Performance Analysis of Gateways with Buffer Constraints

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ABSTRACT

The performance of interconnected networks is highly dependent on the performance of the gateways. Since the finite storage capability of gateways effects the throughput it has to be considered for the analysis of interconnected networks. Different configurations of networks are studied: i) Gateways and the channels of local area networks have no buffer capacity constraints, ii) Only gateways have buffer capacity constraints, iii) Only the channels of local area networks have buffer capacity constraints, iv) Both gateways and the channels of the local area networks have buffer capacity constraints. An approximation method is introduced which allows to compute the throughput for the above network configurations. Examples are given to demonstrate the impacts of gateway buffer capacity on the performance of the network. Approximate results are compared and validated by simulation.

Key Words: Communication Networks, Performance Evaluation, Gateways, Finite Buffers, Throughput

1. Introduction

The interconnection of heterogeneous local area networks is accomplished by dedicated processors, i.e. the gateways, attached to each network. The gateways are the interface between the local and long haul network. They perform necessary protocol conversion, implement flow control algorithms and route packets over the long haul network. Additionally, gateways act as a buffer between networks with different transmission rates. Recent technological advances changed the paradigm of slow long distance communication and (relatively) fast communication in a local area network. The task of a gateway is more difficult when communication has to be maintained between local area networks with different transmission capabilities, e.g., "classical" ethernet type networks exchanging data with high speed local area networks. On the other hand, the imbalance of fast local transmission rates and slow long distance transmission rates may be reversed, e.g., a metropolitan area network backbone with a capacity exceeding by far the transmission rates of the local area networks connected to the backbone. In the near future, several communication systems from different generations will coexist. Internetwork design has to consider the implications of different transmission speeds of the network components. Otherwise, the system will suffer from throughput decrease due to link congestion and packet loss caused by overflow of the gateway buffers.

Few performance studies of gateways in interconnected networks have been done so far. Exley and Merakos [9,10] study two interconnected broadcast networks by simulation and obtain stability conditions

for the network load. They compute values for packet delays under different network access strategies. Lazar and Robertazzi [14] investigate flow control issues using a queueing model of two interconnected networks. Ben-Michael and Rom [6] study two Aloha networks connected via a gateway. By assuming unbounded buffer capacities of gateways they derive analytical formulas for throughput and queueing delay. Varakulsiripunth et. al. [16] analyze a special flow control policy which constrains the amount of traffic accepted by a local area network. They consider the finite buffer space of the gateways and obtain blocking probabilities at the gateways. Heath [11] simulates high speed local area networks and demonstrates the importance of performance decrease due to finite buffer capacity stations. Cheng and Robertazzi [8] give an overview of recent studies on performance analysis of interconnected networks.

In this paper we present analytical solutions for different network configurations to demonstrate the effect of finite buffers of gateways and network access units on the performance of the network. The paper is organized as follows: in section 2 we introduce a classification of different network configurations. For these configurations we develop queueing models in section 3. In section 4 we present analytical solutions for the queueing models. Numerical examples are given in section 5. We demonstrate how the performance of the network is affected if certain parameters are varied. Conclusions are given in section 6. The algorithm we use to solve load dependent queueing networks with finite capacities is explained in detail in the Appendix.

2. System Description

We consider interconnected packet switching networks with several local area networks connected by a long haul network. The local area networks are connected to a long haul network by gateways as shown in Figure 1.
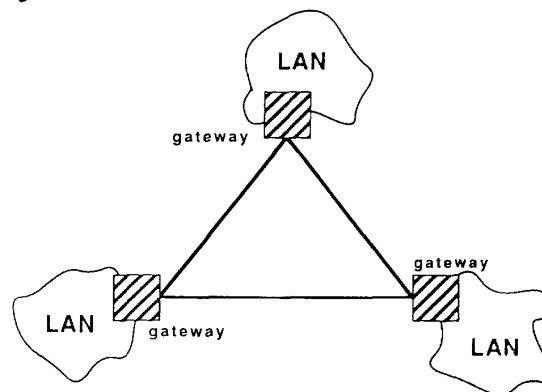

Figure 1. Interconnected Network.

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In the network there are two different types of communication:

i) Communication between hosts of each local area network (*intranetwork traffic*).

ii) Communication between hosts at different local area networks (*internetwork traffic*).

Hosts in the same local area network communicate with each other using a shared broadcast channel. The channel is accessed by the hosts via an interface, so-called network access unit (NAU). Based on communication protocols considered in this study only one packet is allowed to be sent on the channel at a time. If a host wants to transmit a packet to another host in the same local area network it forwards it to its NAU. The access protocol of the local area network decides which packet will be transmitted on the channel next. All packets in NAU of the hosts can be seen as waiting in a global queue for accessing the channel. Though physically still residing at the hosts (in the buffers of the NAU) these packets belong logically to the broadcast channel. Once a packet obtains access to the channel it is immediately transmitted to the destination host, if source and destination hosts belong to the same local area network. If they do not belong to the same local area network, the packet is put into the NAU of the source local area network. The channel sends the packet to the gateway of the source local area network. The gateway then transmits the packet to the gateway of the destination local area network which forwards the packet to the according host through its broadcast channel. The packet has to obtain access to the broadcast channel competing with intranetwork packets.

We investigate four different network configurations based on buffer capacity constraints of channels and gateway buffers.

● **Configuration I: No Capacity Constraints**

Gateways and channels are able to store an infinite number of packets. Even though the infinite buffer capacity assumption is unrealistic we take this configuration into consideration for the sake of comparison with other configurations.

● **Configuration II: Gateways with Capacity Constraints**

If a gateway has a buffer capacity for storing only a limited number of packets for transmission on the long haul network, it is possible that the storage capacity will be exhausted. In this case no more packets are allowed to be forwarded to the gateway because of the buffer overflow problem. All hosts wanting to transmit internetwork packets using this gateway have to wait until a space will become available in the buffer of the gateway. This type of situation occurs if the transmission rate of the local area networks is much higher than the transmission rate of the internetwork. High-speed local networks (HSLN) are a representative example where the high speed of the local area network may cause buffer overflows at the gateways. A low transmission rate for internetwork traffic such as in packet switched satellite networks has the same effect.

● *Configuration III: Channels with Capacity Constraints*

The finite buffer capacity of the channel may cause that each channel is congested. A gateway, for instance, cannot deliver a packet to a host because the access to the channel is not possible due to its congestion. The congestion of the channel can also effect the hosts within each local area network transmitting packets to each other. Heavily loaded local area networks are a representative example for this type of configuration.

● **Configuration IV: Channels and Gateways with Capacity Constraints**

This configuration is a mixture of *configurations II* and *III*. In addition to the performance decrease as in the two above cases, a backlog of untransmitted messages can be observed in a network with a high rate of internetwork transmissions. Suppose, a local area network is congested and packets received by a gateway cannot be forwarded locally. After some time the buffer capacity of the gateway will be exhausted. As an effect, internetwork packets cannot be transmitted to that particular local area network. They

have to remain in their local area network until a space will become available in the congested destination local area network. In this way the congestion may propagate from one local area network to another local area network and may effect the entire internetwork communication. Consequently, the performance of each individual local area network will decrease. In the worst case a deadlock situation may occur where no packets can be transmitted at all.

3. Modeling

We develop queueing models for different configurations described above. The model of the interconnected network consists of M local area networks denoted by LAN_m for $m = 1,...,M$. Each LAN_m contains a station for broadcast channel CH_m and a fixed number R_m of host stations H_{mr} for $r = 1,...,R_m$. Each LAN_m has a gateway station which is composed of an input queue $GW_{m,in}$ and an output queue $GW_{m,out}$ as shown in Figure 2.
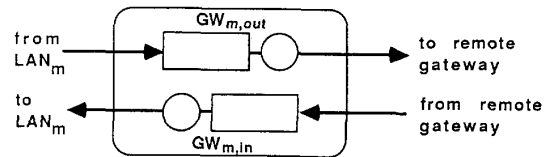

Figure 2. Queueing Model of a Gateway.

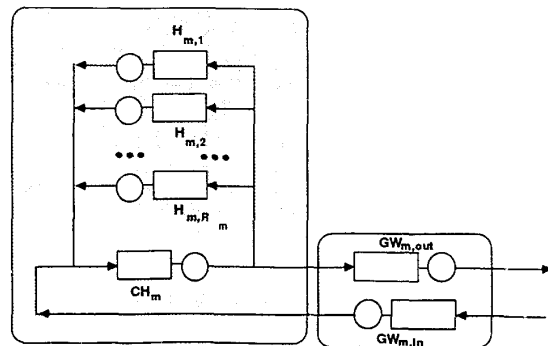The detailed model of a local area network is given in Figure 3.

Figure 3. Queueing Model of a Local Area Network.

All host stations $H_{m1}, H_{m2}, \ldots, H_{mR_m}$ put their packets to the buffer of the according channel CH_m which represents the broadcast channel of LAN_m. From CH_m the packets are routed to either one of the host stations $H_{m1}, H_{m2}, \ldots, H_{mR_m}$ or to the according gateway $GW_{m,out}$. A station of type $GW_{m,in}$ transmits all its output to CH_m. Internetwork connections are established by routing from a station $GW_{m,out}$ to one or more stations $GW_{j,in}$ ($j \neq m$).

The complete model of the interconnected network has N total number of stations where N is composed of:

$$N = \{ \sum_{m=1}^{M} number\ of\ hosts\ in\ LAN_m \ \} + \{number\ of\ channels\}$$

$$+ \{number\ of\ gateway\ queues\}$$

$$= \sum_{m=1}^{M} R_m + M + 2 \cdot M$$

The load of the interconnected network is determined by the fixed

number of packets traversing the network at a time and is denoted by K. In our model packets are routed with fixed probabilities. The service time of all stations is exponentially distributed. The scheduling discipline of all stations is FCFS. All stations CH_m, $GW_{m,in}$ and $GW_{m,out}$ may have a finite buffer size denoted by B_{CH_m}, $B_{GW_{m,in}}$ and $B_{GW_{m,out}}$, respectively. The host stations are assumed to have no buffer constraints ($B_{H_{mr}} = inf$; $m = 1,2,..,M$; $r = 1,2,..,R_m$). Buffer overflows are handled as follows:

> A packet in any station is not allowed to leave if the destination station is full, i.e. the number of packets in the destination station is equal to its buffer capacity. In this case, the packet is blocked in the current station until a packet in the destination station is transmitted and a buffer becomes available.

The complete queueing model of the interconnected network has the structure as given in Figure 4. For the sake of simplicity we give a model with only $M = 3$ local area networks.
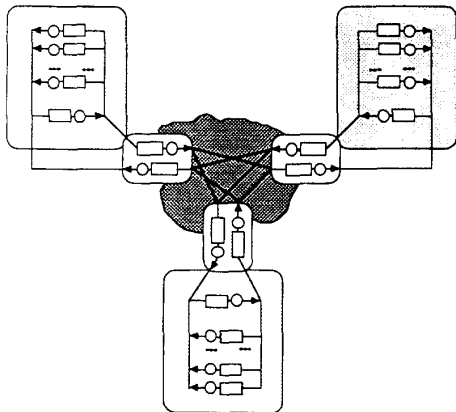

Figure 4. Queueing Model of the Interconnected Network.

4. Performance Analysis

In this section we present analytical solutions providing throughput values for interconnected networks. Since we are primarily interested in studying the performance of interconnected local area networks due to gateway buffer constraints we introduce an approach which allows a separate analysis of intranetwork and internetwork traffic. The analysis of the suggested network configurations will be carried out in two steps. First, we analyze the performance of each local area network independently. Second, we obtain the overall performance of the interconnected network by analyzing the internetwork communication. This approach does not only have the advantage of separating the analysis for different types of traffic, i.e. intra- and internetwork traffic, but also reduces the computational complexity of the analysis and allows us to analyze internetwork traffic under different workload without re-doing the computations for the entire network.

The queueing models of the configurations described in section 2 are different due to buffer size restrictions of gateway and channel stations. The following restrictions apply:

- Configuration I : $B_{CH_m} = B_{GW_{m,in}} = B_{GW_{m,out}} = \infty$
- Configuration II : $B_{GW_{m,in}} < K$; $B_{GW_{m,out}} < K$; $B_{CH_m} = \infty$
- Configuration III : $B_{CH_m} < K$; $B_{GW_{m,in}} = B_{GW_{m,out}} = \infty$
- Configuration IV : $B_{CH_m} < K$; $B_{GW_{m,in}} < K$; $B_{GW_{m,out}} < K$

  for $m = 1,2,...,M$

In the following we discuss the the solutions for each configuration.

4.1. Configuration-I-Networks

Networks of *Configuration I* fulfill the requirements for a product form network [5], i.e., all stations have exponential service times, scheduling is according to FCFS and all buffers are infinite. Hence, analytical methods like mean value analysis, in short form MVA [13], can be applied. In the following we discuss our approach which is based on [7].

For each LAN_m ($1 \leq m \leq M$) we construct one flow-equivalent composite station representing the stations $H_{m1}, H_{m2}, \ldots, H_{mR_m}$ and CH_m. We refer to the flow-equivalent station for LAN_m as *station Lan_m*. The load dependent service rates $\mu_{Lan_m}(k)$ (for $k = 1,2,..., K$) of Lan_m are determined by analyzing the m-th local area network separately, i.e., we set the service times of all stations not belonging to LAN_m equal to zero and compute the throughput values $\lambda_{LAN_m}(k)$, for $k = 1,2,..., K$ using MVA. Then, the values $\lambda_{LAN_m}(k)$ are assigned to the load dependent service rates of the flow-equivalent station $\mu_{Lan_m}(k)$. By this way a reduced network is constructed where the stations belonging to LAN_m are replaced by the flow-equivalent station Lan_m ($1 \leq m \leq M$). The reduced network has the following structure (Figure 5).
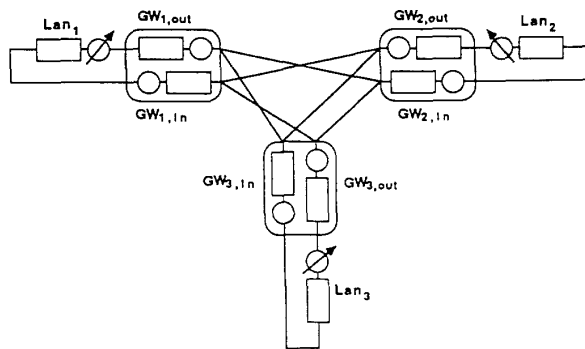

Figure 5. Reduced Queueing Network.

The reduced network can be also analyzed by MVA [13], thus providing exact results. Since all actual systems have finite storage space the results for *configuration-I*-networks can be seen as best case estimations for the investigated networks. The remaining configurations (*II, III* and *IV*) cannot be solved by MVA. Since finite buffer capacity networks do not satisfy the conditions for product form networks we have to apply approximate techniques. However, we will follow the major steps taken in the above procedure, i.e., aggregating the stations of each local area network to a flow-equivalent station and then solving the reduced network from Figure 5.

4.2. Configuration-II-Networks

Here we assume that the buffer size of the gateways is limited ($B_{GW_{m,in}}$; $B_{GW_{m,out}} \leq K$, for $m = 1,2,...,M$). A gateway does not accept packets if its buffer is full. Hence, packets which are ready for transmission to a full station have to remain in the current station until a space becomes available in the full destination buffer, thus keeping the server of the current station idle. We refer to this phenomenon as a *blocking event*. Since the gateway has one queue for incoming traffic ($GW_{m,in}$), as well as for outgoing traffic ($GW_{m,out}$), a full buffer in the gateway may cause blocking at one or more remote gateways $GW_{j,out}$ ($j \neq m$) or at the local channel CH_m.

As for *configuration-I*-networks we first aggregate the stations of each local area network LAN_m, i.e., $H_{m1}, H_{m2}, \ldots, H_{mR_m}$ and CH_m ($1 \leq m \leq M$), to a flow-equivalent station Lan_m. The construction of the

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load dependent stations Lan_m and of the reduced network is carried out exactly as in *configuration I* because the stations of each local area network obey the product form assumptions. However, in the network given in Figure 5 blocking events may occur because of the finite capacity buffers of the gateways. Since blocking causes interdependencies between the stations MVA cannot be applied for performance analysis. Therefore we use the algorithm introduced in the Appendix.

Note that in *configuration-II*-networks only stations CH_m (which are included in the flow-equivalent stations Lan_m in the reduced network) and stations $GW_{m,out}$ may be blocked. Therefore, for *configuration-II*-networks we have to add blocking delay phases to all gateway servers $GW_{m,out}$ and all flow-equivalent stations Lan_m. A formal procedure for delay phase construction is given in the Appendix. We show the result of the phase construction for $GW_{1,in}$ in Figure 6a and for Lan_1 in Figure 6b.
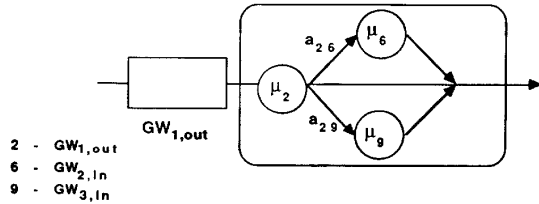

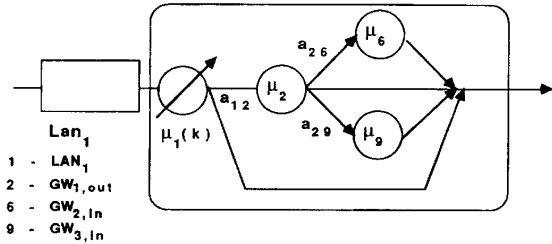Figure 6a. Phases of $GW_{1,out}$ in *Configuration I*.

2 - $GW_{1,out}$
6 - $GW_{2,in}$
9 - $GW_{3,in}$

Lan$_1$

1 - LAN$_1$
2 - $GW_{1,out}$
6 - $GW_{2,in}$
9 - $GW_{3,in}$

Figure 6b. Phases of Lan_1 in *Configuration I*.

Applying the algorithm for load dependent queueing networks with finite buffer capacities given in the Appendix provides us with throughput values for the *configuration-II*-networks.

4.3. Configuration-III-Networks

If the buffer capacity of the broadcast channel in the local area network is finite, throughput degradation occurs for both intranetwork and internetwork traffic, i.e., hosts cannot send packets to the local channel and packets from remote local area networks cannot be delivered to the hosts of the local area network with the congested channel.

As in previous cases we construct the flow equivalent stations Lan_m by analyzing the stations $H_{m1}, H_{m2}, \ldots, H_{mR_m}$ and CH_m separately for each local area network LAN_m ($1 \leq m \leq M$). However, the throughput analysis to obtain $\lambda_{Lan_m}(k)$ (for $k = 1,2,\ldots,K$) cannot be done with standard product form algorithms since the buffer capacities of the stations CH_m are finite. We apply the throughput method for closed queueing networks with exponential service time distributions and finite buffer capacities as described in [2]. With the throughput values we construct the flow-equivalent stations Lan_m with load dependent service rates $\mu_{Lan_m}(k)$ ($1 \leq m \leq M$).

The flow equivalent station Lan_m has a finite buffer capacity which is equal to the buffer capacity of the channel CH_m. The reduced network shown in Figure 5 is analyzed using the algorithm given in the Appendix. However, since only stations Lan_m ($1 \leq m \leq M$) have finite buffer capacity the construction of the delay phases can be simplified. Blocking delays occur only at gateway stations $GW_{m,in}$ waiting for buffer space at Lan_m. All other stations do not have blocking delays. The phase construction for station $GW_{1,in}$ is given in Figure 7.
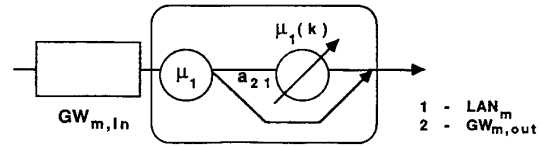

Figure 7. Phases of $GW_{1,in}$ in *Configuration III*.

$GW_{m,in}$ has only one delay phase caused by blocking at CH_m. Note that $GW_{m,out}$, the successor station of CH_m, does not cause blocking at $GW_{m,in}$ since it has infinite buffer capacities. After constructing the delay phases we apply the algorithm given in the Appendix and obtain the throughput values.

4.4. Configuration-IV-Networks

As mentioned before *configuration IV* is a mixture of *configuration-II* and *configuration-III*. However, deadlocks are possible in this configuration. A deadlock is a circular wait of stations each having a packet ready for transmission and waiting for available buffer in the destination stations. The network is analytically tractable only if it is deadlock free. As proven in [12] a network with finite buffer capacities is deadlock free if the sum of the buffer capacities in each cycle of the network exceeds the total number of packets.

The flow equivalent station for each local area network is constructed as described in section 4.3. Note that in this case possibly all stations in the network may cause blocking delays for a station. Applying rules *R1)* and *R2)* from the Appendix we obtain the following phase server for station Lan_1 of Figure 5:

1 - LAN$_1$ 4 - LAN$_2$ 7 - LAN$_3$
2 - $GW_{1,out}$ 5 - $GW_{2,out}$ 8 - $GW_{3,out}$
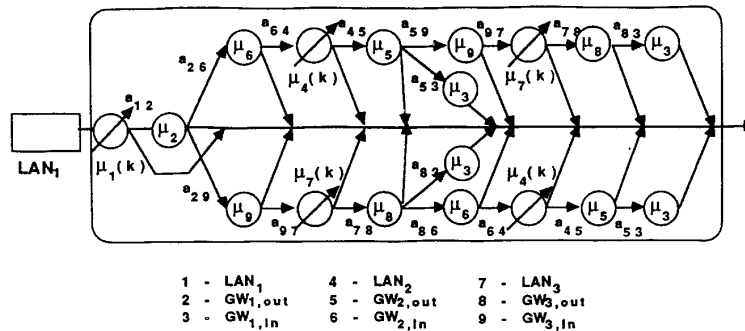3 - $GW_{1,in}$ 6 - $GW_{2,in}$ 9 - $GW_{3,in}$

Figure 8. Phases of Lan_1 in *Configuration IV*.

After the construction of the delay phases for each station we apply the algorithm given in the Appendix and compute the throughput values.

5. Numerical Examples

5.1. Example 1

The example network is a simplified model of the network in Figure 4. Here, we consider an interconnection of two homogeneous local area networks. Each local area network has only three hosts connected to it.
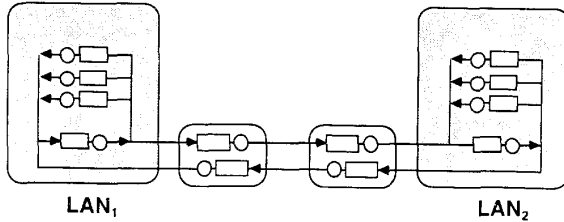
Figure 9. Interconnection of Two Local Area Networks.

The service time of the channel is set to $1/\mu_{CH_m} = 1/3$ ms. Note that according to our discussion in section 2 the service time of the channel includes the time spent in the network access unit. Assuming a packet length of 1000 $Byte/packet$ the service time of the channel corresponds to a network with a maximum transmission rate of 3 $MBit/sec$. The buffer capacity of the channel stations is assumed to be $B_{CH_i}=3$. The ratio of intranetwork traffic and internetwork traffic is set to 3:7 and shows heavy internetwork activity. Intranetwork packets are distributed equally among the host stations. The service time of the gateway stations is assumed to be $1/\mu_{GW_{i,in}} = 1/\mu_{GW_{i,out}} = 1$ (for $m = 1, 2$). The complete list of parameters is summarized in Table 1 and Table 2.

	LAN_m
$1/\mu_{H_{mr}}$ $(r=1,2,3)$	3
$1/\mu_{CH_m}$	1/3
$1/\mu_{GW_{m,in}}$	1
$1/\mu_{GW_{m,out}}$	1

$$m = 1, 2$$

Table 1. Service Times.

p_{ij}	CH_m	$H_{m,r}$ $r=1,2,3$	$GW_{j,in}$ $j \neq m$	$GW_{m,out}$
$H_{m,r}$ $(r=1,2,3)$	1	0	0	0
CH_m	0	0.1	0	0.7
$GW_{m,in}$	1	0	0	0
$GW_{m,out}$	0	0	1	0

Table 2. Transition Probabilities.

We now present examples of different parameter configurations and demonstrate how constraints on the buffer capacity of the stations may decrease the performance of the network.

a) *Configuration I* :

According to the algorithm described in section 4.1 we obtain results given in Figure 10a.
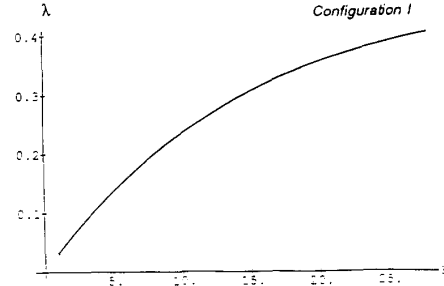
Figure 10a. Throughput for *Configuration I*.

Figure 10a shows the values of the intertraffic throughput, i.e., the throughput seen by each of the gateway stations $GW_{i,j}$ (for $i = 1, 2$ and $j = in, out$), under different network load. Note that these results are exact.

b) *Configuration II* :

We assume the buffer capacities of all gateway stations as:

$$B_{GW_{i,j}} = 2 \qquad \text{for } i = 1, 2 \text{ and } j = in, out$$

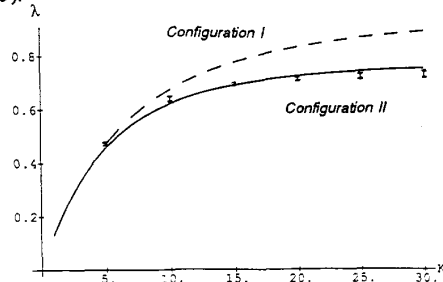The analytical method of section 4.2 provides the following results (Figure 10b).

Figure 10b. Throughput for *Configuration II*.

Our approximate results (†) are compared with the results of configuration I. It can be observed that the difference between the results of *configuration I* and *configuration II* increases if the load of the network increases. This is explained by the increased occurrence of blocking events for networks under heavy load.

c) *Configuration III* :

The buffer capacity constraints for the channel stations are now assumed:

$$B_{CH_i} = 2 \qquad \text{for } i = 1, 2$$
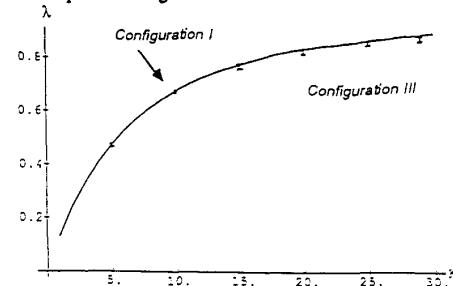
The results are plotted in Figure 10c.

Figure 10c. Throughput for *Configuration III*.

(†) Approximate results are compared with the confidence intervals of simulations. Simulations are done on an IBM 4381 using the RESQ simulation package [SAUE82] with confidence intervals set to 95 %. The confidence intervals in the Figures are denoted by " I ".

The results for *configuration III* are almost identical to results for *configuration I*. Thus, the decrease of performance due to blocking events occurring at the channel station is negligible. For the chosen set of parameters the finite buffer capacity of the channel station does not influence the global performance.

d) *Configuration IV* :

We choose the same buffer capacity constraints we had for the previous configurations:

$$B_{GW_{i,j}} = 2$$

$$B_{CH_i} = 2 \qquad \text{for } i = 1, 2 \text{ and } j = in, out$$

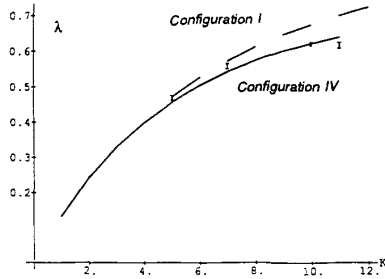The results are plotted in Figure 10d.

Figure 10d. Throughput for *Configuration IV*.

Note that due to the deadlock freedom property performance measures can only be computed up to a maximum load of $K = 11$. It is obvious that the given configuration is not sensitive to buffer constraints of the channel stations in local area networks. However, if finite buffers of the gateway stations are considered as in *configuration II* and *IV* we observe that the overall performance is affected significantly. The performance difference for *configuration-II/IV-* and *configuration-I/III-*networks is illustrated in Figure 10e.
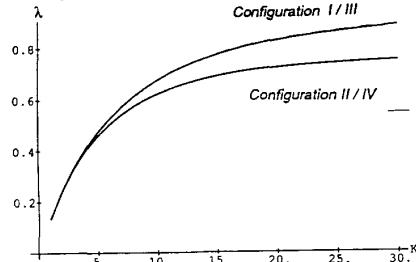

Figure 10e. Summary for *Example 1*.

5.2. Example 2

a) *Basic Model* :

Three local area networks are connected to each other via gateways (Figure 4). The number of hosts in each local area network is set to ten. Including the channel and the gateway stations the entire queueing network has $N = 39$ stations. Our goal for this example is, starting from a set of parameters, to show how the performance of the network changes if certain parameters are changed. We will vary the parameters of the stations of one particular local area network (LAN_1). The parameters for the network are given in Table 3.

B_{CH_m}	10
$B_{GW_{m,in}}, B_{GW_{m,out}}$	2
$1/\mu_{CH_m}$	1/3 ms
$1/\mu_{GW_{m,in}}, 1/\mu_{GW_{m,out}}$	2
$1/\mu_{H_{mr}} \quad (r=1,2,...,10)$	3 ms

$m = 1, 2, 3$

Table 3. Buffer Sizes and Service Times.

Transition probabilities are given by:

p_{ij}	CH_m	$H_{m,r}$ $r = 1,2,...,10$	$GW_{j,in}$ $j \neq m$	$GW_{m,out}$
$H_{m,r} \ r = 1,2,...,10$	1	0	0	0
CH_m	0	0.07	0	0.3
$GW_{m,in}$	1	0	0	0
$GW_{m,out}$	0	0	0.5	0

Table 4. Transition Probabilities.

The network described above is analyzed according to the algorithm given for *configuration-IV*-networks. Throughput results are given in Figure 11a. The dashed line shows throughput values of the corresponding *configuration-I*-network.
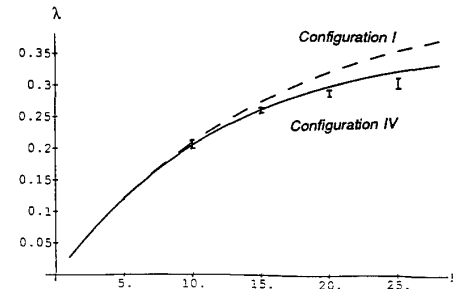

Figure 11a. Throughput of the *Basic Model*.

b) *Modification 1* :

Now assume that LAN_1 is improved in such a way that the maximum transmission rate is increased:

$$\mu_{CH_1} = 40 \ ms^{-1}$$

All other parameters remain unchanged. It can be observed in Figure 11b that the total throughput of the network remains also unchanged.
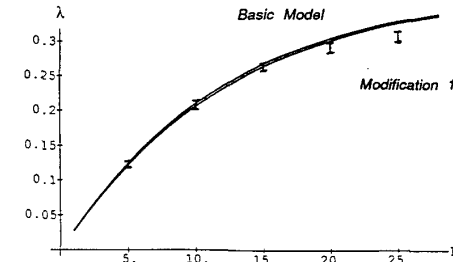

Figure 11b. Throughput of *Modification 1* and the *Basic Model*.

In the following we investigate which parameters of LAN_1 have to be changed in order to achieve a better performance.

c) *Modification 2* :

In addition to the faster channel from *Modification 1* we increase the service rate of the gateway belonging to LAN_1:

- *Modification 2a*: $\mu_{GW_{1,in}} = \mu_{GW_{1,out}} = 2 \ ms^{-1}$
- *Modification 2b*: $\mu_{GW_{1,in}} = \mu_{GW_{1,out}} = 20 \ ms^{-1}$

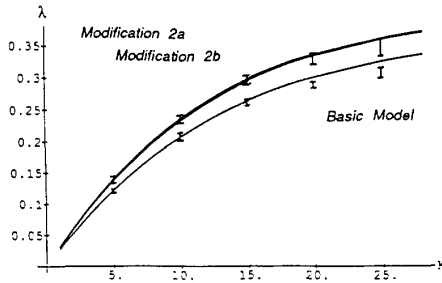Clearly, the throughput values can be improved as demonstrated in Figure 11c.

Figure 11c. Throughput of *Modification 2a/b* and the *Basic Model*.

d) *Modification 3* :

Another way to improve the performance starting from *Modification 1* is to increase the buffer space of the gateway connected to LAN_1. We assume:

$$B_{GW_{1,j}} = 5 \qquad \text{for } j = in, out$$

Other parameters are as given in *Modification 1*. Figure 11d plots the throughput values of *Modification 3* and compares them with those from *Modification 2*.
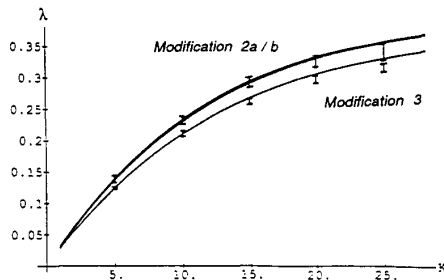

Figure 11d. Throughput of *Modification 2a/b* and *Modification 3*.

It can be seen that an increase of the gateway buffer size as in *Modification 3* does not improve the performance as the reduced service time of the gateway (*Modification 2*). This can be verified by a combination of *Modification 2* and *3* as given in *Modification 4*.

e) *Modification 4* :

We assume the same parameters as in *Modification 1* except:

$$\mu_{GW_{1,in}} = \mu_{GW_{1,out}} = 20 \ ms^{-1}$$

$$B_{GW_{1,j}} = 5 \qquad \text{for } j = in, out$$

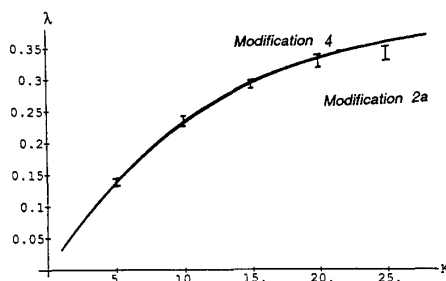Figure 11e plots the throughput values for *Modification 2a* as well as for *Modification 4*.

Figure 11e. Throughput of *Modification 2a* and *Modification 4*.

Different curves cannot be distinguished. Thus, we conclude for *example 2*: if the transmission rate of the channel of a local area network (assuming the parameters given above) is increased without changing other devices of the network, the internetwork throughput can be improved by increasing the service rate of the gateway connected to it. An increase of the gateway buffer size does not have a similar effect. However, in Figure 11c we see that the improvements achieved by speeding up the gateway is limited.

6. **Conclusions**

This study investigated the performance degradation of interconnected computer networks due to finite storage space of the involved stations. We suggested a classification of interconnected networks, developed queueing network models and proposed computational algorithms which give analytical solutions for each class of networks. Numerical examples demonstrated the performance differences of the different classes of networks. Due to the topology of interconnected local area networks the performance of internetwork traffic was shown to be sensitive to variations of the parameters of gateway stations. An improvement of the transmission rate in a local area network was ineffective for internetwork communication unless the parameters of the gateway stations were improved at the same time.

Appendix : Throughput Analysis of Load Dependent Queueing Networks with Finite Buffer Capacities

Model Assumptions:

We consider closed queueing networks with N stations and K total packets. The service time at station i is exponentially distributed with load dependent mean values $1/\mu_i(k)$ (for $i = 1,..., N$ and $k = 1,..., K$). The scheduling discipline at each station is assumed to be FCFS. Each station has a fixed finite buffer capacity B_i where $B_i = (queue\ capacity + 1)$, (for $i = 1,..., N$). Any station whose buffer capacity exceeds the total number of packets in the network can be considered to have infinite capacity ($B_i = \infty$ for some $i = 1, 2, .., N$). A packet which is serviced by the i-th station proceeds to the j-th station with probability p_{ij}, (for $i, j = 1,..., N$), if the j-th station is not full. That is, if the number of packets in the j-th station, k_j, is less or equal to B_j. Otherwise, the packet is blocked in the i-th station until a packet in the j-th station has completed its servicing and a place becomes available.

Algorithm :

The basic idea of the solution algorithm is to replace the finite capacity queueing network, from now on denoted as Φ, by an infinite capacity queueing network, denoted as Γ. In substituting Φ by Γ we have to take the blocking events into account which occur between the stations in Φ, i.e., a packet being served in a station cannot leave the server of this station because the destination station is full. To consider the blocking events we modify the service mechanism of a station such that all delays a packet might undergo due to blocking events in Φ can be represented. Therefore, corresponding delay phases caused by blocking events are appended to the service unit of each station. The frame of the algorithm consists of three steps:

a) Construction of delay phases for each station

b) Computation of service times and branching probabilities for phases

c) Solution of the Network Γ

a) **Construction of delay phases for each station**

Let i be a station of Φ. For each possible blocking delay caused by another station j in Φ (for $i,j = 1,..., N; i \neq j$) we add a service phase to station i. The connection between the added phases and the original server of station i is the same as the transitions between stations in Φ.

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We have to consider that blocking delays may not only be caused by a station's immediate successors but also by stations which occur in each cycle of the network where a particular station is represented. Let i be an arbitrary station in the network. $C_i(l)$ the l-th cycle of stations that starts and ends at station i is defined by:

$$C_i(l) = (i, j_{l_1}, j_{l_2}, ..., i)$$

where j_{l_q} is the q-th station in cycle l of station i. Now let us consider one of these cycles, $(i, j_{l_1}, j_{l_2}, ..., i)$. For instance, assume that there are k_i packets at station i and the number of packets in the network be such that station j_{l_1} through j_{l_r} can be full at the same time. In this case, a packet upon service completion at station i may find station j_{l_1} full, blocking station i's server. Now the question is when this blocked packet will depart from station i. If upon service completion at station j_{l_1} a packet chooses to go to station j_{l_2} which is not full, then the packet at station j_{l_1} will depart and at the same time another packet at station i will join station j_{l_1} unblocking server of the station i. However, if a packet at station j_{l_2} gets blocked because its destination is full then the blocked packet at station i cannot depart. Hence, in the worst case a packet at station i will wait for service completions at stations $j_{l_1}, ..., j_{l_r}$ before leaving station i.

In constructing the phases the following rules must be obeyed [3]:

R1) If two or more cycles are identical up to a certain element then the elements prior to that element are represented only once in the phase construction.

R2) If $\Omega_i(l) = (i, j_{l_1}, j_{l_2}, ..., j_{l_q})$ is a path in cycle $C_i(l)$ starting from station i with
$$\sum_{r=1}^{q-1} B_{j_{l_r}} < K \leq \sum_{r=1}^{q} B_{j_{l_r}},$$ then the stations $(j_{l_q}, j_{l_{q+1}}, ..., i)$ of $C_i(l)$ are not considered in the phase construction for this cycle. In other words, if the sum of station buffer capacities in $\Omega_i(l)$ exceeds the total number of packets then the last station of $\Omega_i(l)$ and all its successors in $C_i(l)$ are not taken into account in the phase construction for station i for that cycle.

b) **Computation of Service Times and Branching Probabilities for Phases**

After the construction of phases we need to determine the parameters such as branching probabilities and service times of the phases.

Since a blocked packet cannot leave a station until space becomes available in the full destination station the time a packet is blocked is equal to the mean remaining service time of the station which causes the blocking. Because of exponential service time distribution the mean remaining service time of a station is given by the mean service time [1]. Therefore, the blocking delay for a load dependent station i caused by station j is determined by $1/\mu_j(B_j)$, the mean service time of station j having a load of B_j packets.

a_{ij} denotes the probability that a packet after a service or blocking delay in phase i enters the phase representing station j. The value of a_{ij} is computed by:

$$a_{ij} = p_{ij} \cdot P_j(k_j = B_j + 1) \qquad \text{for} \quad i, j = 1,..., N \qquad (A.1)$$

where p_{ij} is the transition probability of the original network and $P_j(k_j = B_j + 1)$ are blocking probabilities computed by an iteration which will be discussed next.

For calculation of the probabilities $P_j(k_j = B_j + 1)$ we assume that each station behaves like an isolated station with exponential service time, Poisson arrivals and finite buffer, known as $[M/M/1/N]$ station. In the considered blocking protocol a blocked packet has already been processed at the station it resides at. Thus, the blocked packet belongs logically to the full destination station occupying the (B_j+1)th position in the destination station j which has a buffer capacity of B_j. Hence, we may

approximate the blocking probability by the steady state probability $P_j(k_j = B_j+1)$ of the finite capacity station having its buffer increased by one. The formula for the probability of a finite $[M/M/1/B_j+1]$ station is computed by:

$$P_j(k_j = B_j + 1) = \begin{cases} \rho_j^{B_j+1} \cdot \dfrac{1 - \rho_j}{1 - \rho_j^{B_j+2}} & \text{if } B_j < K \\ 0 & \text{if } B_j \geq K \end{cases} \qquad (A.2)$$

with

$$\rho_j = \frac{\hat{\lambda}_j}{\mu_j(k)} \qquad \text{for} \quad j = 1,..., N, k = 1,..., K \qquad (A.3)$$

where $\hat{\lambda}_j$ is the arrival rate to station j. Since arrivals to station j are rejected once the buffer capacity of the station is exhausted ($k_j = B_j + 1$) we may express $\hat{\lambda}_j$ in terms of the effective arrival rate λ_j as follows:

$$\hat{\lambda}_j = \frac{\lambda_j}{1 - P_j(k_j = B_j + 1)} \qquad \text{for} \quad j = 1,..., N \qquad (A.4)$$

In other words, the effective arrival rate λ_j is the portion of all arrivals which are not rejected because of a full buffer. Equations (A.2) and (A.4) are used as fixpoint iteration to compute the values for $P_j(k_j = B_j + 1)$ for $j = 1,2,..., N$. With the $P_j(k_j = B_j + 1)$ values the probabilities a_{ij} of equation (A.1) can be determined.

c) **Solution of the Network Γ**

Now, we are able to compute the throughput of Γ with the following iteration:

Initially we set all branching probabilities a_{ij} between service and delay phases of each station to zero and, hence, eliminate all delay phases. The network Γ has then the same structure as the network Φ except the stations' buffer capacities are now infinite. Since all stations have exponentially distributed service times with mean value $1/\mu_i(k)$ we obtain the throughput $\lambda^{(0)}$ by applying a product form algorithm such as mean value analysis [41]. The throughput λ_j of each station j is determined by:

$$\lambda_j = \lambda^{(0)} \cdot e_j \qquad \text{for} \quad j = 1,..., N \qquad (A.5)$$

where e_j is the mean number of visits that a packet makes to node j and is given by:

$$e_j = \sum_{i=1}^{N} e_i \cdot p_{ij} \qquad \text{for} \quad j = 1,..., N \qquad (A.6)$$

With the throughput values we compute the fixpoint iteration of equations (A.2) and (A.4) to obtain $P_j(k_j = B_j + 1)$ for $j = 1,2,..., N$. The values for $P_j(B_j + 1)$ are then used to determine the branching probabilities a_{ij} from equation (A.1).

Now, for each multi-phase station j of network Γ we determine the total mean service times $\bar{\mu}_j(k)$, the variance $\bar{\sigma}_j^2(k)$ and the coefficient of variation $\bar{c}_j(k)$. Since the time a packet spends in service phase and each blocking phase of a station are determined by independent random variables with exponential distribution functions, the total time a packet spends in the multi-phase server is itself given by an exponential random variable. The following equations show how to compute $\bar{\mu}_j(k)$, $\bar{\sigma}_j^2(k)$ and $\bar{c}_j(k)$.

$$\frac{1}{\bar{\mu}_j}(k) = \frac{1}{\mu_j(k)} + \sum_{\text{all blocking phases } l} \frac{prob_l}{\mu_l(k)} \qquad (A.7)$$

$$\bar{\sigma}_j^2(k) = \left[\frac{1}{\mu_j(k)}\right]^2 + \sum_{\text{all blocking phases } l} \left[\frac{prob_l}{\mu_l(k)}\right]^2 \qquad (A.8)$$

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$$\bar{c_j}(k) = \sqrt{\bar{\sigma_j^2}(k)} \cdot \bar{\mu_j}(k) \qquad (A.9)$$

$$\text{for} \quad j = 1,\dots, N \quad \text{and} \quad k = 1,\dots, K$$

with

$$prob_l = \prod_{n=0}^{t} a_{s_n s_{n+1}} \qquad (A.10)$$

with $s_0 = j$, $s_t = l$ and $< j, s_1, s_2,\dots, s_{t-1}, l >$ a path of phases in the multi-phase server of station i.

Having calculated the mean value and the coefficient of variation of the service times of all stations we apply the algorithm for queueing networks with general service times and load dependent servers [4] and obtain the throughput value $\lambda^{(n)}$. An iteration test is carried out:

$$| \lambda^{(n)} - \lambda^{(n-1)} | > \varepsilon \qquad (A.11)$$

for n as the number of iterations. If the difference between consecutive throughput values is greater than a threshold value (e.g., $\varepsilon = 10^{-4}$) we continue with the next iteration. Otherwise, the iteration terminates and the final throughput values are obtained.

References

1. I. F. Akyildiz, "Mean Value Analysis for Blocking Queueing Networks", *IEEE Trans. on Software Eng.*, Vol. SE-14, No. 4, April 1988, pp. 418-429.

2. I. F. Akyildiz, "Product Form Approximations for Queueing Networks with Multiple Servers and Blocking", *IEEE Trans. on Comp.*, Vol. 15, No. 1, Jan. 1989, pp. 99-114.

3. I. F. Akyildiz and J. Liebeherr, "Application of Norton's Theorem on Queueing Networks with Finite Capacities", *Proc. INFOCOM '89*, Ottawa, Canada, April 1989, pp. 914-923.

4. I. F. Akyildiz and A. Sieber, "Approximate Analysis of Load-Dependent General Queueing Networks", *IEEE Trans. on Software Eng.*, Vol. 14, No. 11, Nov. 1988, pp. 1537-1545.

5. F. Baskett, K. M. Chandy, R. R. Muntz and G. Palacios, "Open, Closed and Mixed Network of Queues with Different Classes of Customers", *Journal of the ACM*, Vol. 22, Nr. 2, April 1975, pp.248-260.

6. S. Ben-Michael and R. Rom, "Gatewaying Two Aloha Networks", *Proc. INFOCOM '86*, Miami, FL., 1986, pp. 30-33.

7. K. M. Chandy, U. Herzog and L. Woo, "Parametric Analysis of Queueing Network Models, *IBM Journal Res. Dev.*, Vol. 19, Nr. 1, Jan. 1975, pp.43-49.

8. Y.-C. Cheng and T. G. Robertazzi, "Annotated Bibliography of Local Communication System Interconnection", *IEEE J. on Selected Areas in Comm.*, Vol. SAC-5, No. 9, Dec. 1987, pp. 1492-1499.

9. G. Exley and L. Merakos, "Packet Delay Characteristics in Interconnected Random Access Networks", *Proc. GLOBECOM '86*, Houston, TX., 1986, pp. 455-459.

10. G. Exley and L. Merakos, "Throughput-Delay Performance of Interconnected CSMA Local Area Networks", *IEEE J. on Selected Areas in Comm.*, Vol. SAC-5, No. 9, Dec. 1987, pp. 1380-1390.

11. J. R. Heath, "Analysis of Gateway Congestion in Interconnected High-Speed Local Networks", *IEEE Trans. on Comm.*, Vol. 36, No. 8, Aug. 1988, pp. 986-989.

12. S. Kundu and I. F. Akyildiz, "Deadlock Free Buffer Allocation in Closed Queueing Networks", *Queueing Systems Journal*, Vol. 4, Jan. 1989, pp. 47-56.

13. M. Reiser and S. S. Lavenberg, "Mean Value Analysis of Closed Multichain Queueing Networks", *JACM*, Vol. 27, Nr. 2, April 1980, pp. 313-322.

14. A. A. Lazar and T. G. Robertazzi, "Optimal Flow Control of Networks Interconnected via Gateways", *Proc. 19th Annual Conf. Information Science Systems*, Baltimore, MD., March 1985, pp. 263-268.

15. C. H. Sauer, E. A. MacNair, J. F. Kurose, "The Research Queueing Package Version 2", *IBM Thomas J. Watson Research Center*, Yorktown Heights, New York 10598.

16. R. Varakulsiripunth, N. Shiratori, and S. Noguchi, "Congestion Control Policy on Internetwork Gateway", *Proc. ICCC '86*, ed. P. Kuhn, North-Holland, 1986, pp. 659-664.