Service Characterizations for Multi–Hop Multiaccess Wireless Networks

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***Abstract*—The objective of a wireless multiaccess communication system is to distribute limited wireless channel resources efficiently and fairly among its users. Solutions for multiaccess communications have been approached with very different perspectives of the same problem, among them information-theoretic approaches at the physical layer, random access at the media access layer, and packet scheduling at the link or network layer. Different system models with sometimes incongruent assumptions make it difficult to compare or reconcile multiaccess solutions emerging from different areas. In this paper we address these difficulties by presenting system-theoretic characterizations of the available service in multiaccess networks for all three approaches. Using these characterizations we derive performance bounds of multiaccess systems that see bursty traffic. We take advantage of a recently proposed $(\min, \times)$ network calculus, which enables the analysis of networks with time-variable traffic (in bits per second) in terms of the fading channels parameters, i.e., signal-to-noise ratio (in dB). For each of the multiaccess approaches considered, we are able to compute probabilistic performance bounds for multi-hop wireless channels. The numerical results shed light on fundamental tradeoffs offered by each of these approaches.**

I. Introduction

Multiaccess networks are networks where multiple users attempt to access the same channel and compete for channel resources, such as transmission power, bandwidth, codes, etc.. Allocating resources efficiently and fairly is the main design criterion of a multiaccess communication system. In a visionary paper from 1984, Gallager [7] identified three research areas: collision resolution, multiaccess information theory, and spread spectrum, that addressed the same multiaccess problem using different models and approaches. Research in these areas was pursued by distinct communities, proceeding in virtual isolation from each other. The purpose was to contrast and compare multiaccess approaches and to provide a unification of the areas, for the sake of improved multiaccess solutions.

Efforts on cross-layer designs [17] for wireless networks, that exploit physical layer information to improve higher layer mechanisms, helped to reduce barriers between disciplines in wireless communications. However, the lack of methods that analyze different multiaccess models has hampered progress towards a unified view of multiaccess communications.

We present an approach that enables the analysis of different multiaccess communication schemes using a single methodology. Our objective is to devise a model that can express inherent tradeoffs offered by various multiaccess solutions, and, thereby, enable a qualitative comparison of their main similarities and differences. We adopt a network layer view of multiaccess systems where the multiaccess network offers a randomly varying service to incoming bursty traffic flows. For the analysis, we follow a system–theoretic stochastic network calculus approach [11]. We address three approaches to channel access: *network information theory*, *random access*, and *dynamic scheduling*. Whereas Gallager in [7] did not include the third approach, dynamic scheduling has since become a major alternative for allocating channel resources of a multiaccess channel. At the same time, we view spread spectrum as being subsumed as an information-theoretic method.

The information-theoretic approach [5, Ch.15] [9] to multiaccess is concerned with coding schemes that reliably communicate data from a set of transmitters to a single receiver concurrently with no assumptions regarding coordination of transmissions. Ahlswede [1] and Liao [13] presented a coding-theorem for such channels. The capacity region for multiaccess fading channel was investigated by Gallager [8]. Information-theoretic models consider noise and interference between users, however, they generally ignore traffic burstiness. As noted in [7], while ignoring burstiness can be appropriate in point–to–point channels, it is less so in multiaccess channels, where generally only few users are busy at any given time.

Random access is a media access layer approach to multiaccess that is widely used in wireless local-area networks. Above the physical layer, only one user can transmit at a time. In the most basic random access scheme, packets are transmitted without any coordination between users. When a collision occurs, the packets are assumed lost and each packet is retransmitted after a randomly selected back off time. Models of random access networks account for traffic burstiness and interference, but generally ignore the underlying communication process, e.g., noise and fading. Recently, there have been efforts to formulate the random access communication problem in terms of an information–theoretic approach, e.g., [15], [16]. In [15], a new channel coding approach that considers bursty traffic properties and packet collision detection was proposed for random access communication. In [16] an information–theoretic formulation of random access is presented and the set of achievable rates is characterized.

The dynamic packet scheduling approach operates at the link or network layer. A scheduler, that is fully aware of all users' channel states, determines which packet(s) from which user(s) will be transmitted next. The communication channel is viewed as a bit pipe with randomly varying service rates. Scheduling approaches in wireless networks [10] capitalize on a large body of literature addressing various aspects of scheduling, e.g., service differentiation, optimization, stability.

In this paper, we present a model for a performance analysis of all the aforementioned multiaccess methods. To our knowledge, this is the first common model and analysis that can encompass all of these approaches. Our analysis is based on a

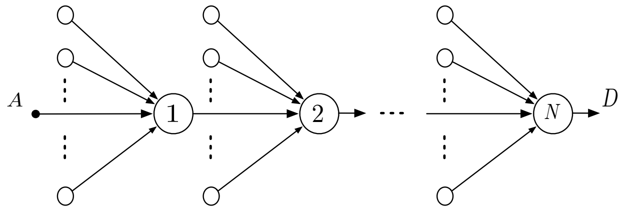


Fig. 1. Multi–hop multiaccess network model.

recently presented stochastic network calculus, referred to as $(\min, \times)$ network calculus, which was developed to evaluate the end–to–end performance of multi–hop wireless networks in terms of the underlying point–to–point fading channel distribution [2]. We derive service characterizations of multiaccess fading channels in terms of stochastic service curves. We then use these service curves to compute probabilistic performance bounds of a traffic flow passing through a cascade of multiaccess channels, as shown in Fig. 1. We view the channel capacity as a time varying function of the underlying instantaneous signal-to-noise ratio (SNR). Our work is related to [4] and [14], which also explore links between information theory and communication networks using network calculus. We note that a joint analysis of different multiaccess methods using network calculus does not exist.

We view the channel capacity as a time varying function of the instantaneous signal-to-noise ratio (SNR). Specifically, with the SNR at the receiver at time epoch u given by γ_u, the instantaneous channel capacity in bits per second per Hz is given by[1] $C(\gamma_u) = \log g(\gamma_u)$, where $g(\gamma_u)$ characterizes the properties of the fading channel. A performance analysis of such system would normally proceed by describing the cumulative service (in bits) in a discrete-time interval $[\tau, t)$ as $S(\tau, t) = \sum_{u=\tau}^{t-1} \log g(\gamma_u)$. However, working with logarithms of random processes is cumbersome and requires approximations or secondary models, e.g., On-Off or Finite State Markov Channels [19]. Instead, $(\min, \times)$ network calculus works with the simpler service description $\mathcal{S}(\tau, t) = \prod_{u=\tau}^{t-1} g(\gamma_u)$. For the analysis, arrivals, measured in bits, are transformed to be compatible with the service description. Let the bivariate process $A(\tau, t)$ describe arrivals in a time interval. The $(\min, \times)$-network calculus uses the transformed arrival process $\mathcal{A}(\tau, t) = e^{A(\tau,t)} = \prod_{u=\tau}^{t-1} g(\gamma_u^a)$, with the interpretation that an arrival at time u is a workload that consumes γ_u^a channel resources. Given arrival and service descriptions in the transform domain, referred to as "SNR domain," one can compute performance bounds that can be readily transferred back to the usual "bit domain" by taking a logarithm.

The ability to transfer network descriptions between different domains facilitates the analysis of networks where traffic and service descriptions are expressed in different layers of the network stack. This property provides the flexibility to tackle an analysis of multi-hop multiaccess communication network.

The presented analysis makes several assumptions and simplifications. For instance, we ignore the control overhead for the multiaccess schemes. Throughout, we use channel capacity limits as time-varying functions, which implies assumptions on the length of the time epoch. We assume that channels in a multi-hop network are independent. We also assume a fixed number of interfering users. As a consequence, we can consider the burstiness of the analyzed flow, but not the burstiness of the interfering flows, thereby ignoring statistical multiplexing gain. Despite these limitations, the analysis and the results are still able to exhibit commonalities and differences of approaches to multiaccess communications, and, may contribute to reconciling and interpreting these approaches.

In Sec. II we give a description of $(\min, \times)$ network calculus. In Sec. III we provide service characterizations for multiaccess models. We provide numerical results for the multi–hop network in Sec. IV, and conclude in Sec. V.

II. $(\min, \times)$ NETWORK CALCULUS

The $(\min, \times)$ network calculus [2] was designed to enable the computation of end–to–end performance bounds of multi–hop wireless networks, where the underlying channels are point–to–point fading channels. The main idea of the $(\min, \times)$ network calculus is to work directly with the fading channel distribution rather than the channel capacity. The calculus takes advantage of the fact that the channel capacity, and hence the offered service, is related to the instantaneous SNR of the underlying fading channel through the logarithmic function via the Shannon capacity limit, $C(\gamma) = W \log(1+\gamma)$, where W is the channel bandwidth and γ is the instantaneous SNR at the receiver. Henceforth, we will work with the normalized capacity $C(\gamma)/W$. We assume block fading where γ remains unchanged during a time slot.

1) Network Model: An N–hop wireless network is modelled by a system of N tandem queues with infinite buffer capacities. We model the traffic which enters the first queue and traverses the entire network as a 'through' flow. We are interested in the end–to–end performance experienced by the through flow. We use the bivariate random processes $A(\tau, t)$ and $D(\tau, t)$ to denote the cumulative arrivals and departures, respectively, measured in bits of the through flow during time interval $[\tau, t)$. $A(\tau, t)$ and $D(\tau, t)$ are real-valued, non-negative processes that are increasing in t with $A(t, t) = D(t, t) = 0$ for all t and $D(0, t) \le A(0, t)$. Then the backlog in the entire network at time $t > 0$ is $B(t) = A(0, t) - D(0, t)$, and the end-to-end delay is $W(t) = \inf\{u \ge 0 : A(0, t) \le D(0, t + u)\}$.

The analysis uses a fluid–flow traffic model and a discrete time domain with equally sized time slots. We further assume that the system starts with empty queues at $t = 0$. The SNR service[2] offered by the fading channel n to flow i, with an instantaneous SNR $\gamma_{i,u}$ during time slot u, is a time–varying random process which is given by

$$\mathcal{S}_{n,i}(\tau, t) = \prod_{u=\tau}^{t-1} g(\gamma_{i,u}) \,. \tag{1}$$

[1] We use the natural logarithm in our computations of channel capacity. Therefore, the channel capacity is in fact $1.44\, C(\gamma_u)$ bits per second per Hz.

[2] We refer to random processes residing in the SNR domain as 'SNR processes', and use calligraphic letters to denote them, e.g., $\mathcal{A}$ and $\mathcal{S}$.

For two SNR processes $\mathcal{X}(\tau,t)$ and $\mathcal{Y}(\tau,t)$, define the $(\min,\times)$ convolution $\mathcal{X}\otimes\mathcal{Y}$ and deconvolution $\mathcal{X}\oslash\mathcal{Y}$ by

$$\mathcal{X}\otimes\mathcal{Y}(\tau,t)\triangleq\inf_{\tau\le u\le t}\left\{\mathcal{X}(\tau,u)\cdot\mathcal{Y}(u,t)\right\}.$$
$$\mathcal{X}\oslash\mathcal{Y}(\tau,t)\triangleq\sup_{u\le\tau}\left\{\frac{\mathcal{X}(u,t)}{\mathcal{Y}(u,\tau)}\right\}.$$

Given SNR service processes $\mathcal{S}_n$ for hop n, the equivalent N–hop SNR service process, denoted by $\mathcal{S}_{\rm net}$, is given by

$$\mathcal{S}_{\rm net}(\tau,t)=\mathcal{S}_1\otimes\mathcal{S}_2\otimes\cdots\otimes\mathcal{S}_N(\tau,t)\,. \quad (2)$$

The analysis proceeds as follows. First, the network traffic processes is transferred to the SNR domain using the exponential function, i.e., $\mathcal{A}=e^A$ and $\mathcal{D}=e^D$. Next, the equivalent SNR network service process $\mathcal{S}_{\rm net}$ is obtained with Eq. (2). Then, bounds for the SNR processes for the end–to–end backlog ($\mathcal{B}$) and delay ($\mathcal{W}$) are computed using $(\min,\times)$ network calculus, where $\mathcal{B}(t)=\mathcal{A}(0,t)/\mathcal{D}(0,t)$, and $\mathcal{W}=W$, since time is maintained in the transfer of domains.

The service offered by a network element n is characterized by the *dynamic SNR server* $\mathcal{S}_n$ which is defined by the following input–output inequality $\mathcal{D}(0,t)\ge\mathcal{A}\otimes\mathcal{S}_n(0,t)$.

2) Performance Bounds: Let $\mathcal{S}_{\rm net}(\tau,t)$ be the network SNR service processes. We define for $s>0$ the function

$$\mathsf{M}_{\rm net}(s,\tau,t)=\sum_{u=0}^{\min(\tau,t)}\mathcal{M}_{\mathcal{A}}(1{+}s,u,t)\,\mathcal{M}_{\mathcal{S}_{\rm net}}(1{-}s,u,\tau), \quad (3)$$

where $\mathcal{M}_Z(s)=E[Z^{s-1}]$, for any complex valued s, is the Mellin transform of the random variable Z [6]. End–to–end performance bounds can be concisely expressed by the following theorem [2]

Theorem 1. *For any $\varepsilon>0$, under the assumptions above, the following are probabilistic performance bounds for a cascade of wireless fading channels.*

- OUTPUT BURSTINESS: $Pr\big(D(\tau,t)>d^{\varepsilon}_{\rm net}\big)\le\varepsilon$, *where*
$$d^{\varepsilon}_{\rm net}(\tau,t)=\inf_{s>0}\left\{\frac{1}{s}\big(\log\mathsf{M}_{\rm net}(s,\tau,t)-\log\varepsilon\big)\right\};$$
- BACKLOG: $Pr\big(B(t)>b^{\varepsilon}_{\rm net}\big)\le\varepsilon$, *where*
$$b^{\varepsilon}_{\rm net}=\inf_{s>0}\left\{\frac{1}{s}\big(\log\mathsf{M}_{\rm net}(s,t,t)-\log\varepsilon\big)\right\};$$
- DELAY: $Pr\big(W(t)>w^{\varepsilon}_{\rm net}\big)\le\varepsilon$, *where*
$$\inf_{s>0}\left\{\mathsf{M}_{\rm net}(s,t+w^{\varepsilon},t)\right\}\le\varepsilon\,.$$

III. MULTIACCESS WIRELESS NETWORKS

The analysis in [2] considers point–to–point fading channel model which is not suitable for multiaccess networks analysis where interfering traffic from other users affects the amount of service received by the through flow. The interference is handled differently in each of the multiaccess approaches described in Section I. We account for the interference by deriving service characterizations for each multiaccess approach.

We assume L homogeneous users transmitting to node n. Each node is associated with a shared fading channel. The arrivals from all interfering users at each hop are independent and identically distributed and their channels have i.i.d. gains.

Due to the bursty nature of traffic, the number of active users (i.e., backlogged) transmitting to node n at any given time is a subset of the total user population. This number is a random process that we denote by $\{M_n(u)\}_{u=0}^{\infty}$, $M_n(u)\in\{1,2,\ldots,L\}$, that is i.i.d. in all time slots $u\in\{0,1,\ldots,t\}$. Given $M_n(u)$, a stochastic performance bound for an arbitrary random process X at a node n with $M_n(u)-1$ interferers during time slot u, can be obtained by conditioning, that is,

$$Pr(X(t)>x)=\sum_{m_n(0)=1}^{L}\cdots\sum_{m_n(t-1)=1}^{L}Pr(X(t)>x|\mathbf{M}_n=\mathbf{m}_n)\cdot Pr(\mathbf{M}_n=\mathbf{m}_n)\,, \quad (4)$$

where the vector $\mathbf{M}_n=(M_n(0),\ldots,M_n(t-1))^T$ and the sample path $\mathbf{m_n}$ are drawn from the distribution of M_n.

Marginalizing in this way is possible but tedious, and will not lead to closed-form results. Instead, we will make the simplifying assumption that a constant number of m_n users are active in each time slot. For any given distribution of M_n, computing Eq. (4) numerically gives the proper bounds.

Next, we derive SNR service characterizations for models of the three multiaccess approaches discussed earlier. The information–theoretic service characterization assumes concurrent transmissions by all users and describes the sum capacity region. Successive decoding can be used to achieve the capacity limit. In the scheduling approach we assume centralized control where one user is selected to transmit at a time. Random access allows users to access the channel whenever they have data to transmit. If a collision occurs, the affected users back off for random times and then retransmit.

A. Information-Theoretic Model

The information-theoretic capacity region for an L–user, flat–fading Gaussian multiaccess fading channel under average power constraint was characterized by Tse and Hanly [18] as

$$C(\mathbf{h},\mathbf{p})=\Big\{\mathbf{r}:\sum_{i\in Q}r_i\le W\log\Big(1+\frac{\sum_{i\in Q}|h_i|^2p_i}{N_0W}\Big),\ \forall Q\subseteq\{1,\ldots,L\}\Big\},$$

for bandwidth W, where $\mathbf{r},\mathbf{p}\in\mathbb{R}^L$ are the rate and power allocation vectors for the L users in the network, $|h_i|^2$ is the channel gain for user i and N_0 is the noise spectral density.

When the number of active users of node n at time slot u is $m_n(u)$, the SNR service process for node n in terms of the normalized sum capacity $C_n(\gamma_u)/W$ is given by $\mathcal{S}^{IT}_n(\tau,t)=\prod_{u=\tau}^{t-1}g(\gamma_u)=\prod_{u=\tau}^{t-1}\big(1+\sum_{i=1}^{m_n(u)}\gamma_{i,u}\big)$, where $\gamma_u\in\mathbb{R}^m$, $\gamma_{i,u}=|h_i|^2p_i/N_0W$ is the instantaneous SNR of user i and $g:\mathbb{R}^m\to\mathbb{R}$. When the power at node n is equally allocated to the $m_n(u)$ users, i.e., $p_i=p_j,\forall i,j$, and with i.i.d channel gains for all users, the $\gamma_{i,u},\forall i$ are i.i.d. In this case, an SNR server for the through flow j at node n is characterized by[3]

$$\mathcal{S}^{IT}_{n,j}(\tau,t)=\prod_{u=\tau}^{t-1} g_j(\gamma_u)=\prod_{u=\tau}^{t-1}\Big(1+\frac{1}{m_n(u)}\sum_{i=1}^{m_n(u)}\gamma_{i,u}\Big)\,,$$

where $g_j(\gamma_u)$ has the distribution

$$Pr(g_j(\gamma_u)\le x)=Pr\Big(\sum_{i=1}^{m_n(u)}|h_i|^2\le\frac{m_n(u)(x-1)}{\bar{\gamma}}\Big).$$

Assuming Rayleigh fading, the above is then the distribution of a sum of $m_n(u)$ i.i.d. exponential random variables. Using the assumption $m_n(u)=m_n$, we determine that $g_j(\gamma)$ has the Gamma distribution with probability density $f_{g_j(\gamma)}(x)=y^{m_n-1}e^{-y}/\Gamma(m_n)$, where, $y=m_n(x-1)/\bar{\gamma}$ and $x\ge 1$. The Mellin transform is then given by the integral

$$\mathcal{M}_{g_j(\gamma)}(s)=\frac{1}{\Gamma(m_n)}\int_0^\infty x^{s-1}y^{m_n-1}e^{-y}dx\,.$$

The above integral can be evaluated numerically. For $m_n=2$, it has the explicit form

$$\mathcal{M}_{g_j(\gamma)}(s)=\frac{1}{\Gamma(2)}e^{\frac{2}{\bar{\gamma}}}\left(\frac{\bar{\gamma}}{2}\right)^s\Big(\Gamma\big(s+1,\frac{2}{\bar{\gamma}}\big)-\frac{2}{\bar{\gamma}}\Gamma\big(s,\frac{2}{\bar{\gamma}}\big)\Big)\,.$$

For i.i.d. fading channels, we use the product property of the Mellin transform to get $\mathcal{M}_{\mathcal{S}^{IT}_{n,j}}(s,\tau,t)=\big(\mathcal{M}_{g_j(\gamma)}(s)\big)^{t-\tau}$.

B. Schedulers for Multiaccess Networks

Consider a centralized scheduler that allocates the wireless channel to a single user at any time slot. With $m_n(u)$ active users at node n, define $\mathbf{v}_n(u)=(v_{n,1}(u),\ldots,v_{n,m_n(u)}(u))$ as the scheduler decision at time u, where $v_{n,i}(u)=1$ if user i is scheduled at time slot u with $\sum_{i=1}^{m_n(u)}v_{n,i}(u)=1$. Let $\gamma^{\mathbf{v}}_u=\prod_{i=1}^{m_n(u)}\gamma_{i,u}^{v_{n,i}(u)}$ be the instantaneous SNR of channel n at time u under schedule $\mathbf{v}_n(u)$. A dynamic SNR server for this multiaccess channel is then given by the service process $\mathcal{S}_n(\tau,t)=\prod_{u=\tau}^{t-1}g(\gamma^{\mathbf{v}}_u)$, where, $g(\gamma_k)=1+\gamma_k$.

1) Opportunistic scheduler: The opportunistic scheduler maximizes channel throughput by scheduling the backlogged user with the best estimated SNR at every time slot [12]. For every n, it selects the schedule $v_{n,i}(u)=1_{\{\gamma_{i,u}>\gamma_{j,u},\forall j\neq i\}}$, for all $u\ge 0$. Defining $\gamma^{\max}_u=\max\{\gamma_{i,u}:i=1,\ldots,m_n(u)\}$, an opportunistic scheduler is characterized by $\gamma^{\mathbf{v}}_u=\gamma^{\max}_u$ for all $u\ge 0$ and its dynamic SNR service for any n is given by

$$\mathcal{S}^{OS}_n(\tau,t)=\prod_{u=\tau}^{t-1}g(\gamma^{\max}_u)=\prod_{u=\tau}^{t-1}(1+\gamma^{\max}_u)\,.$$

Let $m_n(u)=m_n,\forall u$. Using results from order statistics, we obtain the distribution of the m_n^{th} order statistics of $\gamma_{i,u}$, i.e., $F_{\gamma^{\max}_u}(x)$, assuming that channels have i.i.d. fading, as

$$F_{g(\gamma^{\max}_u)}(x)=F^{m_n}_{\gamma_{i,u}}(x-1)=\big(Pr(\gamma_{i,u}\le x-1)\big)^{m_n}\,. \quad (5)$$

[3]This power allocation scheme does not utilize user diversity, but service characterizations for other schemes are also possible.

Assuming a fading channel with i.i.d. channel gain $|h_i|^2$ and average SNR $\bar{\gamma}$, we have $\gamma_{i,u}\overset{d}{=}\gamma_i=\bar{\gamma}|h_i|^2$ for all i, where "$\overset{d}{=}$" expresses equality in distribution, and

$$F_{\gamma_{i,u}}(x-1)=Pr(|h_i|^2\le\frac{x-1}{\bar{\gamma}})\triangleq F_{|h_i|^2}(\frac{x-1}{\bar{\gamma}})\,, \quad (6)$$

for $x\ge 1$. Substituting Eq. (6) in Eq. (5) yields $F_{g(\gamma^{\max}_u)}$.

Assuming that the m_n users have i.i.d. channels, they have access to the channel with probability $\frac{1}{m_n}$. Then under an opportunistic scheduling regime, $\mathcal{S}^{OS}_{n,i}$ is a dynamic SNR server for user i, where

$$\mathcal{S}^{OS}_{n,i}(\tau,t)=\prod_{u=\tau}^{t-1}[g(\gamma^{\max}_u)]^{1/m_n}\triangleq\prod_{u=\tau}^{t-1}g_i(\gamma^{\max}_u)\,,$$

where $g_i(\gamma^{\max}_u)$ has the distribution

$$F_{g_i(\gamma^{\max}_u)}(x)=\Big(F_{|h_i|^2}(\frac{x^{m_n}-1}{\bar{\gamma}})\Big)^{m_n}\,,\quad x\ge 1\,.$$

In the above equation, we used the definition $g_i(\gamma^{\max}_u)=[g(\gamma^{\max}_u)]^{1/m_n}$ and results from order statistics then substituted $\gamma_{i,u}=1+\bar{\gamma}|h_i|^2$. For Rayleigh fading, the Mellin transform of $g_i(\gamma^{\max}_u)$ is given by

$$\begin{aligned}\mathcal{M}_{g_i(\gamma^{\max}_u)}(s)&=\frac{m_n}{\bar{\gamma}}\int_1^\infty y^{\frac{s-1}{m_n}}\Big(1-e^{-\frac{y-1}{\bar{\gamma}}}\Big)^{m_n-1}e^{-\frac{y-1}{\bar{\gamma}}}dy\\&=\sum_{k=0}^{m_n-1}\binom{m_n-1}{k}\frac{(-1)^k m_n\bar{\gamma}^{\frac{s-1}{m_n}}e^{\frac{k+1}{\bar{\gamma}}}}{(k+1)^{\frac{s+m-1}{m_n}}}\Gamma\Big(\frac{s+m_n-1}{m_n},\frac{k+1}{\bar{\gamma}}\Big), \quad (7)\end{aligned}$$

for $s>1$, where $y=x^{m_n}$. The second line is obtained by applying the Binomial theorem $(1-z)^n=\sum_{k=0}^n(-z)^k\binom{n}{k}$ for $|z|<1$, choosing $z=e^{-\frac{y-1}{\bar{\gamma}}}$ and evaluating the resulting integral. Using the assumption of i.i.d. channels, we get $\mathcal{M}_{\mathcal{S}^{OS}_{n,i}}(s,\tau,t)=\big(\mathcal{M}_{g_i(\gamma^{\max}_u)}(s)\big)^{t-\tau}$.

C. Random Access Model

We investigate a service characterization for random access channels that considers the effects of noise and fading as well as users' interference. Recently, Ciucu [4] proposed a network calculus analysis for a slotted–Aloha multiaccess network.

We decompose the problem of analyzing random access networks into two parts. The first deals with the interference aspect, whereas the second deals with noise and fading. For the first part we follow [4] by defining a virtual interference function and we make a similar assumption regarding the saturation of the interfering users. For the second part, we use the SNR service characterization from Eq. (1). We maintain the assumption that the receiver cannot decode a message when a collision occurs and we assume m_n active users at node n at any given time. We denote by the random variable $X_i(u)\in\{0,1\}$ the transmission state of user i at time slot u.

Let $V_n(u)$ be the conditional virtual interference process for flow i during time slot u. Here, $V_n(u)$ represents the interference generated by the other m_n-1 active transmitters within the range of node n in time slot u. Then

$$V_n(u)=1-X_j(u)\cdot\prod_{i=1,i\neq j}^{m_n-1}(1-X_i(u))\,,$$

where $X_i(u), i \neq j$ are i.i.d. Bernoulli random variables with parameter p and $X_j(u)$ is an independent Bernoulli random variable with parameter p^*. Then $V_n(u)$ is also Bernoulli with parameter $1-q$, where $q = p^*(1-p)^{m_n-1}$ is the probability of successful transmission from user j to node n. $V_n(u)$ is 'virtual' since it is assumed that user j is always transmitting.

Assume that all the $X_i(u)$, and hence, $V_n(u)$, are stationary. The interference generated by the random access protocol is captured by the virtual interference process $V_n(u)$ at every time slot u. User i transmits successfully at the channel capacity rate only when all other users are silent. Therefore, the SNR service offered to user i is characterized by

$$\mathcal{S}^{RA}_{n,i}(\tau,t) = \frac{\prod_{u=\tau}^{t-1} g(\gamma_u)}{\prod_{u=\tau}^{t-1}[g(\gamma_u)]^{V_n(u)}} = \prod_{u=\tau}^{t-1}[g(\gamma_u)]^{1-V_n(u)} .$$

The capacity experienced by user i degenerates when $V_n(u) = 1$. On the other hand, when $V_n(u) = 0$, user i receives the full capacity. Since $X_i(u)$ are stationary, we have $V_n(u) \overset{d}{=} V_n$. For i.i.d. fading channels, we also have $\gamma_u \overset{d}{=} \gamma$. Define $G \overset{\triangle}{=} [g(\gamma)]^{1-V_n}$, where $g(\gamma) = 1+\gamma$, and $\gamma > 0$. Then, the distribution of $[g(\gamma)]^{1-V_n}$ for any $x \geq 1$ is given by

$$F_G(x) = Pr([g(\gamma)]^{1-V_n} \leq x) = Pr(g(\gamma) \leq x) \cdot q + 1 - q ,$$

Assuming Rayleigh fading and $g(\gamma) = 1 + \bar{\gamma}|h_i|^2$, we obtain

$$F_G(x) = \left(1 - e^{-\frac{x-1}{\bar{\gamma}}}\right)q + 1 - q = 1 - qe^{-\frac{x-1}{\bar{\gamma}}}$$

with the Mellin transform $\mathcal{M}_G(s) = qe^{\frac{1}{\bar{\gamma}}}\bar{\gamma}^{s-1}\Gamma(s, \bar{\gamma}^{-1})$.

Using the i.i.d. assumptions on γ and X_i, and the product property of the Mellin transform, we obtain

$$\mathcal{M}_{\mathcal{S}^{RA}_{n,i}}(s,\tau,t) = \left[p(1-p)^{m_n-1}e^{\frac{1}{\bar{\gamma}}}\bar{\gamma}^{s-1}\Gamma(s,\bar{\gamma}^{-1})\right]^{t-\tau}.$$

D. Performance Bounds for Multiaccess Channels

We use the SNR service characterizations from Sections III-A to III-C with $m_n = m$ to obtain performance bounds for multi–hop multiaccess networks. These service processes characterize the amount of service offered to the through flow.

We consider a $(\sigma(s), \rho(s))$ bounded through flow [3], where $\frac{1}{s}\log \mathcal{M}_{\mathcal{A}}(s+1,\tau,t) \leq \rho(s)\cdot(t-\tau)+\sigma(s)$, for $s > 1$, traversing a cascade of N multiaccess channels with i.i.d. Rayleigh fading. This arrival model includes Markov-modulated arrivals and exponentially bounded burstiness (EBB) traffic.

Using server concatenation from Eq. (2), a network dynamic SNR server for the opportunistic scheduler is given by [2]

$$\mathcal{M}_{\mathcal{S}^{OS}_{i,\text{net}}}(s,\tau,t) \leq \binom{N-1+t-\tau}{t-\tau}\left(\mathcal{M}_{g_i(\gamma_u^{\max})}(s)\right)^{t-\tau},$$

where, $\mathcal{M}_{g_i(\gamma_u^{\max})}(s)$ is given by Eq. (7). Substituting this in Eq. (3), we get

$$\mathsf{M}_{\text{net}}(s,\tau,t) \leq e^{s(\rho(s)(t-\tau)+\sigma(s))} \sum_{u=[\tau-t]_+}^{\infty} \binom{N-1+t-\tau}{t-\tau} \cdot \underbrace{\left[\sum_{k=0}^{m-1}(-1)^k\binom{m-1}{k}\frac{m\bar{\gamma}^{\frac{-s}{m}}e^{s\rho(s)+\frac{k+1}{\bar{\gamma}}}}{(k+1)^{1-\frac{s}{m}}}\Gamma\left(\frac{m-s}{m},\frac{k+1}{\bar{\gamma}}\right)\right]^u}_{\overset{\triangle}{=}U(s,m)},$$

where $[y]_+ = \max(0, y)$. When $U(s, m) < 1$, the sum converges to $\mathsf{M}_{\text{net}}(s, \tau, t) \leq e^{s(\rho(s)(t-\tau)+\sigma(s))}/(1 - U(s, m))^N$ for $\tau \leq t$, where we used the combinatorial identity

$$\sum_{u=0}^{\infty}\binom{N-1+u}{u}x^u = \frac{1}{(1-x)^N} , \tag{8}$$

for any $0 < x < 1$. Note that $U(s, m) < 1$ can be interpreted as a stability condition for the network. Applying Theorem 1, we get the following probabilistic end–to–end backlog bound

$$b^{\varepsilon}_{\text{net}} = \inf_{s>0}\left\{\sigma(s) - \frac{1}{s}\left(N\log(1-U(s,m)) + \log\varepsilon\right)\right\} .$$

For the delay bound, we compute for $w \geq 0$

$$\mathsf{M}_{\text{net}}(s,t+w,t) \leq e^{s(-\rho(s)w+\sigma(s))}\sum_{u=w}^{\infty}\binom{N-1+u}{u}[U(s,m)]^u$$
$$\leq \frac{e^{s(-\rho(s)w+\sigma(s))}}{(1-U(s,m))^N}\cdot\min\left\{1,[U(s,m)]^w w^{N-1}\right\} . \tag{9}$$

We get the first term in the min by extending the summation down to $u = 0$, and the second term uses the inequality

$$\binom{N-1+u}{u} \leq w^{N-1}\binom{N-1+u-w}{u-w}$$

for $u \geq w$. Then Eq. (9) is obtained by applying Eq. (8) to both cases. Then Eq. (9) can be evaluated numerically.

Probabilistic performance bounds for the other two approaches differ in the form of function $U(s, m)$. For the information–theoretic model and $m = 2$, we compute

$$U^{IT}(s,2) = \frac{e^{s\rho(s)}e^{\frac{2}{\bar{\gamma}}}(\frac{\bar{\gamma}}{2})^{1-s}}{\Gamma(2)}\left(\Gamma\left(2-s,\frac{2}{\bar{\gamma}}\right) - \frac{2}{\bar{\gamma}}\Gamma\left(1-s,\frac{2}{\bar{\gamma}}\right)\right) .$$

For the random access model, we obtain

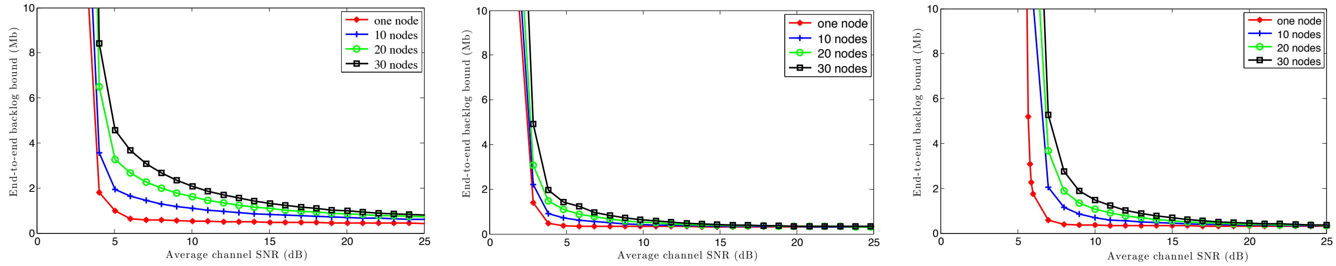
$$U^{RA}(s,m) = e^{s\rho(s)}p^*(1-p)^{m-1}e^{\frac{1}{\bar{\gamma}}}\bar{\gamma}^{-s}\Gamma\left(1-s,\frac{1}{\bar{\gamma}}\right) . \tag{10}$$

Inserting each of these two functions in the performance bounds from Theorem 1 gives bounds for the other models.

IV. Numerical Results

We computed numerical results for a cascade of N multiaccess Rayleigh fading channels with $m = 2$ active users at every hop and a transmission bandwidth of $W = 20$ kHz. The arrivals are $(\sigma(s), \rho(s))$ bounded with deterministic bursts and rates, with $\sigma(s) = 50$ kb and $\rho(s) = 30$ kbps. The probabilistic bounds are determined with violation probability $\varepsilon = 10^{-4}$.

Fig. 2 shows the end–to–end backlog bound of multi–hop multiaccess channels as a function of the average channel SNR for different number of hops under the three different service models derived in the previous section. We use the same model parameters for all three graphs. Furthermore, for Fig. 2(c) we set $p^* = 1$ and $p = 0.2$. The selection of $p^* = 1$ maximizes the capacity received by the through flow at the expense of the other flows and will result in the most optimistic bounds. It is worth noting that when $\bar{\gamma}$ increases the effect of p diminishes since $\bar{\gamma}^{-s}$ becomes dominant in Eq. (10).



(a) Information-theoretic channel model.

(b) Opportunistic scheduler model.

(c) Random access model with $p = 0.2$, $p^* = 1$.

Fig. 2. End-to-end backlog bound ($b^{\varepsilon}_{\text{net}}$) vs. average channel SNR ($\bar{\gamma}$) for multihop multiaccess Rayleigh fading channels using $m = 2$, and with $\varepsilon = 10^{-4}$, $W = 20$ kHz, $(\sigma(s), \rho(s))$ bounded traffic with $\sigma(s) = 50$ kb and $\rho(s) = 30$ kbps, and for $N = 1, 10, 20, 30$.

The backlog bounds in Figs. 2(a), 2(b) and 2(c) exhibit similar trends, but also show notable differences. For sufficiently high average SNR ($\bar{\gamma} > 20$ dB) we observe that the backlog remains low for all the three models for every N. The opportunistic scheduler outperforms the other two models which is a result of the user diversity gain. We also notice that random access performs very close to the opportunistic scheduler at high $\bar{\gamma}$. This is expected since random access is known to perform well at a low utilization. The performance of the information–theoretic model with equal power allocation is lower than that for the opportunistic scheduler and the effect of N on the performance is more visible. This is mainly due to the naive power allocation policy.

When $\bar{\gamma}$ is decreased the channel becomes saturated and the backlog increases sharply resulting in network instability. We observe that the blow–up point for the information–theoretic model is around 4 dB, for the random access model around 6 dB and the opportunistic scheduler outperform the other two at just under 3 dB. On the other hand, the opportunistic scheduler has the highest complexity among the three and generates the most channel state feedback overhead. The information–theoretic model assumes channel coding and incurs coding overhead. Random access is the simplest scheme, and is therefore an attractive approach for networks with low utilization. It is worth noting that the provided comparison is qualitative since we are comparing backlog bounds, rather than exact backlog, for the three multiaccess schemes. Nevertheless, this comparison is reasonable since all three bounds were obtained using same methodology and under the same assumptions which suggest that the three bounds have comparable tightness.

V. Conclusion

In this work, we investigated a system–theoretic network calculus approach for multi–hop multiaccess wireless networks analysis. We considered three distinct approaches to multiaccess communication in the literature: information–theoretic, random access and dynamic scheduling. We provided service characterizations for the three approaches. The service characterizations enable us to conduct a qualitative comparison using end–to–end performance bounds. The numerical results exposed an inherent trade–off between performance and complexity at low SNR. For higher SNR and low utilization, it appears that a simple random access is sufficient. Our numerical results suggest that a hybrid multiaccess design that incorporates a combination of different multiaccess approaches may have merit.

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