3.7 $x(t) = 2\sin(2\pi t - 3) + \sin(6\pi t)$

HW 7
ECEN 314
solved by
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$T = 1 \qquad \omega_0 = 2\pi$

We could solve the integral:

$$X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}\,dt$$

But it leads to a huge mess. Instead,
remember $x(t) = \sum_{k=-\infty}^{k=\infty} X[k]e^{jk\omega_0 t}$

$$2\sin(2\pi t - 3) = \frac{2}{2j}\left(e^{-j3}e^{j2\pi t} - e^{j3}e^{-j2\pi t}\right)$$

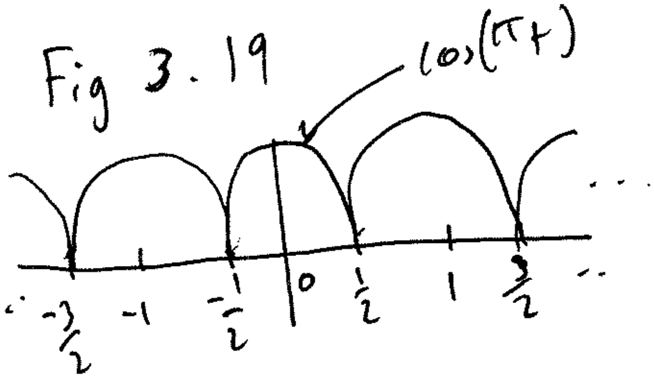
$$\sin(6\pi t) = \frac{1}{2j}\left(e^{j6\pi t} - e^{-j6\pi t}\right)$$

$$x(t) = \cancel{\frac{2}{2j}}\ \frac{1}{j}e^{-j3}\underbrace{e^{j2\pi t}}_{k=1} - \frac{1}{j}e^{j3}\underbrace{e^{-j2\pi t}}_{k=-1} + \frac{1}{2j}\underbrace{e^{j6\pi t}}_{k=3} - \frac{1}{2j}\underbrace{e^{-j6\pi t}}_{k=-3}$$

$$X[k] = \begin{cases} -\frac{1}{2j} = \frac{j}{2}, & k=-3 \\ -\frac{e^{j3}}{j} = je^{j3}, & k=-1 \\ -je^{j3}, & k=1 \\ \frac{-j}{2}, & k=3 \end{cases}$$

3.8

Fig 3.19



Find FS

so $T = 1 \Rightarrow \omega_0 = 2\pi$

$$X[k] = \int_0^1 \cos(\pi t)\, e^{-jk2\pi t}\, dt$$

$$\cos(\pi t) = \frac{1}{2}\left(e^{j\pi t} + e^{-j\pi t}\right)$$

$$X[k] = \frac{1}{2}\int_0^1 e^{j\pi t} e^{-jk2\pi t}\, dt + \frac{1}{2}\int_0^1 e^{-j\pi t} e^{-jk2\pi t}\, dt$$

$$= \frac{1}{2}\left[\frac{1}{j(\pi - 2k\pi)}\right]\left[e^{j(\pi - k2\pi)} - 1\right]$$

$$+ \frac{1}{2}\left[\frac{1}{-j(\pi + 2k\pi)}\right]\left[e^{-j(\pi + k2\pi)} - 1\right]$$

3.9

a) Determine $x(t)$ from

$$X[k] = -j\,\delta[k-2] + j\,\delta[k+2] + 2\delta[k-3] + 2\delta[k+3], \quad \omega_0 = \pi$$

$k=2$ $\quad$ $k=-2$ $\quad$ $k=3$ $\quad$ $k=-3$

$$x(t) = \sum_{k=-\infty}^{k=\infty} e^{jk\omega_0 t} \cdot X[k]$$

$$= -je^{j2\pi t} + je^{-j2\pi t} + 2e^{j3\pi t} + 2e^{-j3\pi t}$$

$$= -j\left(e^{j2\pi t} - e^{-j2\pi t}\right) + 2\left(e^{j3\pi t} + e^{-j3\pi t}\right)$$

$$= \frac{1}{2j} \cdot 2\left(e^{j2\pi t} - e^{-j2\pi t}\right) + \frac{1}{2} \cdot 4\left(e^{j3\pi t} + e^{-j3\pi t}\right)$$

$$= 2\sin(2\pi t) + 4(\sin(3\pi t))$$

3.9 b)

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]\, e^{jk\omega_0 t} \qquad \omega_0 = \pi/2$$

$$= \sum_{k=-4}^{k=4} X[k]\, e^{jk\pi/2\, t} \qquad X(k) = e^{-j\pi/2\, k}$$

$$= \sum_{k=-4}^{k=4} e^{j\frac{\pi}{2}k(t-1)} \qquad \text{let } k' = 4+k$$

$$= \sum_{k'=0}^{k'=8} e^{j\frac{\pi}{2}(k'-4)(t-1)} = e^{-j2\pi(t-1)} \sum_{k'=0}^{8} e^{j\frac{\pi}{2}k'(t-1)}$$

$$= e^{-j2\pi(t-1)} \left[\frac{1 - e^{j\pi/2 \cdot 9(t-1)}}{1 - e^{j\pi/2\,(t-1)}} \right]$$

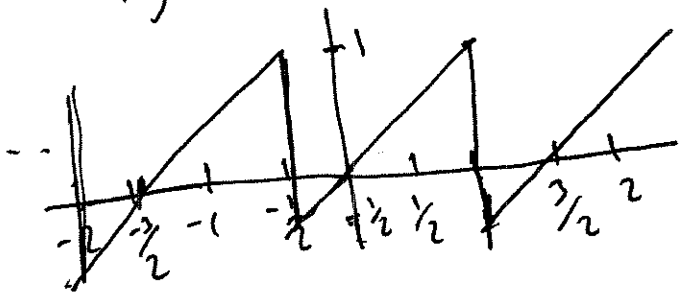
$$= e^{-j2\pi(t-1)} \cdot \frac{e^{j\frac{\pi}{4}9(t-1)}}{e^{j\frac{\pi}{4}(t-1)}} \cdot \frac{2j}{2j} \left[\frac{e^{-j\frac{\pi}{4}9(t-1)} - e^{j\frac{\pi}{4}9(t-1)}}{e^{-j\frac{\pi}{4}(t-1)} - e^{j\frac{\pi}{4}(t-1)}} \right]$$

$$= \cancel{e^{j\pi(t-1)(-2+9/4-\frac{1}{4})}} \to 1 \quad \frac{\sin\left(9\frac{\pi}{4}(t-1)\right)}{\sin\left(\frac{\pi}{4}(t-1)\right)}$$

$$= \frac{\sin\left(9\pi(t-1)/4\right)}{\sin\left(\pi(t-1)/4\right)}$$

3.10

Fig 3.29



$T = \frac{3}{2} \quad \omega_0 = \frac{4\pi}{3}$

$x(t) = t$

$$X[k] = \frac{2}{3} \int_{-\frac{1}{2}}^{1} t e^{-jk\omega_0 t} \, dt$$

if $k = 0$

$$X[0] = \frac{2}{3} \int_{-\frac{1}{2}}^{1} t^2 \, dt = \frac{t^2}{3} \Big|_{-\frac{1}{2}}^{1} = \frac{1}{3} - \frac{1/4}{3}$$

$$= \frac{3/4}{3} = \frac{1}{4}$$

otherwise

$$X[k] = \frac{2}{3} \int_{-\frac{1}{2}}^{1} t e^{-jk\omega_0 t} \, dt$$

solve by parts

$$\frac{2}{3}\int_{-\frac{1}{2}}^{1} t\, e^{-jk\omega_0 t}\, dt$$

$$u = t \qquad dv = e^{-jk\omega_0 t}\, dt$$

$$du = dt \qquad v = -\frac{1}{jk\omega_0} \cdot e^{-jk\omega_0 t}$$

$$X[k] = \frac{2}{3}\left[\frac{-t}{jk\omega_0} \cdot e^{-jk\omega_0 t} + \frac{1}{jk\omega_0}\int e^{-jk\omega_0 t}\, dt\right]_{-\frac{1}{2}}^{1}$$

$$= \frac{2}{3}\left[\frac{-t}{jk\omega_0} e^{-jk\omega_0 t} + \frac{1}{k^2\omega_0^2} e^{-jk\omega_0 t}\cancel{dt}\right]_{-\frac{1}{2}}^{1}$$

$$= \frac{2}{3}\left[-\frac{1}{jk\omega_0} e^{-jk\omega_0} - \frac{1}{2jk\omega_0} e^{jk\omega_0/2} + \frac{1}{k^2\omega_0^2} e^{-jk\omega_0 t} - \frac{1}{k^2\omega_0^2} e^{jk\omega_0/2}\right]$$

$$= \frac{-2}{3jk\omega_0}\left(e^{-jk\omega_0} + \frac{1}{2} e^{jk\omega_0/2}\right) + \frac{2}{3k^2\omega_0^2}\left(e^{-jk\omega_0 t} - e^{jk\omega_0/2}\right)$$

3.50

a) $x(t) = \sin(3\pi t) + \cos(4\pi t)$

$$= \frac{1}{2j}\left(e^{j3\pi t} - e^{-j3\pi t}\right) + \frac{1}{2}\left(e^{j4\pi t} + e^{-j4\pi t}\right)$$

$x(t) = \sum X[k] e^{j\omega_0 k t}$ $\quad T = 2$

$\omega_0 = \frac{2\pi}{T} = \pi$

if $k = 3$, $X[k] = \frac{1}{2j} = \frac{-j}{2}$

if $k = -3$, $X[k] = -\frac{1}{2j} = \frac{j}{2}$

if $k = 4$, $X[k] = \frac{1}{2}$

if $k = -4$, $X[k] = \frac{1}{2}$

if $k =$ anything else $X[k] = 0$

3.50

b) $x(t) = \sum_{m=-\infty}^{\infty} \delta(t - m/3) + \delta(t - 2m/3)$

Let's look at

$$\delta(t - m/3) = 1 \quad \text{if} \quad t - m/3 = 0$$

$t = \frac{m}{3}$ whenever t is ~~a~~ a multiple of $\frac{1}{3}$

$$\sum_{m=-\infty}^{\infty} \delta(t - m/3) = \cancel{1}\,\infty$$

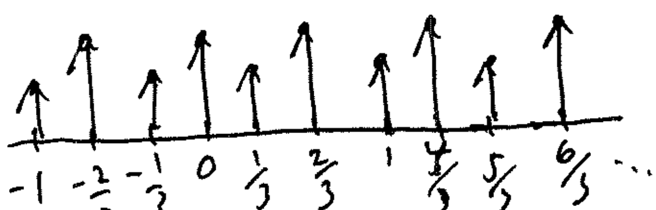
now look at

$$\delta(t - 2m/3) = 1 \quad \text{if} \quad t - 2m/3 = 0$$

$t = 2\frac{m}{3}$, whenever t is a multiple of $\frac{2}{3}$

$$\sum_{n=-\infty}^{\infty} \delta(t - 2m/3) = \cancel{1}\,\infty$$

so let's plot $x(t)$



$$T = \frac{2}{3}$$

$$\omega_0 = \frac{2\pi}{T} = 3\pi$$

$$X[k] = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^{2/3} \left(\delta\left(t - \tfrac{1}{3}\right)\cancel{2} + 2\delta(t)\right) e^{-jk\omega_0 t}\, dt$$

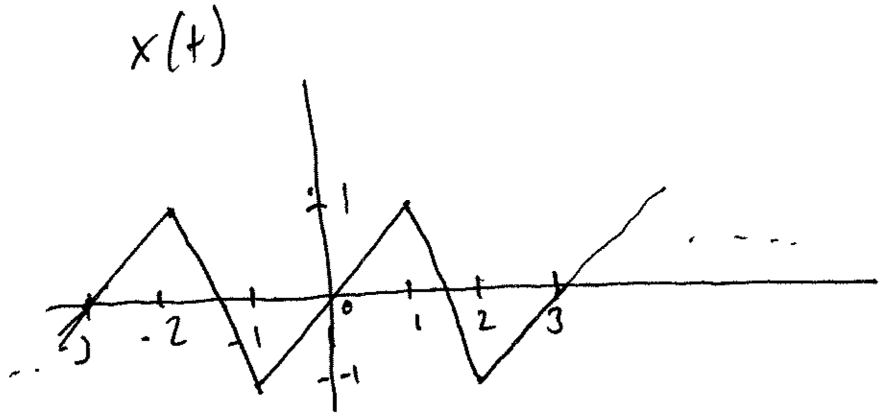
b) continued

$$X[k] = \frac{3}{2}\int_0^{2/3} \delta\left(t-\frac{1}{3}\right)e^{-jk\omega_0 t}\,dt + 3\int_0^{2/3}\delta(t)e^{-jk\omega_0 t}\,dt$$

$$= \frac{3}{2}e^{-jk\omega_0\left(\frac{1}{3}\right)} + 3e^{-jk\omega_0(0)}\,\cancel{dt}$$

$$= \frac{3}{2}e^{-jk\pi} + 3$$

3.50 f)



$T = 3$

$\omega_0 = \frac{2\pi}{3}$

if $-1 < t < 1$

$x(t) = t$

if $1 < t < 2$

$x(t) = 3 - 2t$

$$X[k] = \int_{t_0}^{t_0+T} x(t) e^{-jk\omega_0 t} dt$$

$$= \int_{-1}^{2} x(t) e^{-jk\omega_0 t} dt$$

$$= \int_{-1}^{1} t e^{-jk\omega_0 t} dt + \int_{1}^{3} (3-2t) e^{-jk\omega_0 t} dt$$

These two integrals are done by parts

3.51

a) $X[k] = j\delta[k-1] - j\delta[k+1] + \delta[k-3] + \delta[k+3]$

$$x(t) = \sum_{k=-\infty}^{k=\infty} X[k] e^{+jk\omega_0 t}$$

$$x(t) = j\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) + \left(e^{j3\omega_0 t} + e^{-j3\omega_0 t}\right)$$

$$= \frac{1}{2j} \cdot (-2)\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) + \frac{1}{2} \cdot 2\left(e^{j3\omega_0 t} + e^{-j3\omega_0 t}\right)$$

$$= -2\sin(\omega_0 t) + 2\cos(3\omega_0 t)$$

for part a) $\omega_0 = 2\pi$ so

$$x(t) = -2\sin(2\pi t) + 2\cos(6\pi t)$$

for part b) $\omega_0 = 4\pi$ so

$$x(t) = -2\sin(4\pi t) + 2\cos(12\pi t)$$

3.51 c)

$$X[k] = \left(-\frac{1}{3}\right)^{|k|} \qquad \omega_0 = 1$$

$$x(t) = \sum_{k=-\infty}^{k=\infty} \left(-\frac{1}{3}\right)^{|k|} e^{jkt}$$

$$= \sum_{k=-\infty}^{k=0} \left(-\frac{1}{3}\right)^{-k} e^{jkt} + \sum_{k=0}^{k=\infty} \left(-\frac{1}{3}\right)^{k} e^{jkt} - 1$$

accounts for double sum $k=0$

let $k' = -k$

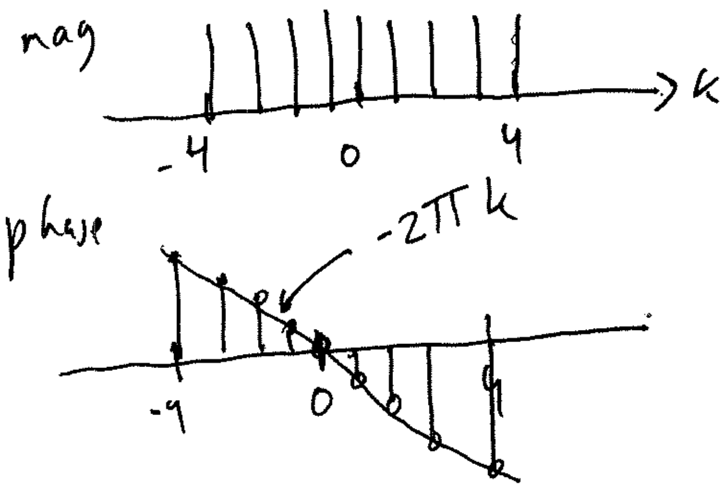
$$= \sum_{k'=\infty}^{k'=0} \left(-\frac{1}{3}\right)^{k'} e^{-jk't} + \sum_{k=0}^{k=\infty} \left(-\frac{1}{3}\right)^{k} e^{jkt}$$

$$= \sum_{k'=0}^{k'=\infty} \left[\left(-\frac{1}{3}\right) e^{-jt}\right]^{k'} + \sum_{k=0}^{k=\infty} \left(\left(-\frac{1}{3}\right) e^{jt}\right)^{k}$$

$$= \frac{1}{1 + \frac{1}{3} e^{-jt}} + \frac{1}{1 + \frac{1}{3} e^{jt}}$$

3.51

e) $X[k]$ Fig p 3.51(b) $\omega_0 = 2\pi$



$$x(t) = \sum_{k=-4}^{k=4} e^{-j2\pi k} \cdot e^{j2\pi k t}$$

this is solved similarly to problem 3.9 ~~8.9~~ b)