BLIND IMAGE RESTORATION VIA RECURSIVE FILTERING USING DETERMINISTIC CONSTRAINTS

Deepa Kundur *Dimitrios Hatzinakos*

Department of Electrical and Computer Engineering
University of Toronto, Toronto, Ontario, Canada M5S 1A4
e-mail: dimitris@comm.toronto.edu

ABSTRACT

Classical linear image restoration techniques assume that the linear shift invariant blur, also known as the point-spread function (PSF), is known prior to restoration. In many practical situations, however, the PSF is unknown and the problem of image restoration involves the simultaneous identification of the true image and PSF from the degraded observation. Such a process is referred to as *blind deconvolution.* This paper presents a novel blind deconvolution method for image restoration. The method is flexible for incorporating different constraints on the true image. An example of the method is given for situations in which the imaged scene consists of a finite support object against a uniformly grey background. The only information required are the nonnegativity of the true image and the support size of the original object. For situations in which the exact object support is unknown, a novel support-finding algorithm is proposed.

1. INTRODUCTION

Image restoration refers to the task of recovering an image from a degraded observation. Although classical linear image restoration techniques have been thoroughly studied [1], the more difficult problem of *blind image restoration* has numerous research possibilities.

In applications such as artificial satellite imaging, remote sensing, and medical imaging, improved image quality is often costly or physically impossible to obtain. In addition, little is known about the image to be restored, and it is often difficult to calculate or measure the PSF explicitly. The problem of simultaneously estimating the PSF (or its inverse) and restoring an unknown image is called *blind deconvolution.* The goal is to obtain a shifted scaled version of the original image. Theoretically, the scale and shift of an image are not recoverable in general by blind deconvolution algorithms [3].

Initial research into blind deconvolution of images assumed that the degradation of the image resulted from linear camera motion or an out-of- focus camera lens. Based on these models a parametric form for the PSF was derived, and the parameter values were estimated using the frequency domain nulls of the degraded image [2].

More recent methods estimate the image and PSF simultaneously in the restoration process [3]-[10]. They can be grouped into four classes based on their assumptions about the true image and PSF. The major drawback of existing blind deconvolution methods for images is that they suffer from poor convergence properties; the algorithms converge to local minima [4], [5], [7], [8] or are so computationally demanding [3], [9] that they are impractical for imaging applications.

This paper presents a novel technique for the class of nonparametric deterministic image constraints methods that overcomes the limitations of existing techniques. The general method involves the iterative minimization of a convex cost function. The image is restored by filtering the blurred image to produce an image estimate which is restricted to lie on a convex set representing the known deterministic constraints of the true image. The approach can incorporate a variety of constraints on the image such as pixel amplitude bounds, support, maximum energy, and smoothness, among others.

This paper focusses the particular situation in which an object of finite extent is imaged against a uniformly grey background. The edges of the object are assumed to be completely or almost completely included within the observed frame. This often occurs in applications such as astronomy and medical imaging. Statistical knowledge of the original image or a parametric model of the PSF are not needed. The only information required for restoration is the nonnegativity of the true image, and support size of the original object. This particular algorithm, referred to as the Nonnegativity and Support constraints Recursive Inverse Filtering (NAS-RIF) technique, involves numerically minimizing a convex cost function. All other methods incorporate the minimization of nonconvex cost functions; the advantage of the proposed NAS-RIF technique is that convergence to the global minimum is guaranteed, even in the presence of noise. In addition, the proposed technique shows faster convergence speed than existing iterative techniques and does not require heavy memory requirements. The superior performance of the NAS-RIF algorithm is demonstrated by computer simulations and comparisons with existing methods of its class.

The proposed technique and the methods of [7]- [9] belong to the class of nonparametric finite support blind image restoration methods. They are included for comparisons and make the following assumptions to achieve blind image restoration. The degradation process is assumed to be represented by the following linear model:

$$g(x,y) = f(x,y) * h(x,y) + n(x,y)$$

where $f(x,y)$ is the true image, $h(x,y)$ is the PSF, $n(x,y)$ is the additive noise, $g(x,y)$ is the degraded image, (x,y) is the discrete pixel coordinate, and $*$ represents two-dimensional linear convolution. The true image is required to be nonnegative with known finite support (the support is defined as the smallest rectangle containing the entire object). In applications such as astronomy, this information is sometimes available. In our method, it can be estimated using

0-7803-3192-3/96 $5.00©1996 IEEE

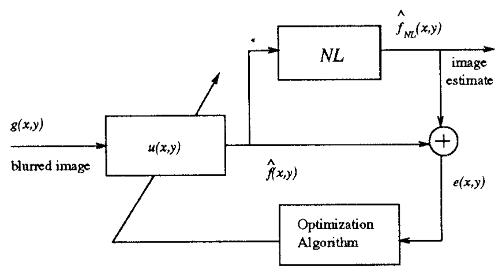

Figure 1. Proposed Blind Deconvolution Scheme for Images

a novel technique introduced in section 3.. In addition to the assumptions stated above, the methods of [7]-[9] require that the blur also be nonnegative with known finite support for proper restoration. In contrast, the only assumption our algorithm makes about the blur is that its inverse exists.

2. THE PROPOSED METHOD

2.1. The Blind Deconvolution Approach

The proposed NAS-RIF technique consists of a variable FIR filter $u(x,y)$ with the blurred image $g(x,y)$ as input. The output of this filter represents an estimate of the true image $\hat{f}(x,y)$. This estimate is passed through a nonlinear filter which uses a non-expansive mapping to project the estimated image into the space representing the known characteristics of the true image. The difference between this projected image $\hat{f}_{NL}(x,y)$ and $\hat{f}(x,y)$ is used as the error signal to update the variable filter $u(x,y)$. Figure 1 gives an overview of the scheme.

We concentrate on the particular algorithm for which the image is assumed to be nonnegative with known support, so the *NL* block of Figure 1 represents the projection of the estimated image onto the set of images that are nonnegative with given finite support. This requires replacing the negative pixel values within the region of support with zero, and pixels values outside the region of support with the background grey-level L_B. The cost function used in the restoration procedure follows:

$$\begin{aligned} J(u) &= \sum_{\forall(x,y)} e^2(x,y) + \gamma\left[\sum_{\forall(x,y)} u(x,y) - 1\right]^2 \\ &= \sum_{(x,y)\in D_{sup}} \hat{f}^2(x,y)\left[\frac{1-\text{sgn}(\hat{f}(x,y))}{2}\right] \\ &+ \sum_{(x,y)\in \overline{D}_{sup}} [\hat{f}(x,y) - L_B]^2 + \gamma\left[\sum_{\forall(x,y)} u(x,y) - 1\right]^2 \quad (1) \end{aligned}$$

where $\hat{f}(x,y) = g(x,y) * u(x,y)$, D_{sup} is the set of all pixels inside the region of support, and $\overline{D}_{sup}$ is the set of all pixels outside the region of support. The variable γ in third term of equation 1 is nonzero only when L_B is zero, ie., the background colour is black. The third term is used to constrain the parameters away from the trivial all-zero global minimum for this situation.

It can be shown that equation 1 is convex [11], so that convergence of the algorithm to the global minimum is possible using a variety of numerical optimization routines. The conjugate gradient minimization routine is used for minimization of J because its speed of convergence is much faster than other descent routines such as the steepest-descent method. The recursive algorithm, referred to as the NAS- RIF method is summarized in Table 1.

Table 1. Summary of the proposed NAS-RIF algorithm.

I) Definitions:

- $\hat{f}_k(x,y)$: estimate of true image at kth iteration
- $u_k(x,y)$: FIR filter parameters of dimension $N_{xu} \times N_{yu}$ at iteration k
- δ: tolerance used to terminate the algorithm
- $J(\underline{u}_k)$: cost function at parameter setting $\underline{u}_k$
- $\nabla J(\underline{u}_k)$: gradient of J at $\underline{u}_k$
- $< \cdot, \cdot >$: scalar product
- Note: underlined letters represent lexicographically ordered vectors of their two-dimensional counterparts.

II) Set initial conditions ($k = 0$):
Set FIR filter $u_k(x,y)$ to all zeros with a unit spike in the middle
Set tolerance $\delta > 0$

III) At iteration (k): $k = 0, 1, 2, ...$

1) $\hat{f}_k(x,y) = u_k(x,y) * g(x,y)$

2) $\hat{f}_{NL}(x,y) = NL[\hat{f}(x,y)]$

3) Minimization Routine to update FIR filter parameters.
For example: (conjugate gradient routine)

3a) $[\nabla J(\underline{u}_k)]^T = [\frac{\partial J(u_k)}{\partial u(1,1)} \frac{\partial J(u_k)}{\partial u(1,2)} \cdots \frac{\partial J(u_k)}{\partial u(N_{xu},N_{yu})}]$
where

$$\frac{\partial J(\underline{u}_k)}{\partial u(i,j)} = 2\sum_{(x,y)\in D_{sup}} \hat{f}_k(x,y)\left[\frac{1-\text{sgn}(\hat{f}_k(x,y))}{2}\right] g(x-i+1, y-j+1) + 2\sum_{(x,y)\in \overline{D}_{sup}} [\hat{f}_k(x,y) - L_B] g(x-i+1, y-j+1) + 2\gamma\left[\sum\nolimits_{\forall(x,y)} u_k(x,y) - 1\right]$$

3b) $\beta_{k-1} = (< \nabla J(\underline{u}_k) - \nabla J(\underline{u}_{k-1}), \nabla J(\underline{u}_k) >)/ (< \nabla J(\underline{u}_{k-1}), J(\underline{u}_{k-1}) >)$

3c) If $k = 0$, $d_k = -\nabla J(\underline{u}_k)$
Otherwise, $\underline{d}_k = -\nabla J(\underline{u}_k) + \beta_{k-1} d_{k-1}$

3d) Perform a line minimization such as `dlinmin.c` in [13] to find t_k such that
$J(\underline{u}_k + t_k \underline{d}_k) \leq J(\underline{u}_k + t\underline{d}_k)$ for all $t \in \mathbf{R}$

3e) $\underline{u}_{k+1} = \underline{u}_k + t\underline{d}_k$

4) $k = k + 1$

5) If $J(\underline{u}_k) < \delta$, stop. Otherwise, go to **1**.

2.2. Uniqueness of the Solution

Under ideal conditions of an infinite extent filter $u(x, y)$, and in the absence of additive noise, the solution to the algorithm may not be unique. For example, if the background colour of the image is black, the true image is invertible, and the support of the image and PSF are identical, a restoration which globally minimizes the cost function can be the true image, the PSF or many erroneous intermediate solutions. The possibility of these erroneous solutions is one of the dilemmas of blind deconvolution algorithms. With the lack of sufficient information, it is difficult to often overcome this problem.

Sufficient conditions for uniqueness of the solution for the NAS-RIF algorithm are developed in [11], and are analogous to persistence of excitation for system identification.

3. DETERMINATION OF THE SUPPORT

A method for assessing the optimal support size automatically and objectively is proposed. It uses the hold-out (HO) method used for model validation in data analysis. The proposed support-finding algorithm is inspired by the constraint assessment algorithm of [12], but is modified for blind image restoration.

Competing assumptions on the true image, such as different support sizes, can be assessed using the hold-out method. A support size for the true image is assumed. The image estimate pixels $\hat{f}(x, y)$ outside the assumed region of support are collectively called the *estimation set*; they are used to obtain an estimate of the true image. This is accomplished by minimizing a criterion, called the *estimation error*, which incorporates only the pixels within the estimation set. Specifically, the proposed blind deconvolution algorithm is applied using the assumed support and excluding the nonnegativity constraint. The set of pixels within the assumed region of support is called the *validation set*, and is used to assess the "correctness" of the assumed support size. This is performed by computing the *validation error* which measures the energy of negative pixels of the image estimate within the assumed region of support. The assumed support which produces the minimum validation error is selected as the true image support. The algorithm follows in Table 2.

If the assumed support is exact or larger than the actual support a reasonable estimate of the true image can be obtained. Since the true image is nonnegative, the validation error for such an image estimate should be small. Thus, the assumed support which minimizes the validation error is intuitively a good estimate of the actual support.

Table 2. Summary of the proposed support finding algorithm.

Assume an equally spaced grid of support parameter values (L_x, L_y) from $(1, 1)$ to the size of the blurred image (N_{xg}, N_{yg}).

1) Assume a rectangular support S with dimensions (L_x, L_y) from the grid. If all values in the grid have been selected before, either

1. Go to step **5** if the exhausted grid contains successive elements.
2. Form a finer grid centred about $(L_{x,min}, L_{y,min})$ (the parameters giving the minimum of the validation error found so far), and select a parameters (L_x, L_y) out of this new grid.

2) Based on the assumed support S, find the restoration filter $u^*(x, y)$ by using the conjugate gradient algorithm, to minimize the following estimation error function: $J(\underline{u}) = \sum_{(x,y)\in\overline{S}}[\hat{f}(x,y) - L_B]^2 + \gamma\left[\sum_{\forall(x,y)} u(x,y) - 1\right]^2$ where $\hat{f}(x, y) = u(x, y) * g(x, y)$ and $\overline{S}$ is the region outside the assumed support.

3) Calculate the validation error based on the minimizing filter parameters $u^*(x, y)$ of the estimation error of step **2**.

$$V(S) = \frac{1}{\| S \|} \sum_{(x,y)\in S} \hat{f^*}^2(x, y) \left[\frac{1 - \text{sgn}(\hat{f}^*(x, y))}{2}\right] \quad (2)$$

where $\| \cdot \|$ denotes the number of elements in the argument set, and the "restored" image estimate $\hat{f}^*(x, y) = u^*(x, y) * g(x, y)$.

4) Save the parameters $(L_{x,min}, L_{y,min})$, which give the minimum value of $V(S)$ found so far. Go to step **1**.

5) Select the support parameters that minimize $V(S)$ as the optimal support size for restoration.

4. SIMULATION RESULTS AND COMPARISONS

The results of the proposed algorithm and the IBD algorithm described in [7] and modified in [8] are shown in figure 2. The original toy and binary images shown in Figures 2(a) and 3(a) of support 119×81 and 15×65 were blurred using a 21×21 truncated Gaussian PSF, and noise was added for a blurred signal-to-noise ratio (BSNR) of 70 dB. The degraded images are displayed in Figures 2(b) and 3(b). The proposed support-finding algorithm estimated the support of the toy image as 120×84, and the binary image as 15×65. Based on these supports, the NAS-RIF restorations and mean square error (MSE) plots are shown in Figures 2(c),(e), and 3(c),(e). The proposed NAS-RIF method converged to a very good estimate of the solution in approximately 300 iterations for the toy image and 100 iterations for the binary image. The results of the IBD method are shown in Figures 2(d),(f) and 3(d),(f). The image estimate which showed the minimum energy of negative pixels within the region of support and pixels deviating from the background grey-level was used as the restored image for both sets of results of the IBD algorithm. The restorations of the IBD algorithm, shown in Figures 2(d) and 3(d), are the image estimates at the 1000*th* and 3000*th* iterations, respectively.

Although the IBD algorithm produces comparable results to the NAS-RIF algorithm for simple binary images, it fails to converge to a reliable image estimate for more complicated grey-scale images. The algorithm often exhibits instability and, at times, begins to diverge even as it appears to be converging to a good solution. Simulation results of the IBD algorithm for exact support size at various different initial conditions and noise parameter values α produced similar results as those shown.

The algorithm of [9] produced good results for very small images; however, for the images shown in this paper, it was too computationally time consuming to produce a good estimate. The order of the algorithm per iteration is $O(N_f^4)$, where N_f is the number of pixel values of the image estimate. In contrast, the NAS-RIF method has order $O(N_f N_u N_{ls,k})$ per iteration, where N_u is the number of FIR filter parameters of $u(x, y)$ and $N_{ls,k}$ is the number

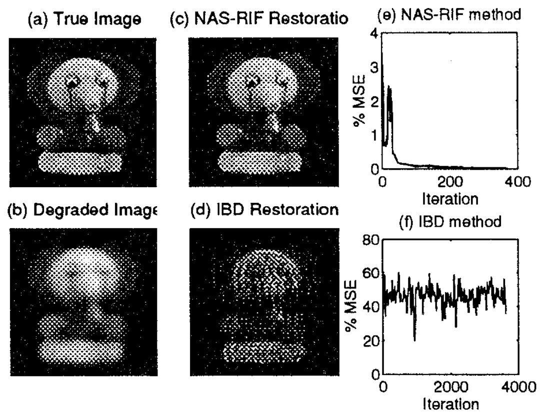

Figure 2. Simulation Results of the proposed NAS-RIF and existing IBD Restoration Methods for the toy image.

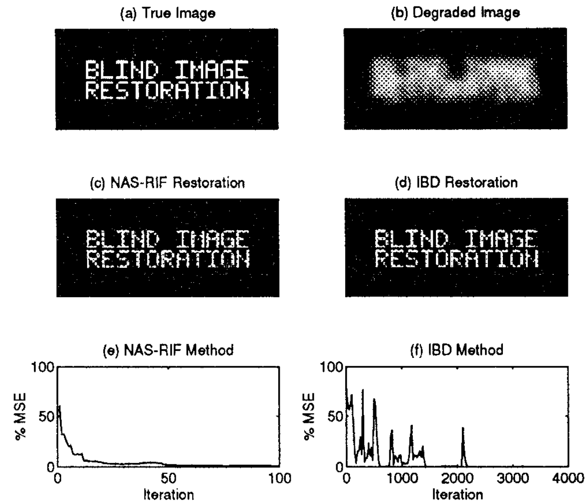

Figure 3. Simulation Results of the proposed NAS-RIF and existing IBD Restoration Methods for the binary image.

of line searches required in `dlinmin.c` [13] at the kth iteration, and the IBD method has order $O(N_f \log_2(N_f))$ per iteration. Since the number of filter parameters is usually much smaller than the image size, the NAS-RIF method requires much fewer computations than the IBD method to produce a good estimate in general.

5. CONCLUSION

A general method for blind deconvolution of images is presented which is flexible in incorporating different image constraints, and shows superior convergence properties to existing methods. An example of the technique for nonnegativity and support constraints, the NAS-RIF method, is discussed and compared to existing techniques. The proposed NAS-RIF algorithm has superior convergence properties than existing methods of its class. Simulation results demonstrate the more reliable performance and faster convergence of the method.

In situations in which the support of the true image is unknown, a support-finding algorithm is proposed. The algorithm shows promise for practical blind image restoration.

REFERENCES

[1] A. K. Katsaggelos, ed., *Digital Image Restoration.* New York: Springer-Verlag, 1991.

[2] M. Cannon, "Blind deconvolution of spatially invariant image blurs and phase," *IEEE Trans. Acoust. , Speech, Signal Process.*, vol. 24(1), pp. 58-63, Feb. 1976.

[3] D. C. Ghiglia, L. A. Romero and G. A. Mastin, "Systematic approach to two-dimensional blind deconvolution by zero-sheet separation," *J. Opt. Soc. Am. A*, vol. 10(5), pp. 1024-1036, May 1993.

[4] R. L. Lagendijk, J. Biemond and D. E. Boekee, "Identification and restoration of noisy blurred images using the expectation-maximization algorithm," *IEEE Trans. Acoust. , Speech, Signal Process.*, vol. 38(7), July 1990.

[5] S. J. Reeves and R. M. Mersereau, "Blur identification by the method of generalized cross-validation," *IEEE Trans. Image Processing*, vol. 1(3), 1992, pp. 301-311.

[6] G. Jacovitti and A. Neri, "A Bayesian approach to 2D non minimum phase AR identification," *Fifth AASP Workshop on Spectrum Estimation and Modelling*, 1990, pp. 79-83.

[7] G. R. Ayers and J. C. Dainty, "Iterative blind deconvolution method and its applications," *Opt. Letters*, vol. 13(7), pp. 547-549, 1988.

[8] B. L. K. Davey, R. G. Lane and R. H. T. Bates, "Blind deconvolution of noisy complex-value image," *Optics Communications*, vol. 69(5,6), pp. 353-356, Jan. 1989.

[9] B. C. McCallum, "Blind deconvolution by simulated annealing," *Optics Communications*, vol. 75(2), pp. 101-105, Feb. 1990.

[10] D. Kundur and D. Hatzinakos, "A novel recursive filtering method for blind image restoration," *Proceedings of the IASTED International Conference on Signal and Imaging Processing – SP- 95*, pp. 428-431, Nov. 1995.

[11] D. Kundur, "Blind deconvolution of still images using recursive inverse filtering," M.A.Sc. thesis, University of Toronto, Department of Electrical and Computer Engineering, 1995.

[12] S. J. Reeves and M. Mersereau, "Automatic assessment of constraint sets in image restoration," *IEEE Trans. Image Processing*, vol. 1(1), pp. 119-123, Jan. 1992.

[13] W. H. Press, S. A. Toukolsky, W. T. Vetterling and B. P. Flannery, *Numerical Recipes in C, The Art of Scientific Computing*, 2nd ed., Cambridge University Press, New York: 1992.