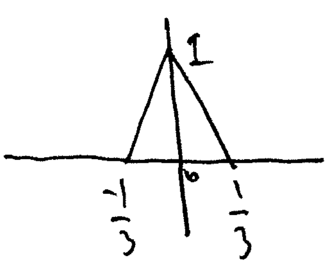


Notice the answer in the book has typos

f) sketch

$x(3t) + x(3t+2)$

from a) from b)



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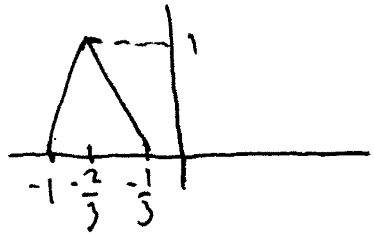

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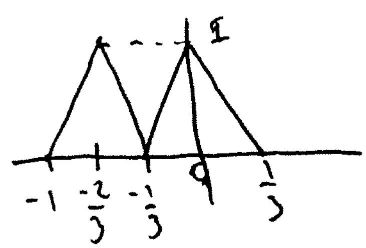


1.15 Discrete - time signal

$$x[n] = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & |n| > 2 \end{cases}$$

Find $y[n] = x[3n-2]$

So

$$y[n] = \begin{cases} 1 & -2 \le 3n-2 \le 2 \\ 0 & |3n-2| > 2 \quad \text{(i.e. otherwise)} \end{cases}$$

$$3n - 2 \le 2 \Rightarrow 3n \le 4$$

$$n \le \frac{4}{3}$$

since n is discrete then

$$n \le 1$$

$$3n - 2 \ge -2 \Rightarrow 3n \ge 0$$

$$n \ge 0$$

So

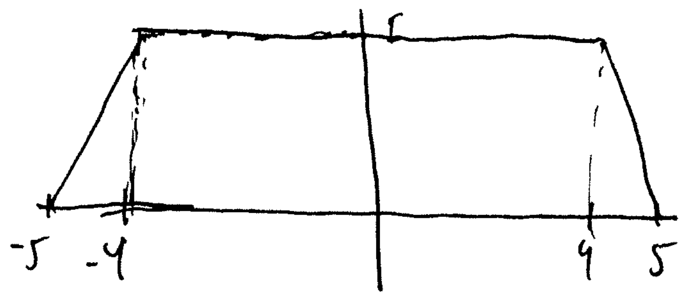
$$y[n] = \begin{cases} 1 & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

1.51

From $x(t)$ in fig P1.47 sketch

$$y(t) = x(10t - 5)$$

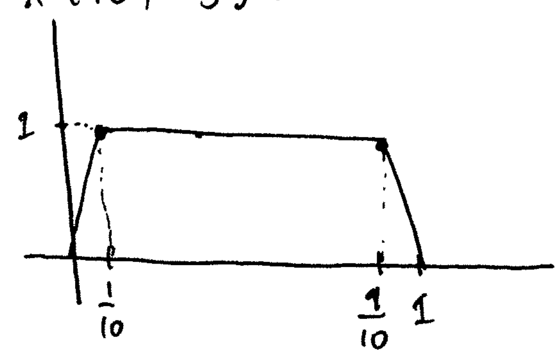
fig P1.47



I'm going to follow a similar procedure as with problem 1.14

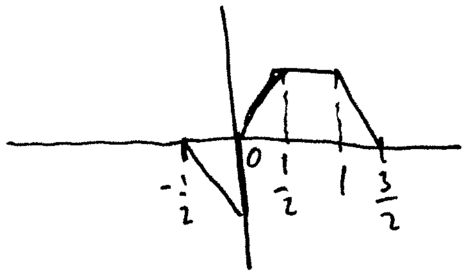
$$10t - 5 = -5 \Rightarrow 10t = 0 \Rightarrow 0$$
$$10t - 5 = -4 \Rightarrow 10t = 1 \Rightarrow t = \frac{1}{10}$$
$$10t - 5 = 4 \Rightarrow 10t = 9 \Rightarrow t = \frac{9}{10}$$
$$10t - 5 = 5 \Rightarrow 10t = 10 \Rightarrow t = 1$$

$y(t) = x(10t - 5) =$

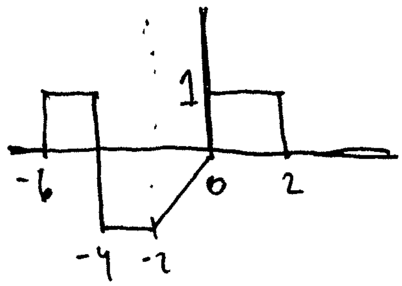


1.52 f) Sketch $z(t) = x(2t)\, y(\frac{1}{2}t+1)$

$x(2t) =$



$y(\frac{1}{2}t+1)$



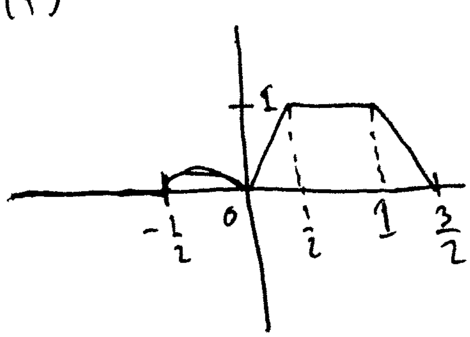
when $t = (0, 2)$ then $z(t) = x(2t)$

because in that range $y(\frac{1}{2}t+1) = 1$

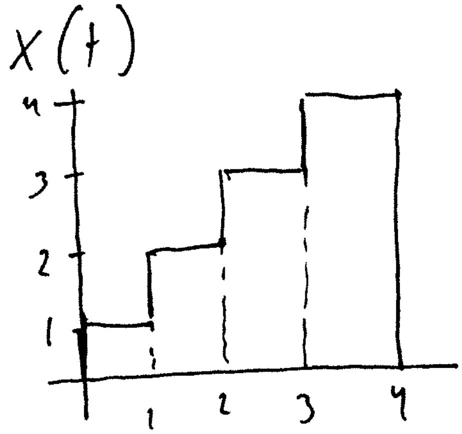
when $t = (-\frac{1}{2}, 0)$ then $x(2t) = -2t - 1$

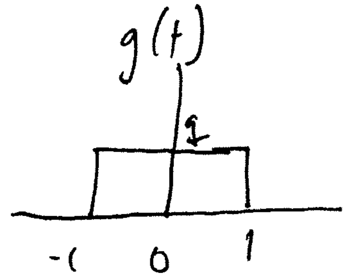
$y(\frac{1}{2}t+1) = \frac{1}{2}t$

$z(t) =$ $\qquad z(t) = -t^2 - \frac{1}{2}t$



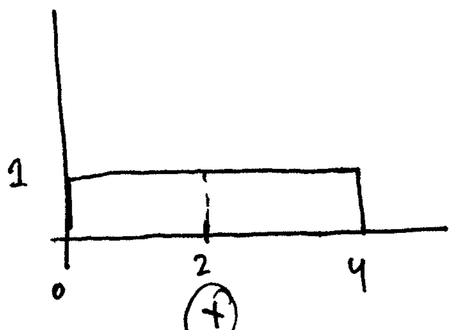
1.53





Express $x(t)$ in terms of $g(t)$

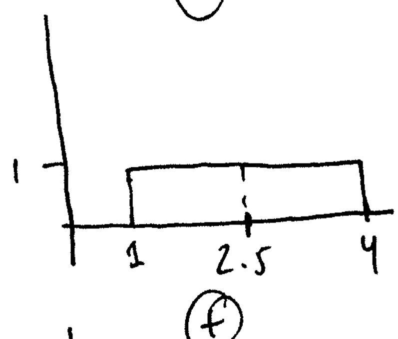
Notice that $x(t) =$



shift $g(t)$ by -2
stretch $g(t)$ by $\frac{1}{2}$

$= g\left(\frac{1}{2}(t-2)\right)$

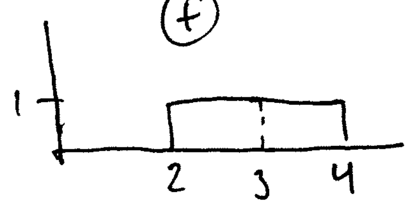
$+$



shift $g(t)$ by -2.5
stretch $g(t)$ by $\frac{1}{1.5} = \frac{1}{\left(\frac{3}{2}\right)} = \frac{2}{3}$

$= g\left(\frac{2}{3}(t-2.5)\right)$

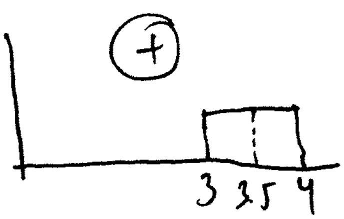
$+$



shift $g(t)$ by -3

$= g(t-3)$

$+$



shift $g(t)$ by -3.5
stretch $g(t)$ by 2

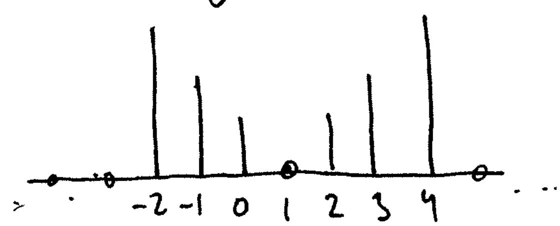
$= g\left(2\left(t-\frac{3}{5}\right)\right)$

So

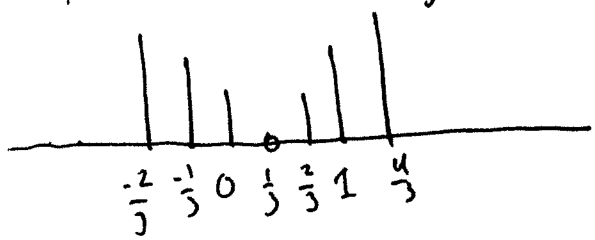
$$x(t) = g\left(\frac{1}{2}(t-2)\right) + g\left(\frac{2}{3}(t-2.5)\right) + g(t-3) + g\left(2\left(t-\frac{3}{5}\right)\right)$$

1.56 b) sketch $x[3n-1]$

first shift x by 1 to the right

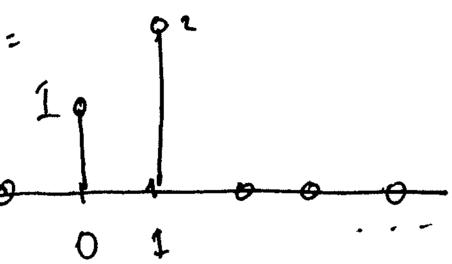


then scale by 3



eliminate non integer n

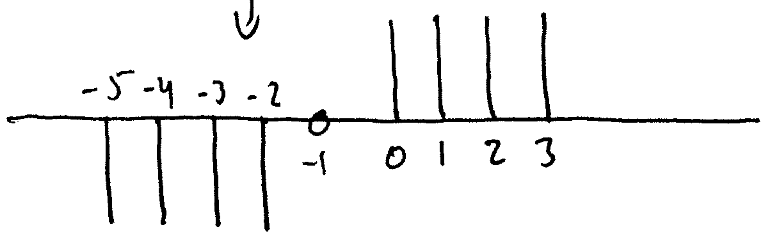
$x[3n-1] =$



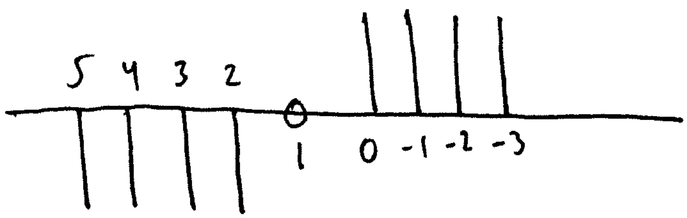
1.56 c)

sketch $y[1-n] = y[-n+1]$

first shift by 1 to the left

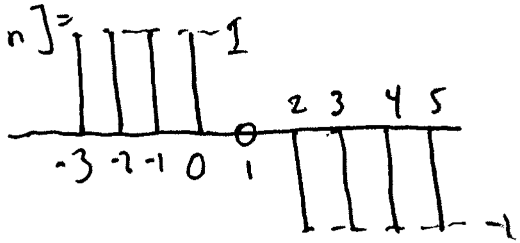


scale by -1



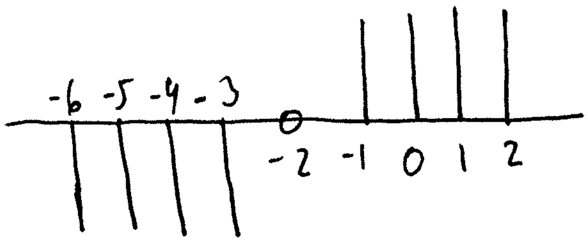
arrange it so n is increasing from left to right

$y[1-n] =$

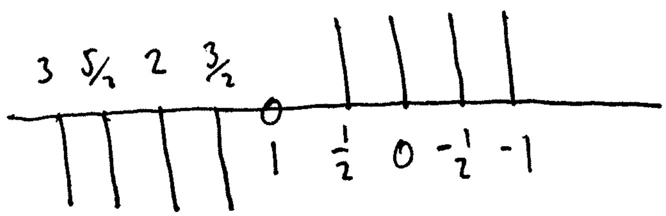


1.56 d) $y[2-2n] = y[-2n+2]$

first shift by 2 to the left

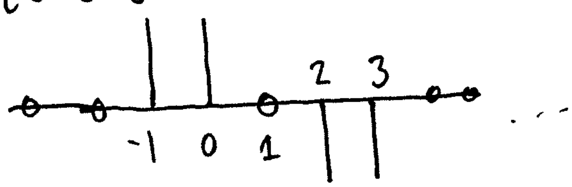


then scale by -2



Eliminate non-integer n and arrange so n is increasing

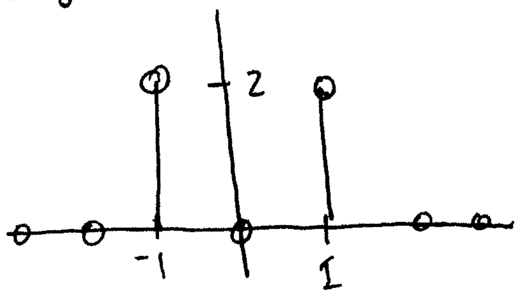
$y[2-2n]$



1.56 f)

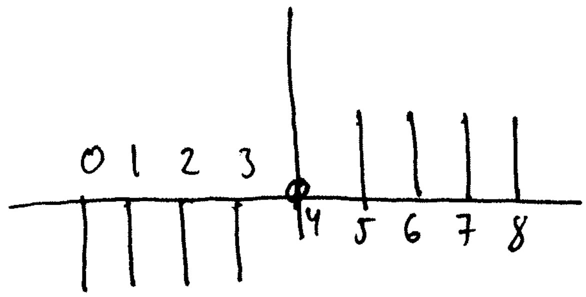
$x[2n] + y[n-4]$

$x[2n] =$



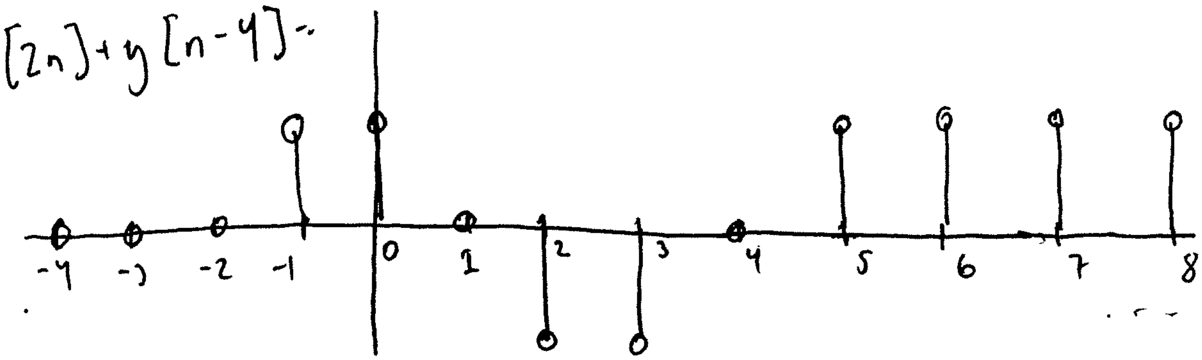
+

$y[n-4]$



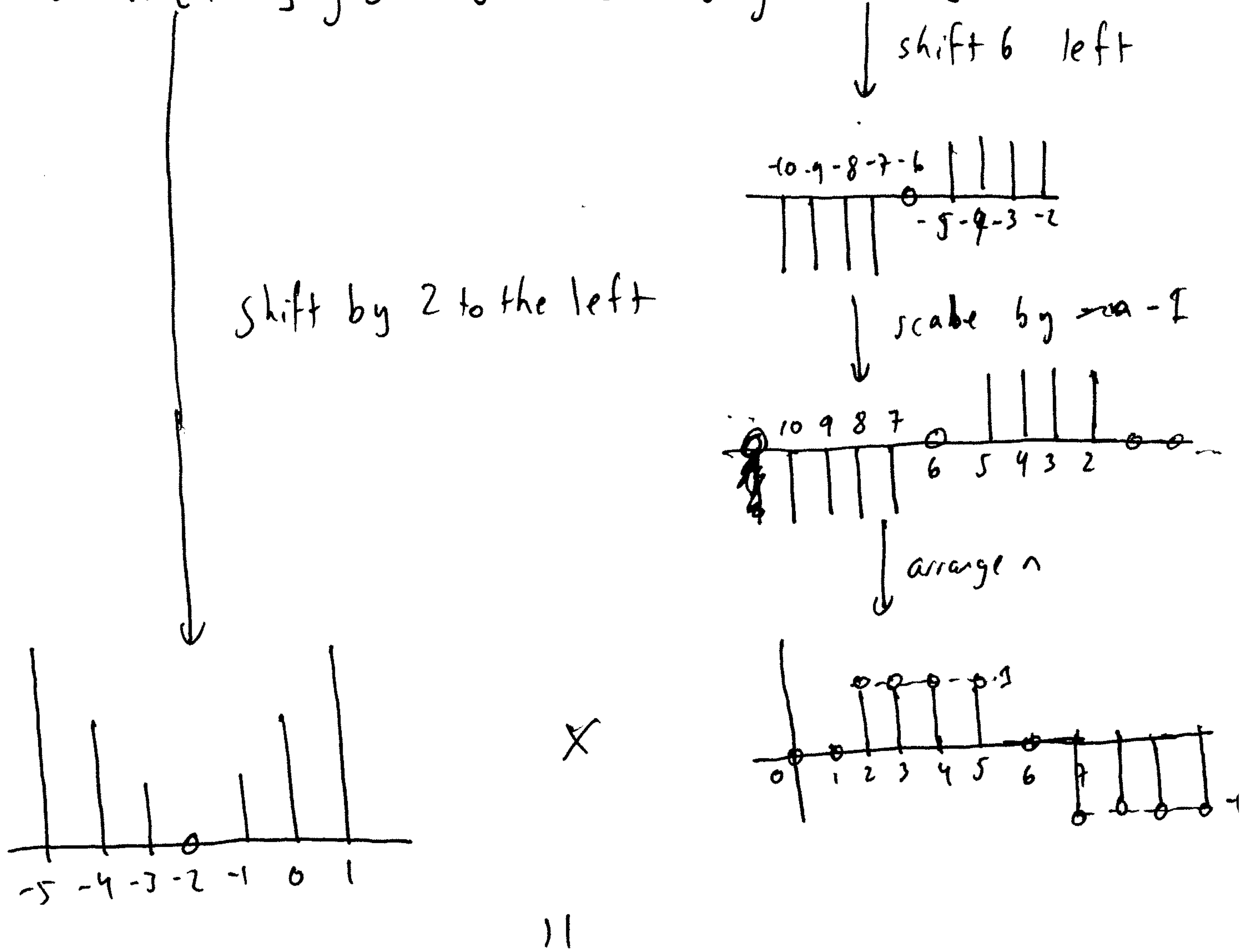
||

$x[2n] + y[n-4] =$



1.56

k) $x[n+2]\,y[6-n] = x[n+2]\,y[-n+6]$



$x[n+2]\,y[6-n] =$ zero signal