Homework 8 Part 1

Solved by Sanjay Nair
sanjaynair@neo.tamu.edu

3.14

a) $x(t) = e^{2t}u(-t)$

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}\,dt$$

$$= \int_{-\infty}^{\infty} e^{2t}u(-t)e^{-jwt}\,dt$$

$$= \int_{-\infty}^{0} e^{2t}e^{-jwt}\,dt = \int_{-\infty}^{0} e^{t(2-jw)}\,dt$$

$$= \frac{1}{2-jw}\left[e^{t(2-jw)}\right]_{-\infty}^{0} = \frac{1}{2-jw}\left[1-0\right]$$

$$= \frac{1}{2-jw} = \frac{-1}{jw-2}$$

b) $x(t) = e^{-|t|}$

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} \, dt$$

$$= \int_{-\infty}^{\infty} e^{-|t|} e^{-jwt} \, dt = \int_{-\infty}^{0} e^{t} e^{-jwt} \, dt + \int_{0}^{\infty} e^{-t} e^{-jwt} \, dt$$

$$= \int_{-\infty}^{0} e^{t(1-jw)} \, dt + \int_{0}^{\infty} e^{t(-1-jw)} \, dt$$

$$= \frac{1}{1-jw} \left[e^{t(1-jw)} \right]_{-\infty}^{0} + \frac{1}{-1-jw} \left[e^{t(-1-jw)} \right]_{0}^{\infty}$$

$$= \frac{1}{1-jw} \left[1 - e^{\infty(1-jw)} \right] + \frac{1}{-1-jw} \left[e^{\infty(-1-jw)} - 1 \right)$$

$$= \frac{1}{1-jw} - \frac{1}{-1-jw} = \frac{1+jw}{1+w^2} - \frac{(-1+jw)}{1+w^2} = \frac{2}{1+w^2}$$

c) $x(t) = e^{-2t}u(t-1)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

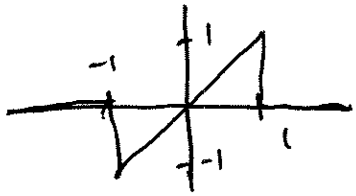
$$= \int_{-\infty}^{\infty} e^{-2t}u(t-1)e^{-j\omega t}\,dt = \int_{1}^{\infty} e^{-2t}e^{-j\omega t}\,dt$$

$$= \int_{1}^{\infty} e^{-t(2+j\omega)}\,dt$$

$$= -\frac{1}{2+j\omega}\left[e^{-t(2+j\omega)}\right]_1^{\infty}$$

$$= -\frac{1}{2+j\omega}\left[0 - e^{-(2+j\omega)}\right] = \frac{e^{-(2+j\omega)}}{2+j\omega}$$

d) $x(t) =$



$$\cancel{X}(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} t\, e^{-j\omega t}\, dt$$

$u = t$ $\qquad dv = e^{-j\omega t}\, dt$

$\frac{du}{dt} = 1 \Rightarrow du = dt \qquad v = \frac{-1}{j\omega} e^{-j\omega t}$

$$= \left[\frac{-t}{j\omega} e^{-j\omega t} + \frac{1}{j\omega} \int e^{-j\omega t}\, dt \right]_{-1}^{1}$$

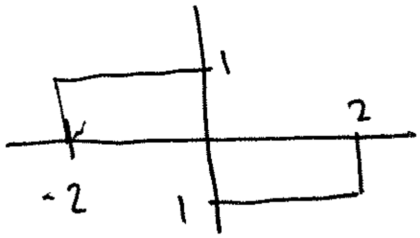
$$= \left[\frac{-t}{j\omega} e^{-j\omega t} + \frac{1}{\cancel{j}\omega^2} e^{-j\omega t} \right]_{-1}^{1}$$

$$= \frac{-1}{j\omega}\left[+e^{-j\omega} + e^{j\omega} \right] + \frac{1}{\omega^2}\left[e^{-j\omega \cancel{t}} - e^{j\omega \cancel{t}} \right]$$

$$= \frac{1}{j\omega}\left[e^{+j\omega} - e^{-j\omega} \right] - \frac{1}{\omega^2}\left[e^{j\omega \cancel{t}} - e^{-j\omega \cancel{t}} \right]$$

$$= \frac{2}{\omega} \sin\omega - \frac{j2}{\omega^2} \sin\omega$$

e) $x(t)$



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$= \int_{-2}^{0} e^{-j\omega t}\,dt - \int_{0}^{2} e^{-j\omega t}\,dt$$

$$= \frac{1}{-j\omega}\left[e^{-j\omega t}\right]_{-2}^{0} + \frac{1}{j\omega}\left[e^{-j\omega t}\right]_{0}^{2}$$

$$= -\frac{1}{j\omega}\left[1 - e^{2j\omega}\right] + \frac{1}{j\omega}\left[e^{-2j\omega} - 1\right]$$

$$= \frac{1}{j\omega}\left[e^{2j\omega} + e^{-2j\omega}\right]$$

$$= \frac{2}{j\omega}\cos(2\omega)$$

3.15

a) $X(jw) = \begin{cases} 2\cos w, & |w| < \pi \\ 0, & |w| > \pi \end{cases}$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}\,dw$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \cos w\, e^{jwt}\,dw$$

$$\int \cos w\, e^{jwt}\,dw = \frac{1}{2}\int e^{jw(1+t)}\,dw + \frac{1}{2}\int e^{jw(t-1)}\,dw$$

$$= \frac{1}{2(jw(1+t))}e^{jw(1+t)} + \frac{1}{2(jw(t-1))}e^{jw(t-1)}$$

$$\cancel{x(t) = \frac{1}{\pi}\left[\frac{1}{2(jw(1+\pi))}e^{jw(1+\pi)} - \frac{1}{2jw(1-\pi)}e^{jw(}\right.}$$

$$x(t) = \left[\frac{1}{2j(1+t)\pi}\cdot\left(e^{j\pi(1+t)} - e^{-j\pi(1+t)}\right) + \frac{1}{2j(t-1)\pi}\cdot\left(e^{j\pi(t-1)} - e^{-j\pi(t-1)}\right)\right]$$

$$= \frac{\sin(\pi(t+1))}{\pi(t+1)} + \frac{\sin(\pi(t-1))}{\pi(t-1)}$$

3.15

b) $X(jw) = 3\delta(w-4)$

$$x(t) = \frac{1}{2\pi} 3 \int_{-\infty}^{\infty} \delta(w-4) e^{jwt} dw$$

$$= \frac{3}{2\pi} e^{j4t}$$

3.15

c) $X(j\omega) = \pi e^{-|\omega|}$

$$x(t) = \frac{1}{2}\int_{-\infty}^{0} e^{+\omega} e^{\cancel{-}j\omega t}\, d\omega + \frac{1}{2}\int_{0}^{\infty} e^{-\omega} e^{j\omega t}\, d\omega$$

$$= \frac{1}{2}\int_{-\infty}^{0} e^{\omega(1+jt)}\, d\omega + \frac{1}{2}\int_{0}^{\infty} e^{\omega(-1+jt)}\, d\omega$$

$$= \frac{1}{2(1+jt)}\left[e^{\omega(1+jt)}\right]_{-\infty}^{0} + \frac{1}{2(-1+jt)}\left[e^{\omega(-1+jt)}\right]_{0}^{\infty}$$

$$= \frac{1}{2(1+jt)} - \frac{1}{2(-1+jt)} = \frac{(1-jt)-(-1-jt)}{2(1+t^2)}$$

$$= \frac{2}{2(1+t^2)} = \frac{1}{1+t^2}$$