2.1, 2.2, 2.3, 2.5, 2.6, 2.32, 2.33 (a,c), 2.34 (a,e,k)
2.39 (a,b,n), 2.40 (a,k,p)

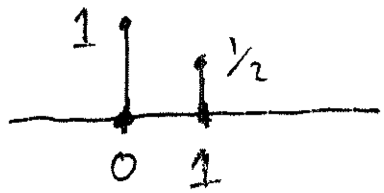
Solved by Sanjay Nair
sanjaynair@neo.tamu.edu

2.1.-

$$y[n] = x[n] + \frac{1}{2}x[n-1]$$

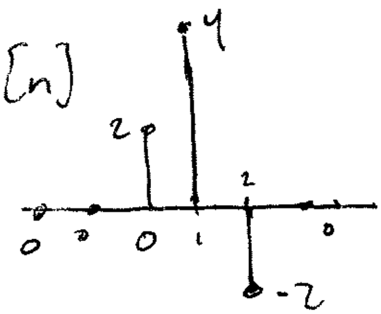
so

h[n]



$$x[n] = \begin{cases} 2, & n=0 \\ 4, & n=1 \\ -2, & n=2 \\ 0, & \text{else} \end{cases} \longrightarrow$$

x[n]



for this particular x[n] we'll have

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{k=\infty} x[k]\cdot h[n-k]$$

$$= \sum_{k=0}^{k=2} x[k]\cdot h[n-k]$$

$h[n-k] \neq 0$ if $0 \leq n-k \leq 1$

$k \leq n$ $\quad$ $n-k \leq 1$

$0 \leq n$ $\quad$ $n \leq 1+k$

$n \leq 3$

(continues)

2.1 continued

$$y[0] = \sum_{k=0}^{0} x[k]h[-k] = x[0]\cdot x[0] = 2\cdot 1 = 2$$

$$y[1] = \sum_{k=0}^{k=1} x[k]h[1-k] = x[0]\cdot h[1] + x[1]\cdot h[0]$$
$$= 2\cdot \tfrac{1}{2} + 4\cdot 1 = 5$$

$$y[2] = \sum_{k=0}^{k=2} x[k]h[2-k] = x[0]\cdot h[2] + x[1]\cdot h[1] + x[2]h[0]$$
$$= 2\cdot 0 + 4\cdot \tfrac{1}{2} + -2\cdot 1 = 2-2 = 0$$

$$y[3] = \sum_{k=0}^{k=2} x[k]h[3-k] = x[0]h[3] + x[1]h[2] + x[2]h[1]$$
$$= 0 + 0 - 2\cdot \tfrac{1}{2} = -1$$

$$y[n] = \begin{cases} 2, & n=0 \\ 5, & n=1 \\ 0, & n=2 \\ -1, & n=3 \\ 0, & \text{otherwise} \end{cases}$$

2.2

(a) $y[n] = u[n] * u[n-3]$

$$= \sum_{k=-\infty}^{\infty} u[k] \cdot u[n-3-k] = \sum_{k=0}^{\infty} 1 \cdot u[n-3-k]$$

$u[n-3-k] = 1$ if $n-3-k \geq 0 \Rightarrow n \geq 3+k \geq 3$

$$\Downarrow$$

$$-k \geq 3-n$$

$$k \leq n-3$$

$$y[n \geq 3] = \sum_{k=0}^{n-3} 1 = \underbrace{1+1+1+\ldots+1}_{n-2 \text{ times}} = n-2$$

$$y[n] = \begin{cases} 0 & n < 3 \\ n-2 & n \geq 3 \end{cases}$$

2.2 b)

$$y[n] = \left(\frac{1}{2}\right)^n u[n-2] * u[n]$$

$$= \sum_{k=-\infty}^{k=\infty} \left(\frac{1}{2}\right)^k u[k-2] \cdot u[n-k]$$

$$= \sum_{k=2}^{k=\infty} \left(\frac{1}{2}\right)^k \cdot u[n-k]$$

$$u[n-k] = 1 \quad \text{if} \quad n-k \geq 0 \Rightarrow n \geq k \geq 2$$

$$\Downarrow$$

$$-k \geq -n$$

$$k \leq n$$

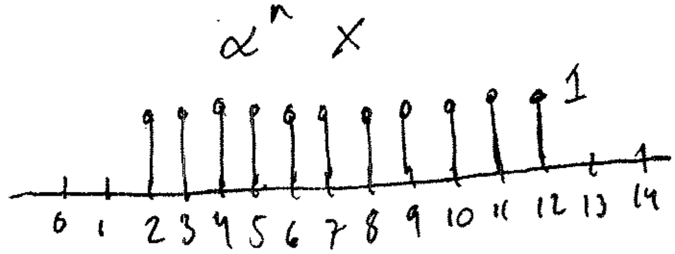
$$y[n \geq 2] = \sum_{k=2}^{n} \left(\frac{1}{2}\right)^k = \frac{1}{2} - \left(\frac{1}{2}\right)^n$$

$$y[n] = \begin{cases} 0, & n < 2 \\ \frac{1}{2} - \left(\frac{1}{2}\right)^n, & n \geq 2 \end{cases}$$

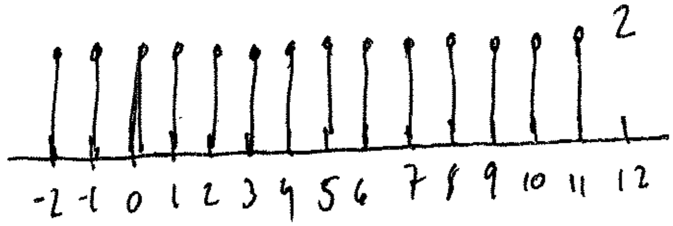
c)

$$y[n] = \underbrace{\alpha^n \{u[n-2] - u[n-13]\}}_{\text{call this } x[n]} * \underbrace{2\{u[n+2] - u[n-12]\}}_{\text{call this } h[n]}$$

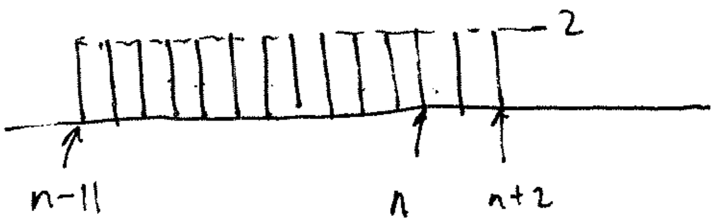
$x[n]$ can be sketched as:



$h[k]$ can be sketched as:



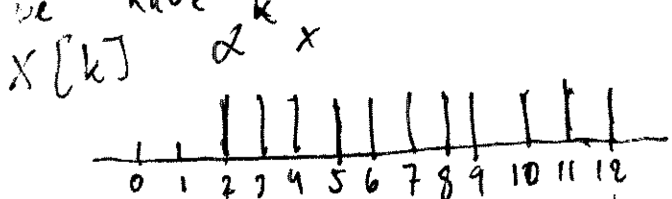
so $h[n-k]$ can be sketched as:



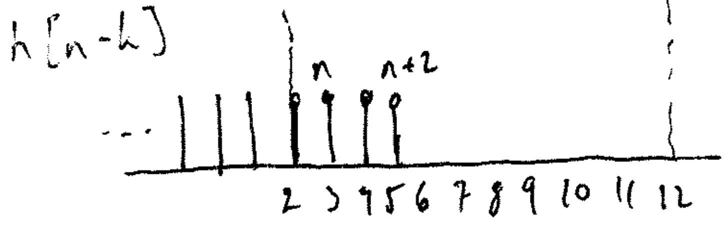
so $y[n] = 0$ if $n < 0$ or if $n \geq 24$

let's consider the case of $0 \leq n \leq 10$

we have



$$y[n] = \sum_{k=2}^{k=n+2} \alpha^k \cdot 2 = 2 \sum_{k=2}^{n+2} \alpha^k$$



$$\sum_{k=0}^{n-1} ar^k = a\,\frac{1-r^n}{1-r}$$

$$y[n] = \sum_{k=2}^{n+2} 2\cdot\alpha^k = \sum_{k=0}^{n+2} 2\cdot\alpha^k - \sum_{k=0}^{k=1} 2\cdot\alpha^k$$

$$= 2\cdot\frac{1-\alpha^{n+3}}{1-\alpha} - 2\cdot\frac{1-\alpha^2}{1-\alpha}$$

$$= 2\cdot\frac{(1-\alpha^{n+3})-(1-\alpha^2)}{(1-\alpha)}$$

$$= 2\cdot\frac{\alpha^2-\alpha^{n+3}}{1-\alpha}\cdot\frac{(-1)}{(-1)} = \frac{2\cdot(\alpha^{n+3}-\alpha^2)}{\alpha - 1}$$

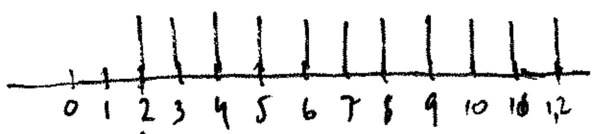
$$= \frac{2(\alpha^{n+3})}{\alpha}\cdot\frac{(1-\alpha^{-1-n})}{(1-\alpha^{-1})} = 2\alpha^{n+2}\,\frac{1-\alpha^{-1-n}}{1-\alpha^{-1}}$$

$$y[n] = 2\alpha^{n+2}\left(\frac{1-\alpha^{-1-n}}{1-\alpha^{-1}}\right) \quad \text{if} \quad 0 \le n \le 10$$

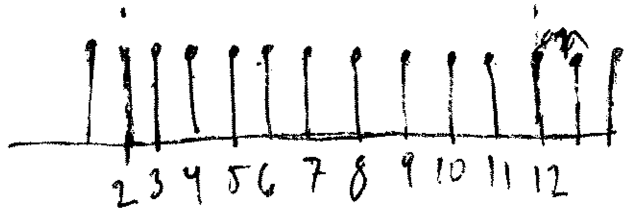
now let's consider $11 \le n \le 13$

$x[k] = \alpha^k \quad x$

$$y[n] = \sum_{k=2}^{k=12} 2\cdot\alpha^k$$



$h[n-k]$



$$y[n] = \sum_{k=2}^{k=12} 2 \cdot \alpha^k = \sum_{k=0}^{k=12} 2 \cdot \alpha^k - \sum_{k=0}^{k=1} 2 \cdot \alpha^k$$

$$= 2 \cdot \frac{1-\alpha^{13}}{1-\alpha} - 2 \cdot \frac{1-\alpha^{2}}{1-\alpha}$$

$$= 2 \cdot \frac{\alpha^2 - \alpha^{13}}{1-\alpha} = 2\, \frac{\alpha^{13} - \alpha^2}{\alpha - 1} = \frac{2\alpha^{13}}{\alpha} \cdot \frac{(1-\alpha^{-11})}{(1-\alpha^{-1})}$$

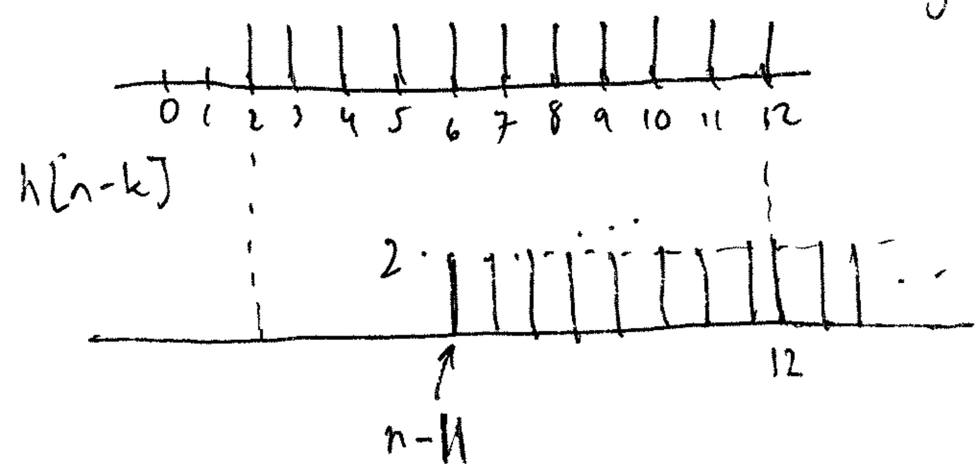
$$= 2\alpha^{12} \frac{(1-\alpha^{-11})}{(1-\alpha^{-1})}$$

$$y[n] = 2\alpha^{12}\left(\frac{1-\alpha^{-11}}{1-\alpha^{-1}}\right) \quad \text{if} \quad 11 \le n \le 13$$

finally, consider $14 \le n \le 23$

$x[k]$ α^k times

$$y[n] = \sum_{k=n-11}^{k=12} 2 \cdot \alpha^k$$



$$y[n] = \sum_{k=n-11}^{12} 2\cdot\alpha^k = \sum_{k=0}^{12} 2\cdot\alpha^k - \sum_{k=0}^{n-12} 2\cdot\alpha^k$$

$$= 2\cdot\frac{1-\alpha^{13}}{1-\alpha} - 2\cdot\frac{1-\alpha^{n-11}}{1-\alpha}$$

$$= 2\cdot\frac{\alpha^{n-11}-\alpha^{13}}{1-\alpha} = 2\cdot\frac{\alpha^{13}-\alpha^{n-11}}{\alpha-1}$$

$$= \frac{2\alpha^{13}}{\alpha}\cdot\frac{(1-\alpha^{n-24})}{(1-\alpha^{-1})} = 2\alpha^{12}\left(\frac{1-\alpha^{n-24}}{1-\alpha^{-1}}\right)$$

$$y[n] = 2\alpha^{12}\left(\frac{1-\alpha^{n-24}}{1-\alpha^{-1}}\right)$$

$$y[n] = \begin{cases} 0, & n < 0 \\ 2\alpha^{n+2}\left(\frac{1-\alpha^{-1-n}}{1-\alpha^{-1}}\right), & 0 \le n \le 10 \\ 2\alpha^{12}\left(\frac{1-\alpha^{-11}}{1-\alpha^{-1}}\right), & 11 \le n \le 13 \\ 2\alpha^{12}\frac{1-\alpha^{n-24}}{1-\alpha^{-1}}, & 14 \le n \le 23 \\ 0, & n \ge 24 \end{cases}$$

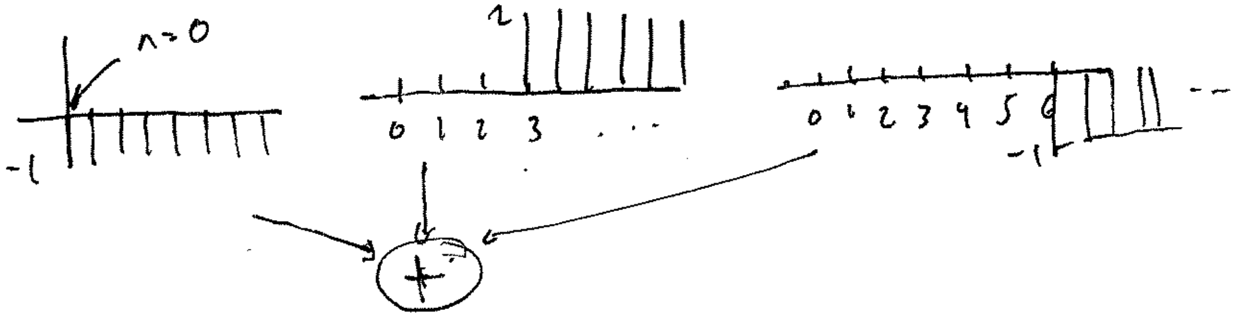
the book has a typo

2.2

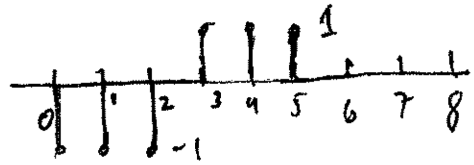
d) $y[n] = \underbrace{\left(-u[n] + 2u[n-3] - u[n-6]\right)}_{\text{call this } x[n]} * \underbrace{\left(u[n+1] - u[n-10]\right)}_{\text{call this } h[n]}$

$x[n]$ is

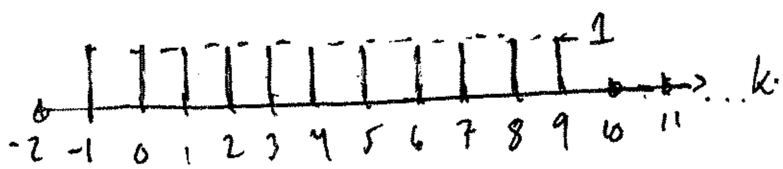
$-u[n] \quad + \quad 2u[n-3] \quad - \quad u[n-6]$



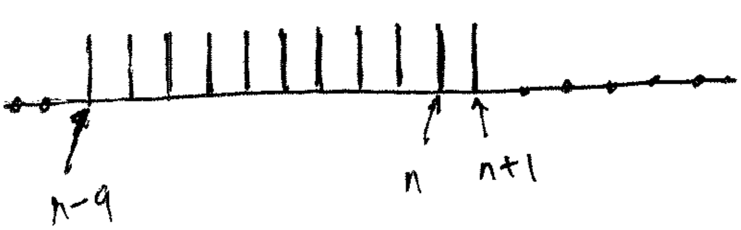
$x[n] =$



$h[k]$ is



so $h[n-k]$ is



so

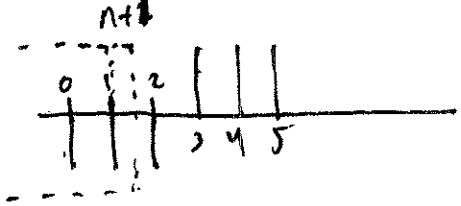
$y[n] = 0$ if

$n \leq -2 \quad (n < -1)$

$n \geq 15 \quad (n > 14)$

let's look at $-1 \le n \le 1$

x[k]

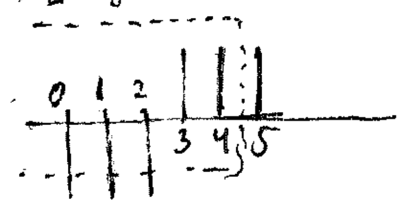


$$y[n] = \sum_{k=0}^{n+1} (-1) = (-1)\cdot(n+2) = -(n+2)$$

$$y[n] = -(n+2) \quad \text{if} \quad -1 \le n \le 1$$

now if $2 \le n \le 4$

x[k]



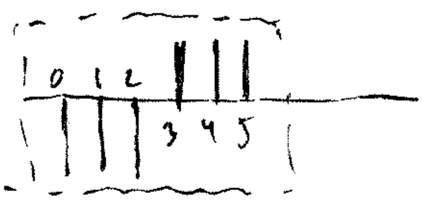
$$y[n] = -3 + \sum_{k=3}^{n+1} 1$$

$$= -3 + \sum_{k=0}^{n+1} 1 - \sum_{k=0}^{k=2} 1 = -3 - 3 + (n+2) = n-4$$

$$y[n] = n-4 \quad \text{if} \quad 2 \le n \le 4$$

if $5 \le n \le 9$

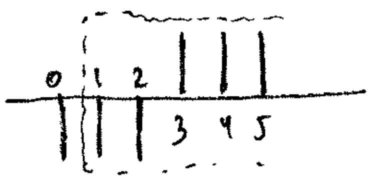
x[k]



$$y[n] = -3 + 3 = 0$$

$$y[n] = 0 \quad \text{if} \quad 5 \le n \le 9$$

if $10 \le n \le 11$



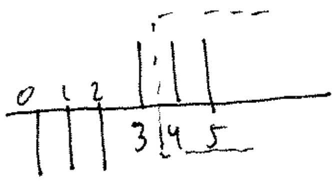
$$y[n] = \sum_{k=n-9}^{k=2} -1 + \sum_{k=3}^{k=5} 1$$

$$= \sum_{k=0}^{k=2} -1 + \sum_{k=0}^{n-10} 1 + \sum_{k=3}^{k=5} 1 = \sum_{k=0}^{n-10} 1 = n-9$$

$$y[n] = n-9 \quad \text{if} \quad 10 \le n \le 11$$

if $12 \le n \le 14$

$x[k]$



$$y[n] = \sum_{k=n-9}^{5} 1 = \sum_{k=0}^{k=5} 1 - \sum_{k=0}^{n-10} 1$$

$$= 6 - (n-9) = 15 - n$$

$$y[n] = 15 - n \quad \text{if} \quad 12 \le n \le 14$$

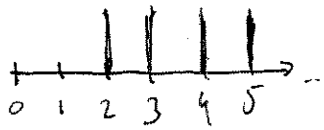
$$y[n] = \begin{cases} 0, & n - 1 \\ -(n+2), & -1 \le n \le 1 \\ n-4, & 2 \le n \le 4 \\ 0, & 5 \le n \le 9 \\ n-9, & 10 \le n \le 11 \\ 15-n, & 12 \le n \le 14 \\ 0, & n > 14 \end{cases}$$

2.2

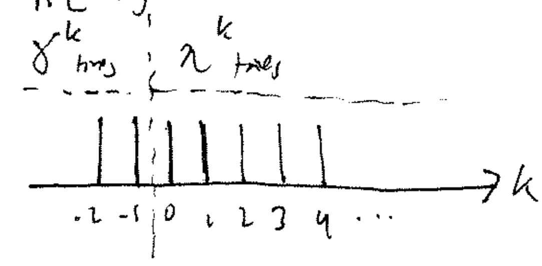
e) $y[n] = u[n-2] * h[n]$ where

$$h[n] = \begin{cases} \gamma^n, & n < 0, \ |\gamma| > 1 \\ \eta^n, & n \geq 0, \ |\eta| < 1 \end{cases}$$

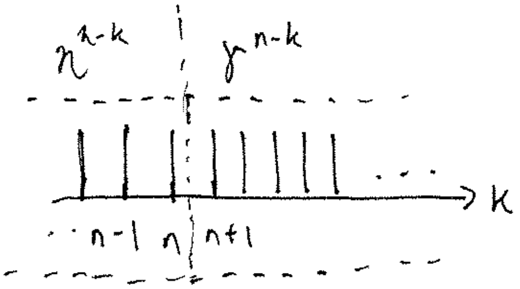
$u[n-2]$



$h[k] =$



$h[n-k]$



so if $n < 2$

$$y[n] = \sum_{k=2}^{k=\infty} \gamma^{n-k} = \sum_{l=2-n}^{\infty} \gamma^{-l} = \sum_{l=2-n}^{l=\infty} \left(\frac{1}{\gamma}\right)^{l}$$

let $-l = n-k$

so if $k=2$: $-l = n-2$, $l = 2-n$

if $k = \infty$: $-l = n - \infty$, so $l = \infty$

$$\sum_{l=2-n}^{l=\infty}\left(\frac{1}{\gamma}\right)^{l}=\sum_{l=0}^{l=\infty}\left(\frac{1}{\gamma}\right)^{l}-\sum_{l=0}^{l=2-n-1}\left(\frac{1}{\gamma}\right)^{l}$$

$$\sum_{k=0}^{\infty} ar^{k}=\frac{a}{1-r} \quad \text{if } |r|<1$$

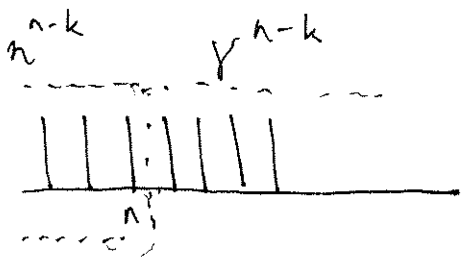
since $|\gamma|>1$ we can use this equation because $\left|\frac{1}{\gamma}\right|<1$

$$=\frac{1}{1-\left(\frac{1}{\gamma}\right)}-\frac{1-\left(\frac{1}{\gamma}\right)^{2-n}}{1-\left(\frac{1}{\gamma}\right)}=\frac{\left(\frac{1}{\gamma}\right)^{2-n}}{1-\left(\frac{1}{\gamma}\right)}=\frac{\gamma^{n-2}}{1-\gamma^{-1}}$$

$$=\frac{\gamma}{\gamma}\cdot\frac{\gamma^{n-2}}{1-\gamma^{-1}}=\frac{\gamma^{n-1}}{\gamma-1} \quad \text{if } n<2$$

$$y[n]=\frac{\gamma^{n-1}}{\gamma-1} \quad \text{if } n<2$$

now if $n \geq 2$



the n^{n-k} terms start getting involved

$$y[n]=\sum_{k=2}^{k=n} \cancel{n}^{\,n-k}+\sum_{l=0}^{l=\infty}\left(\frac{1}{\gamma}\right)^{l}$$

we know $\sum_{l=0}^{l=\infty} \left(\frac{1}{\gamma}\right)^{l} = \frac{1}{1-\left(\frac{1}{\gamma}\right)}$

$$\sum_{k=2}^{k=n} \eta^{n-k} = \sum_{m=0}^{m=n-2} \eta^{m} = \frac{1-\eta^{n-1}}{1-\eta}$$

consider $m = n-k$

if $k = 2$ if $k = n$

$m = n-k$ $m = 0$

$m = n-2$

$$y[n] = \frac{1-\eta^{n+1}}{1-\eta} + \frac{1}{1-\left(\frac{1}{\gamma}\right)} \qquad \text{if } n \geq 2$$

$$y[n] = \begin{cases} \frac{\gamma^{n-1}}{\gamma-1} & \text{if } n < 2 \\ \frac{1}{1-\left(\frac{1}{\gamma}\right)} + \frac{1-\eta^{n+1}}{1-\eta} & n \geq 2 \end{cases}$$

2.3

$x(t) = u(t) \qquad h(t) = e^{-t}u(t)$

$$y(t) = \int_{-\infty}^{\infty} u(\tau) \cdot h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \underset{\uparrow\ \tau > 0}{u(\tau)}\, e^{-(t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_0^{\infty} e^{-(t-\tau)} u(t-\tau)$$

$u(t-\tau) = 1 \quad \text{if} \quad t - \tau > 0$

so

we can pull out a $u(t)$ $\rightarrow$ $\boxed{t > \tau > 0}$ $\qquad -\tau > -t$, $\tau < t$

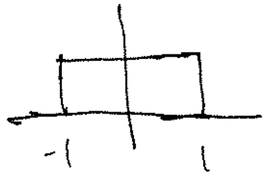
$$y(t) = u(t) \int_0^t e^{\tau - t}\, d\tau = u(t) \cdot \left(e^{\tau - t}\right)_0^t$$

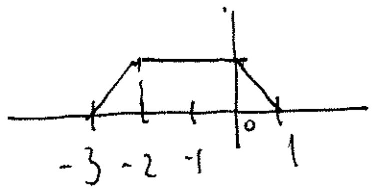
$$= u(t) \cdot \left(e^{t-t} - e^{0-t}\right) = u(t)\left(e^0 - e^{-t}\right)$$

$$= u(t) \cdot \left(1 - e^{-t}\right)$$

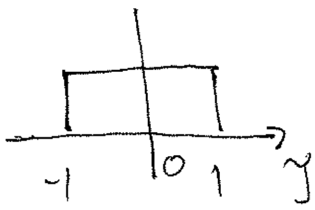
2.5

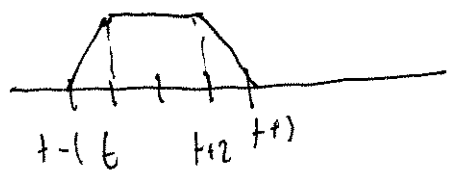
$x(t)$ $h(t)$





so $x(\tau)$ $h(t-\tau)$

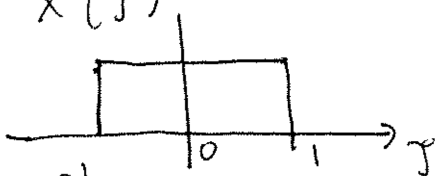




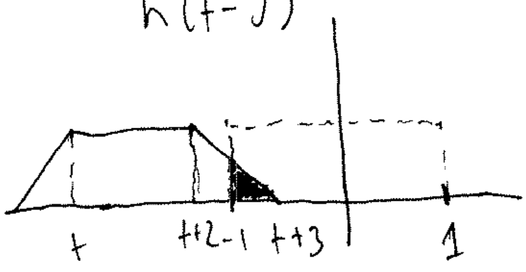
from the begining we can say that $y(t) = 0$ if $t < -4$ $t > 2$

now consider $-4 \le t < -3$

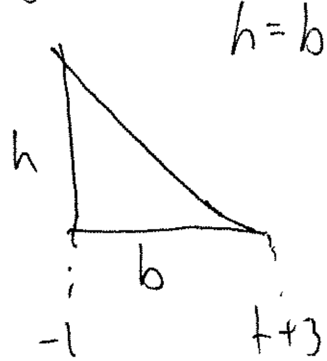
$x(\tau)$



$h(t-\tau)$



$y(t) =$ area of shaded triangle



$h = b = (t+3) - (-1)$

$= t+4$

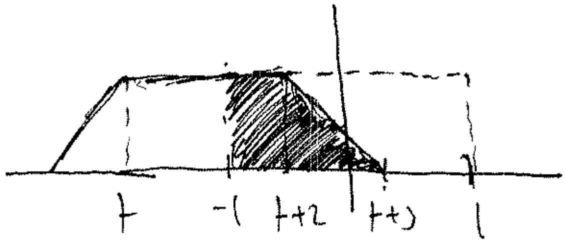
$\text{area} = \frac{hb}{2} = \frac{(t+4)^2}{2}$

$y(t) = \frac{t^2}{2} + 4t + 8$ if $-4 \le t < -3$

now consider

$-3 \le t < -2$

$h(t-\tau)$

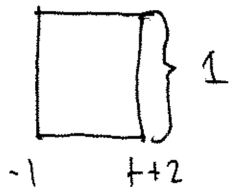


$y(t) =$ shaded area

$=$ box area $+$ triangle area

triangle area $= \frac{1}{2}$

box area $=$ area

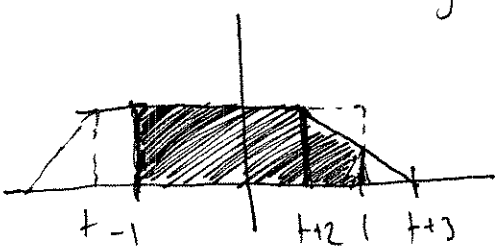


$= (t+3) \cdot 1 = t+3$

$$y(t) = t+3 + \frac{1}{2} = t + \frac{7}{2} \quad \text{if} \quad -3 \le t < -2$$

consider $-2 \le t < -1$

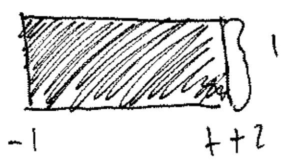
$h(t-\tau)$



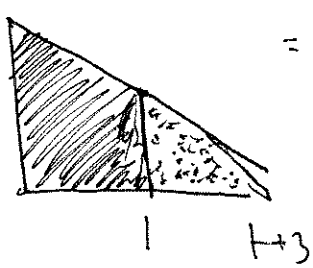
$y(t) =$ shaded area

$y(t) =$ box area $+$ trap area

box area $= t+3$



trap area $= \frac{1}{2} -$ dotted area



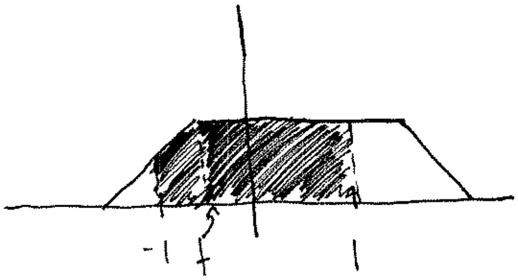
$$= \frac{1}{2} - \frac{(t-2)^2}{2} = \frac{1}{2} - \frac{t^2 - 4t + 4}{2}$$

$$= -\frac{t^2}{2} - 2t - \frac{3}{2}$$

$$y(t) = t+3 - \frac{t^2}{2} - 2t - \frac{3}{2} = -\frac{t^2}{2} - t + \frac{3}{2} \quad \text{if} \ -2 \le t < -1$$

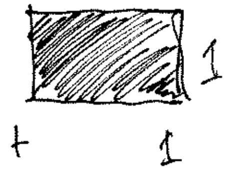
now consider $-1 \le t < 0$

$h(t-\tau)$

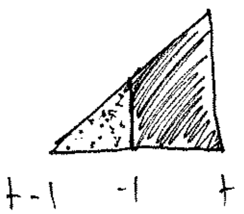


$y(t) =$ shaded area
$=$ box area + trap area

box area $= (1-t)\cdot 1 = 1-t$



trap area $= \frac{1}{2} -$ dotted area

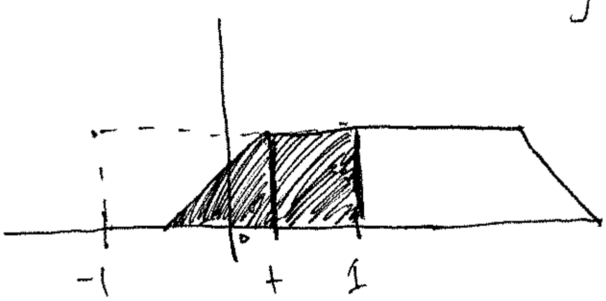


$$= \frac{1}{2} - \frac{(-1-(t-1))^2}{2} = \frac{1}{2} - \frac{(-t)^2}{2} = \frac{1}{2} - \frac{t^2}{2}$$

$$y(t) = 1-t+\frac{1}{2}-\frac{t^2}{2} = -\frac{t^2}{2} - t + \frac{3}{2}$$

consider $0 \le t < 1$

$h(t-\tau)$



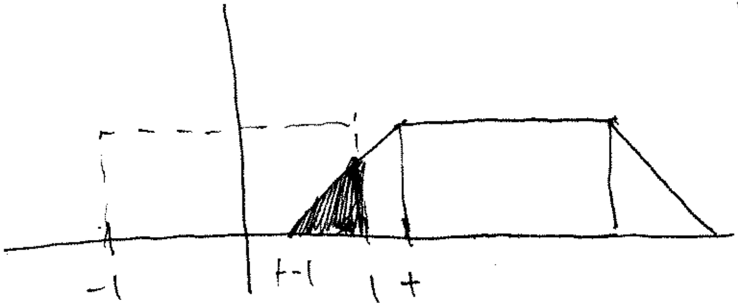
$y(t) =$ shaded area
$=$ box area + triangle area

$$= (1-t) + \frac{1}{2}$$

$$= \frac{3}{2} - t$$

$$y(t) = \frac{3}{2} - t \quad \text{if } 0 \le t < 1$$

consider $1 \leq t < 2$



$y(t) =$ shaded area

$=$ triangle area

$$= \frac{(1-(t-1))^2}{2} = \frac{(-t+2)^2}{2}$$

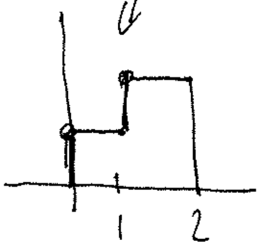
$$= \frac{t^2}{2} - \frac{4t}{2} + \frac{4}{2}$$

$$y(t) = \frac{t^2}{2} - 2t + 2 \quad \text{if } 1 \leq t < 2$$

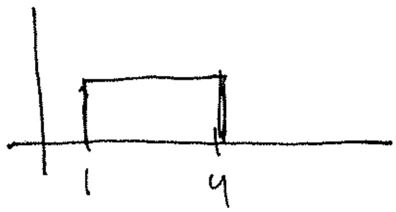
$$y(t) = \begin{cases} 0, & t < -4,\ t > 2 \\ \frac{1}{2} t^2 + 4t + 8, & -4 \leq t < -3 \\ t + \frac{7}{2}, & -3 \leq t < -2 \\ -\frac{t^2}{2} - t + \frac{3}{2}, & -2 \leq t < 0 \\ \frac{3}{2} - t, & 0 \leq t < 1 \\ \frac{t^2}{2} - 2t + 2, & 1 \leq t < 2 \end{cases}$$

2.6

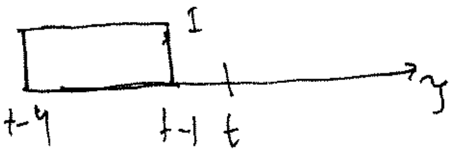
$x(t) = u(t) + u(t-1) - 2u(t-2)$



$h(t) = u(t-1) - u(t-4)$



$h(t-\tau)$

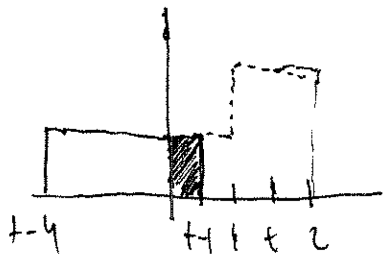


so clearly

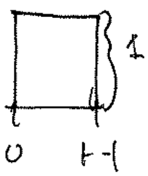
$y(t) = 0$ if $t < 1$ or $t \geq 6$

consider $1 \leq t < 2$

$h(t-\tau)$

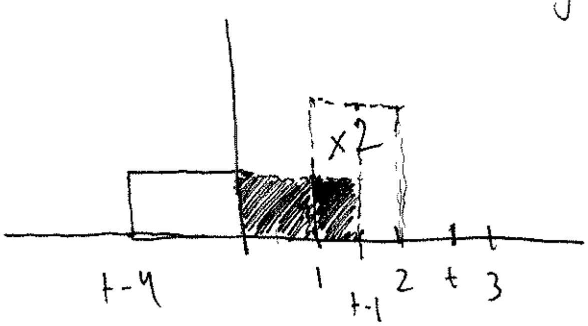


$y(t) =$ box area

$=$ area  $= t-1$

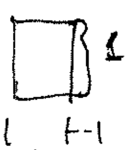
consider $2 \leq t < 3$

$h(t-\tau)$



$y(t) =$ shaded area after bonuses

$=$ square area $+ 2\times$ box area

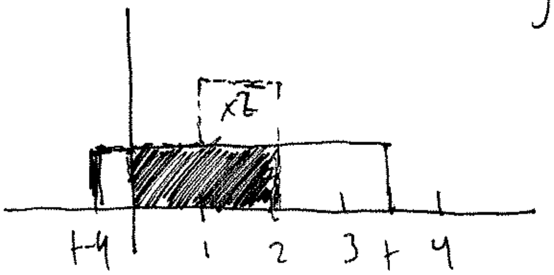
$= 1 + 2\cdot$area 

$= 1 + 2(t-1-1) = 1 + 2t - 4$

$y(t) = 2t - 3$ if $2 \leq t < 3$

consider $3 \le t < 4$

$h(t-\tau)$



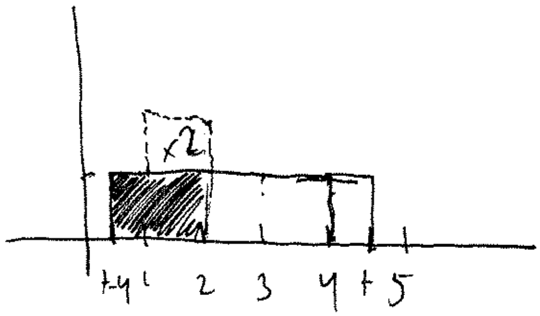
$y(t)$ = shaded area after bonuses

= square area + 2 · square area

$= 1 + 2 \cdot 1 = 3$

$y(t) = 3$ if $3 \le t < 4$

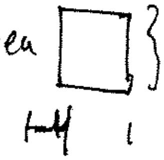
consider $4 \le t < 5$



$y(t)$ = shaded area after bonuses

$y(t)$ = box area + 2 · square area

= area



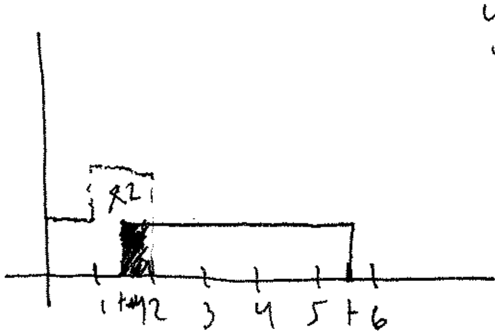
$+ 2 \cdot 1$

$= 1-(t-4)+2$

$= -t+5+2 = 7-t$

$y(t) = 7-t$ if $4 \le t < 5$

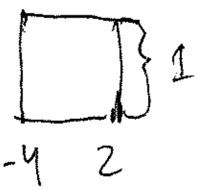
consider $5 \le t < 6$



$y(t)$ = shaded area after bonuses

$y(t) = 2 \cdot$ box area

$= 2 \cdot$ area

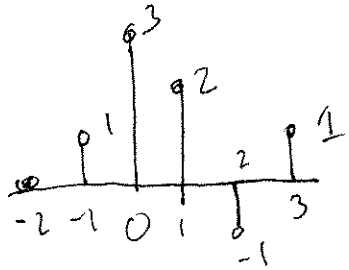


$= 2(2-(t-4)) = 2(-t+6) = 12-2t$

$y(t) = 12-2t$ if $5 \le t < 6$

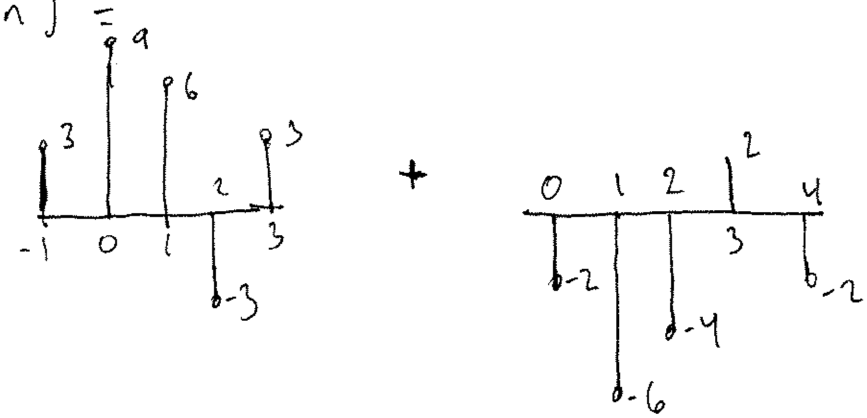
2.32

$h[n]$

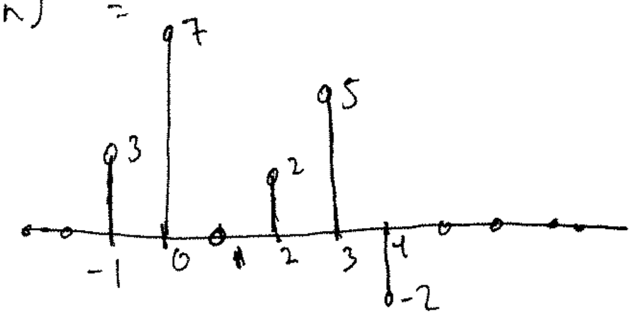


(a) $x[n] = 3\delta[n] - 2\delta[n-1]$

$y[n] =$

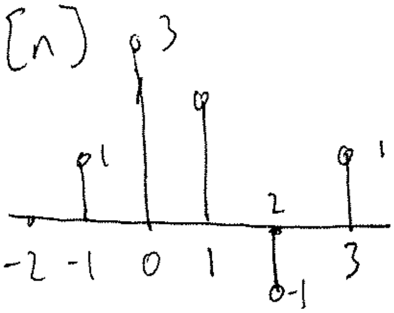


$y[n] =$



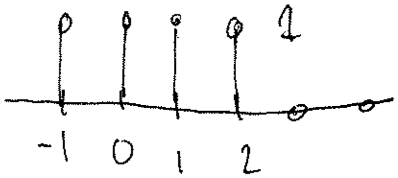
2.32 (b)

$h[n]$

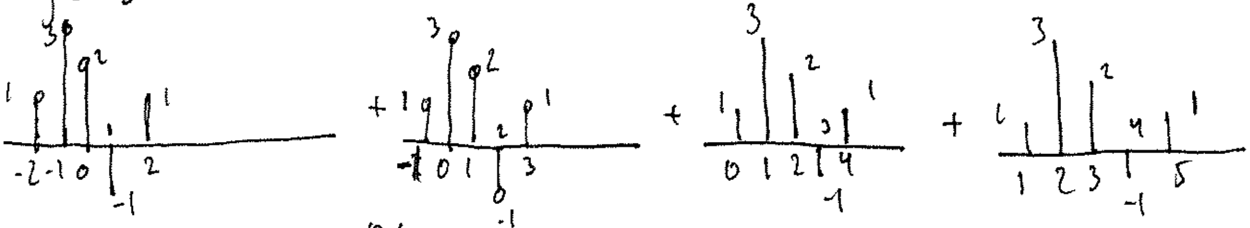


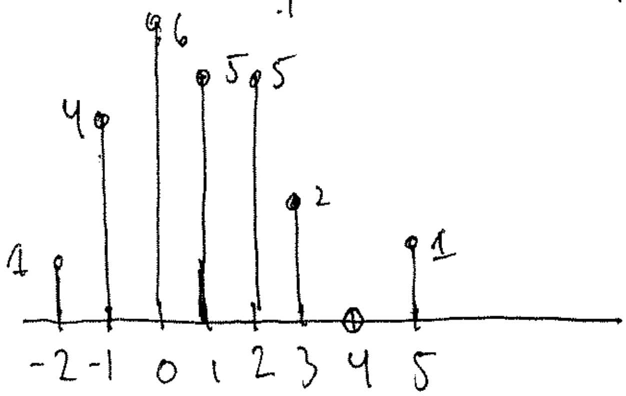
$$x[n] = u[n+1] - u[n-3]$$

$=$



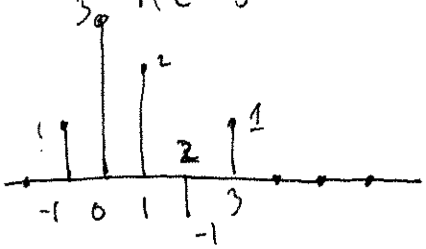
$y[n] =$



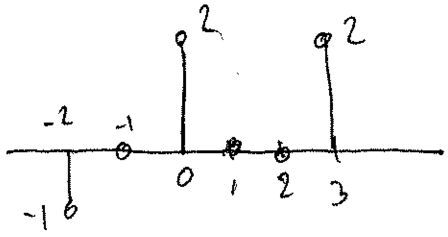


2.32

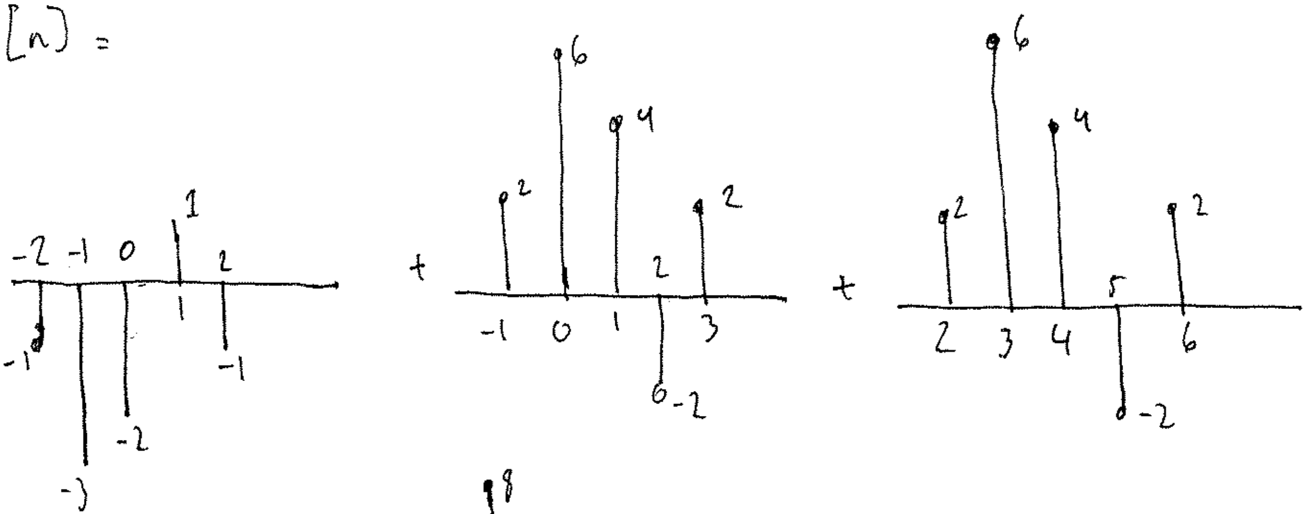
(c) h[n]



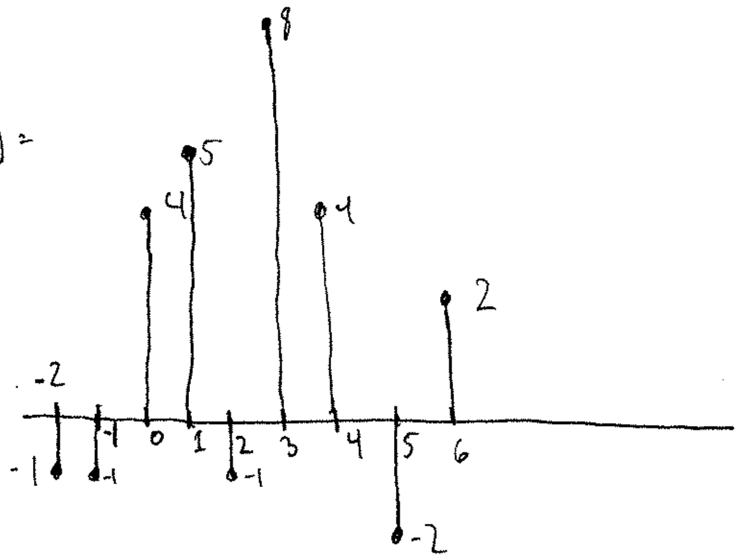
x[n]



y[n] =



y[n] =



2.33

(a) $y[n] = u[n+3] * u[n-3]$

$$y[n] = \sum_{k=-\infty}^{k=\infty} u[k+3] \cdot u[n-k-3]$$

$$u[k+3] = 1 \quad \text{if} \quad k+3 \geq 0 \Rightarrow k \geq -3$$

$$y[n] = \sum_{k=-3}^{k=\infty} u[k+3] \cdot u[n-k-3]$$

$$u[n-k-3] = 1 \quad \text{if} \quad n-k-3 \geq 0$$

$$n-k \geq 3$$

$$n \geq 3+k \qquad\qquad -k \geq 3-n$$

$$n \geq 0 \qquad\qquad k \leq n-3$$

$$y[n \geq 0] = \sum_{k=-3}^{k=n-3} 1 = \sum_{k=0}^{k=n} 1 = n+1$$

$$y[n] = \begin{cases} 0 & n < 0 \\ n+1 & n \geq 0 \end{cases}$$

2.33

(c)

$$y[n] = \left(\frac{1}{4}\right)^n u[n] * u[n+2]$$

$$= \sum_{k=-\infty}^{k=\infty} \left(\frac{1}{4}\right)^k u[k] \cdot u[n-k+2]$$

$$u[k] = 1 \quad \text{if} \quad k \geq 0$$

$$= \sum_{k=0}^{k=\infty} \left(\frac{1}{4}\right)^k u[n-k+2]$$

$$u[n-k+2] = 1 \quad \text{if} \quad n-k+2 \geq 0$$

$$n \geq -2+k \qquad -k \geq -n-2$$

$$n \geq -2 \qquad k \leq n+2$$

$$y[n \geq -2] = \sum_{k=0}^{k=n+2} \left(\frac{1}{4}\right)^k = \frac{1-\left(\frac{1}{4}\right)^{n+3}}{1-\left(\frac{1}{4}\right)} = \frac{1-\left(\frac{1}{4}\right)^{n+3}}{\frac{3}{4}}$$

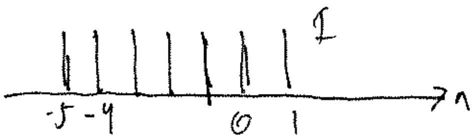
$$= \frac{4}{3} \cdot \left(1-\left(\frac{1}{4}\right)^{n+3}\right)$$

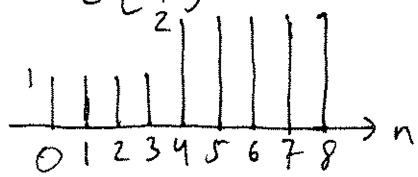
$$y[n] = \begin{cases} 0 & \text{if } n < -2 \\ \frac{4}{3}\left(1-\left(\frac{1}{4}\right)^{n+3}\right) & \text{if } n \geq -2 \end{cases}$$

2.34

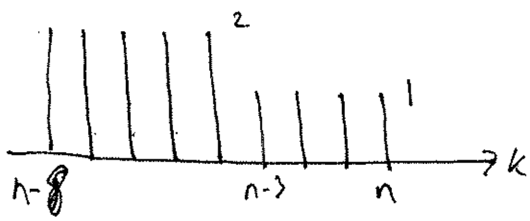
(a) $m[n] = x[n] * z[n]$

$x[n]$ $\qquad\qquad$ $z[n]$





$z[n-k]$



$m[n] = 0$ if
$n \le -6$
$n \ge 10$

consider $-5 \le n \le -2$

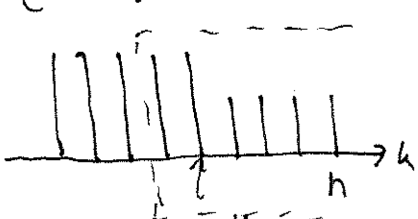
$z[n-k]$

$$m[n] = \sum_{k=-5}^{n} (1) = \sum_{k=0}^{n+5} (1) = n+6$$

$m[n] = n+6$ if $-5 \le n \le -2$

consider $-1 \le n \le 1$

$z[n-k]$



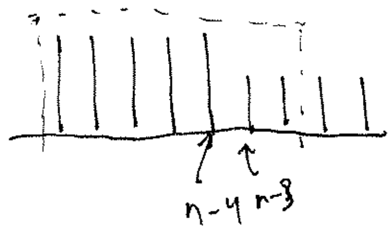
$$m[n] = 4 + \sum_{k=-5}^{n-4} 2 = 4 + \sum_{k=0}^{n+1} 2 = 4 + 2(n+2)$$

$m[n] = 2n + 8$ if $-1 \le n \le 1$

consider

$2 \le n \le 3$

$z[n-k]$



$$m[n] = \sum_{k=-5}^{n-4} (2) + \sum_{k=n-3}^{k=1} (1)$$

$$= \sum_{k=0}^{n+1} (2) + \sum_{k=0}^{k=n-4} (1)$$

$$= 2(n+2) + 1(n-4)$$

$m[n] = 3n+1$ if $2 \le n \le 3$

if $n = 4$

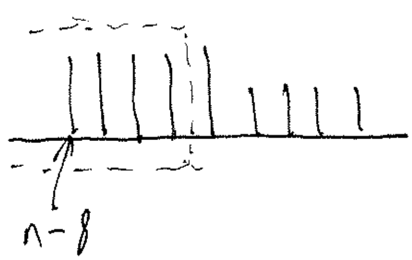
$z[n-k]$

$m[n] = 2 \cdot 5 + 1 = 11$

$m[n] = 11$ if $n = 4$

if $5 \le n \le 9$

$z[n-k]$



$$m[n] = \sum_{k=n-8}^{k=1} (2) = \sum_{k=n-9}^{k=0} (2)$$

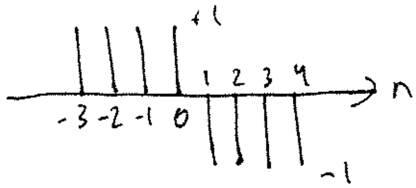
$$= 2(n-10)$$

$m[n] = 2n-20$ if $5 \le n \le 9$

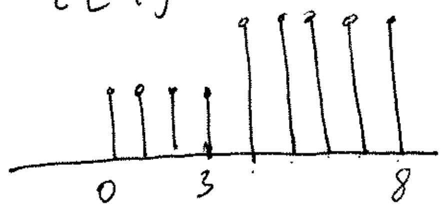
2.34
(e)

$$m[n] = y[n] * z[n]$$

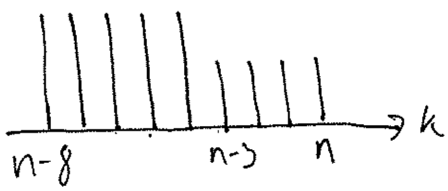
$y[n]$



$z[n]$



$z[n-k]$

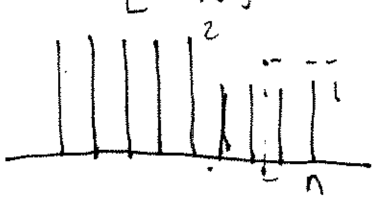


$m(n) = 0$ if

$n \le -4$ or $n \ge 13$

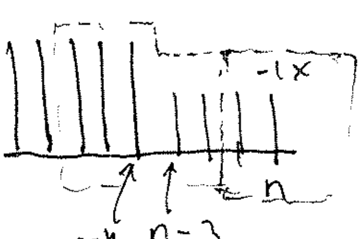
consider $-3 \le n \le 0$

$z[n-k]$



$$m[n] = \sum_{k=-3}^{k=n}(1) = \sum_{k=0}^{k=n+3}(1) = n+4$$

consider $1 \le n \le 4$

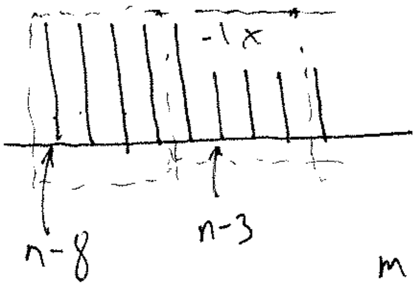


$$m[n] = \sum_{k=1}^{n}(-1) + \sum_{k=n-3}^{k=0}(1) + \sum_{k=-3}^{k=n-4}(2)$$

$$m[n] = \sum_{k=0}^{n-1}(2) - (n) + (n-2)$$

$$m[n] = 2n - n + n - 2 = 2n - 2 \quad \text{if } 1 \le n \le 4$$

consider $5 \le n \le 7$



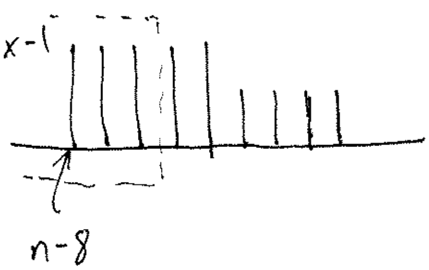
$$m[n] = \sum_{k=n-8}^{0}(2) - \sum_{k=1}^{n-4}(2) - \sum_{k=n-3}^{4}(1)$$

$$m[n] = 2(n-7) - 2(n-4) - (n-6)$$

$$= 2n-14-2n+8-n+6$$

$$m[n] = -n \qquad \text{if } 5 \le n \le 7$$

consider $8 \le n \le 12$



$$m[n] = \sum_{k=n-8}^{4}(-2) = \sum_{k=n-12}^{0}(-2) = -2(n-11)$$

$$m[n] = 22-2n \qquad \text{if } 8 \le n \le 12$$

2.39

(a) $y(t) = \left(u(t) - u(t-2)\right) * u(t)$

$$y(t) = \int_{-\infty}^{\infty} \left(u(\tau) - u(\tau-2)\right) \cdot u(t-\tau)\, d\tau$$

$u(\tau) - u(\tau - 2) = 1$ if $0 < \tau < 2$

$$y(t) = \int_0^2 u(t-\tau)\, d\tau$$

$u(t-\tau) = 1$ if $t - \tau > 0$

$t > \tau$ $-\tau > -t$

we can pull out a $u(t)$ term $\rightarrow t > 0$ $\tau < t$

if $0 < t < 2$

$$y(t) = \int_0^t d\tau = t$$

if $2 < t$

$$y(t) = \int_0^2 d\tau = 2$$

$$y(t) = \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } 0 < t < 2 \\ 2 & \text{if } 2 < t \end{cases}$$

2.39

(b)

$$y(t) = e^{-3t}u(t) * u(t+3)$$

$$= \int_{-\infty}^{\infty} e^{-3\tau} u(\tau) \cdot u(t-\tau+3)\, d\tau$$

$\tau > 0$

$$= \int_{0}^{\infty} e^{-3\tau} u(t-\tau+3)\, d\tau$$

$u(t-\tau+3) = 1$ if $t-\tau+3 > 0$

$t > \tau - 3$ $\qquad$ $-\tau > -t-3$

$t > -3$ $\qquad$ $\tau < t+3$

$$y(t > -3) = \int_{0}^{t+3} e^{-3\tau}\, d\tau$$

$$= \left[\frac{e^{-3\tau}}{-3}\right]_0^{t+3} = -\frac{1}{3}\left[e^{-3t-9} - 1\right]$$

$$y(t) = \begin{cases} 0, & t < -3 \\ \frac{1}{3}\left(e^{-3t-9} - 1\right), & t > -3 \end{cases}$$

2.39

(a) $y(t) = e^{-\gamma t} u(t) * e^{\beta t} u(-t)$

$$y(t) = \int_{-\infty}^{\infty} e^{-\gamma \tau} u(\tau) \cdot e^{\beta(t-\tau)} u(\tau - t)\, d\tau$$

$u(\tau) = 1$ if $\tau > 0$

$$= \int_{0}^{\infty} e^{-\gamma \tau} \cdot e^{\beta(t-\tau)} u(\tau - t)\, d\tau$$

$u(\tau - t) = 1$ if $\tau - t > 0$

$-t > -\tau$ $\quad$ $\tau > t$

$t < \tau$

$t < 0$

$$y(t) = \int_{t}^{\infty} e^{-\tau(\gamma+\beta) + \beta t}\, d\tau$$

$$= e^{\beta t} \int_{t}^{\infty} e^{-\tau(\gamma+\beta)}\, d\tau = e^{\beta t} \cdot \left[\frac{e^{-\tau(\gamma+\beta)}}{-(\gamma+\beta)} \right]_{t}^{\infty}$$

$$y(t) = e^{\beta t} \cdot \frac{1}{\gamma+\beta} \left(e^{-t(\gamma+\beta)} \right) \quad \text{if } t < 0$$

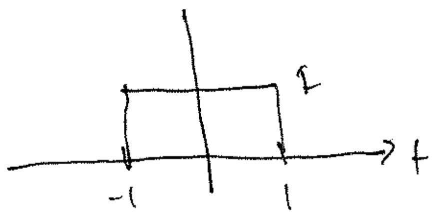
$= 0$, otherwise

2.40

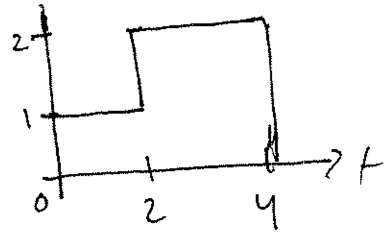
(a)

$m(t) = x(t) * y(t)$

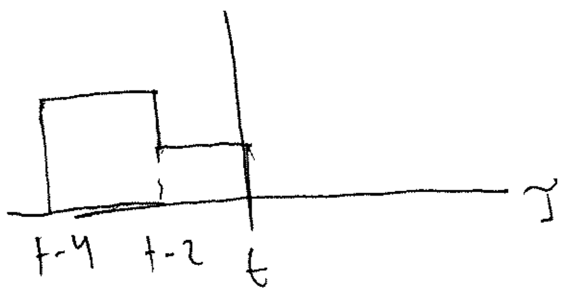
$x(t) =$



$y(t) =$



$y(t-\tau)$



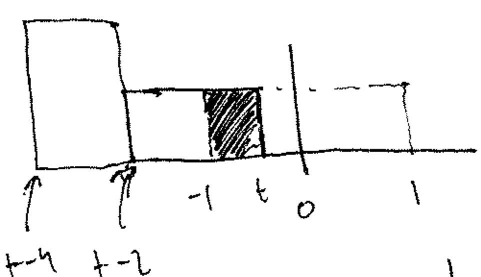
so

$m(t) = 0$ if $t < -1$

or if $t > 5$

consider $-1 < t < 1$

$y(t-\tau)$

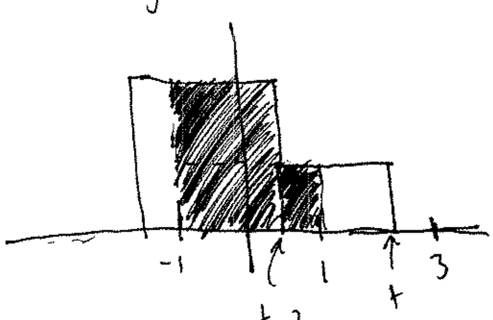


$m(t) = \int_{-1}^{t} 1\, d\tau = t + 1$

$m(t) = t+1$ if $-1 < t < 1$

consider $1 < t < 3$

$y(t-\tau)$



$m(t) =$ shaded area

$= \int_{-1}^{t-2} 2 \cdot d\tau + \int_{t-2}^{1} 1\, d\tau$

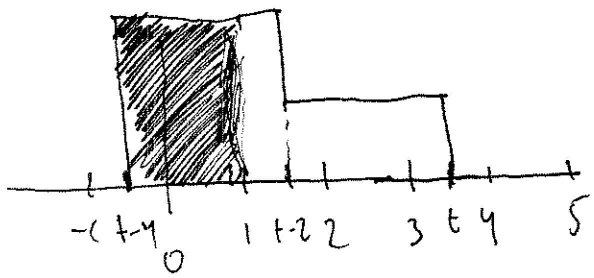
$= 2(t-1) + (1-t+2)$

$= 2t - 2 + 1 - t + 2 = t + 1$

$m(t) = t+1$ if $1 < t < 3$

consider $\quad 3 < t < 5$

$y(t-\gamma)$



$m(t) =$ shaded area

$$= 2\int_{t-4}^{1} d\gamma = 2\left(1-(t-4)\right)$$

$$= 2(1-t+4)$$

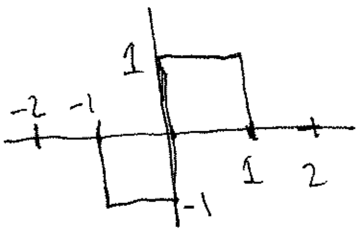
$$m(t) = 10-2t \quad 3<t<5$$

$$m(t) = \begin{cases} 0, & t<-1,\ t>5 \\ t+1, & -1<t<3 \\ 10-2t, & 3<t<5 \end{cases}$$

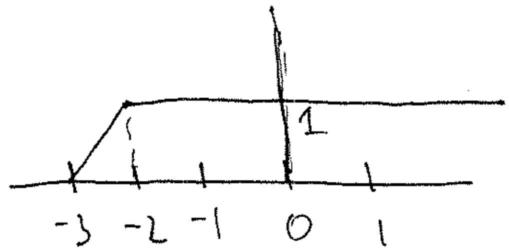
2.40

(k) $m(t) = z(t) * b(t)$

$z(t)$

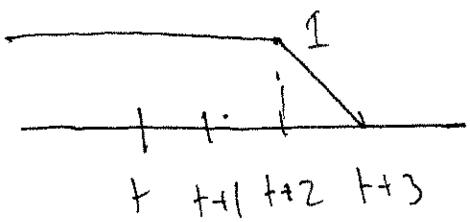


$b(t)$



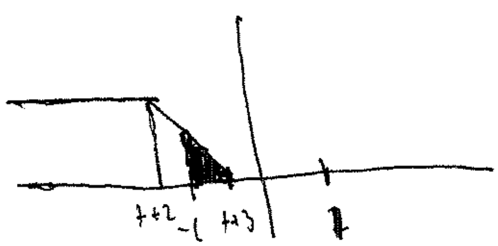
$b(t-\tau)$

$m(t) = 0$ if $t < -4$

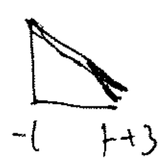


consider

$-4 < t < -3$



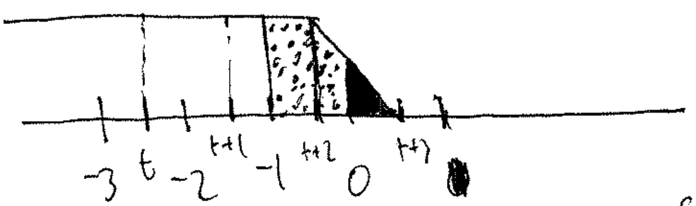
$m(t) = -$ shaded are

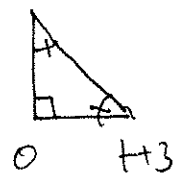
$= -$ area 

$$= -\frac{(t+4)^2}{2} = -\frac{t^2}{2} - 4t - 8$$

consider $-3 < t < -2$

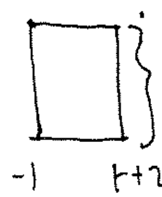
$m(t) =$ shaded area $-$ dotted area



shaded area $=$ area 

shaded area $= \frac{(t+3)^2}{2} = \frac{t^2+6t+9}{2}$

dotted area $=$ ~~square~~ box area $+$ trap area

box area $=$ area 

$= t+3$

trap area $= \frac{1}{2} -$ shaded area

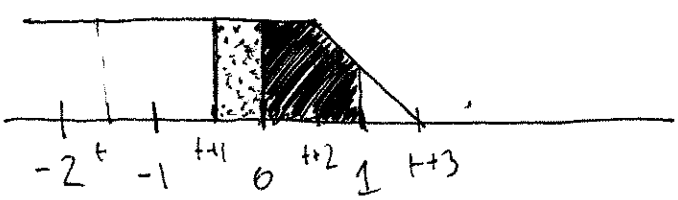
$m(t) =$ shaded area $-$ (box area $+$ trap area)

$$= \frac{t^2+6t+9}{2} - \left(t+3 + \frac{1}{2} - \frac{t^2+6t+9}{2}\right)$$

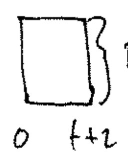
$$= t^2+6t+9 - t - 3 - \frac{1}{2} = t^2+5t+\frac{5}{2}$$

consider $\quad -2 < t < -1$

$m(t) =$ shaded area $-$ dotted area



shaded area $=$ box_1 area $+$ trap area

box_1 area $=$ area  $= t+2$

trap area $= \dfrac{1}{2} -$ area  $= \dfrac{1}{2} - \dfrac{(t+2)^2}{2} = \dfrac{1}{2} - \dfrac{t^2+4t+4}{2}$

dotted area $=$ box_2 area $=$ area  $= -(t+1)$

$m(t) =$ shaded area $-$ dotted area

$$= (t+2) + \frac{1}{2} - \frac{t^2+4t+4}{2} + (t+1)$$

$$= -\frac{t^2}{2} + \frac{3}{2}$$

$$m(t) = \frac{3}{2} - \frac{t^2}{2} \qquad \text{if } -2 < t < -1$$

consider

$-1 < t$

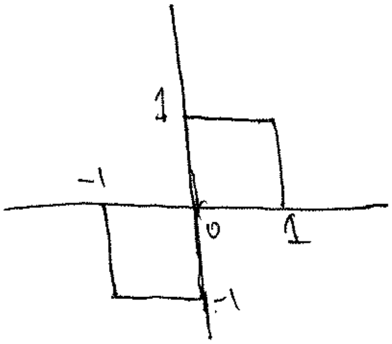
$m(t) =$ shaded area − dotted area

$m(t) = 0 \quad \text{if} \quad t > -1$

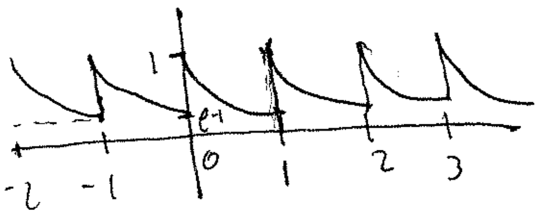
$$m(t) = \begin{cases} 0, & t < -4, \; t > -1 \\ -\frac{t^2}{2} - 4t - 8, & -4 < t < -3 \\ t^2 + 5t + \frac{5}{2}, & -3 < t < -2 \\ \frac{3}{2} - \frac{t^2}{2}, & -2 < t < -1 \end{cases}$$

2.40

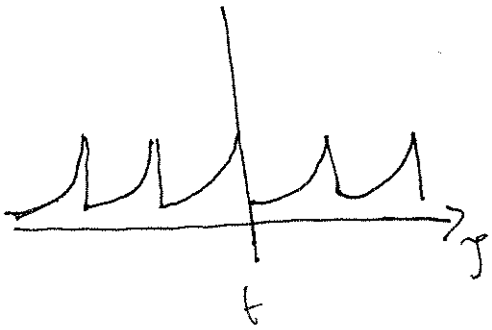
(p) $m(t) = z(t) * d(t)$



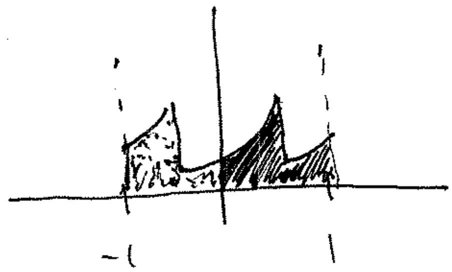
$d(t)$



$d(t-\tau)$



let's look at the -1 to 1 range of $d(t-\tau)$ we have



$m(t) =$ shaded area - dotted area

since it's evident that shaded area = dotted area then

$m(t) = 0$ always