Service Characterizations for Multi-Hop Multiaccess Wireless Networks

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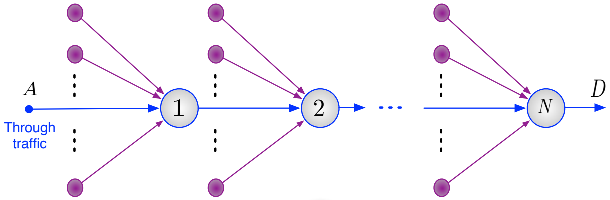
Multihop Multiaccess Wireless Network



- Nodes are store and forward relays
- Channel capacity at each node is shared among multiple users
- Service offered to A is given by the multiaccess fading channel capacity

Our interest: End-to-end delay and backlog

Motivation

R. Gallager (1985): A Perspective on Multiaccess Channels:
"For the last ten years there have been at least three bodies of research on multiaccess channels, each proceeding in virtual isolation from the others and each using totally different models."

1. collision resolution,
2. multiaccess information theory,
3. spread spectrum.

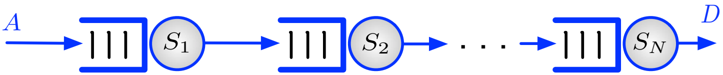
Our take: Models that can analyze different types of multiaccess networks enable a unified view of multiaccess communications.

Multiaccess Wireless Channels

1. Multiaccess information theory
    - physical layer
    - coding schemes for reliable many–to–one communication
    - permits concurrent transmissions with no coordination
2. Random access or collision resolution
    - MAC layer
    - no coordination of transmissions
    - collision when two or more users transmit simultaneously
3. Dynamic scheduling
    - link/network layer
    - centralized scheduler makes decision which user can transmit

$\Rightarrow$ **different interference types lead to different service models**

Network Model



- Fluid-flow traffic, discrete time
- Interfering flows are i.i.d. and their channels have i.i.d. gains
- Rayleigh fading channels
- Channel capacity with SNR γ_i at time i is $C(\gamma_i) = \log g(\gamma_i)$, where $g(\gamma_i)$ encodes the effect of the multiaccess mechanism.
- Control overhead is ignored

Q: How do different multiaccess approaches affect the end–to–end network layer performance for the through flow?

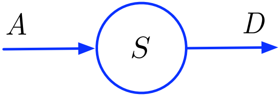
Our Approach

- **Goal:** Reconcile and compare multiaccess solutions at physical, MAC, and link/network layer.

- **Analysis approach:** Network calculus

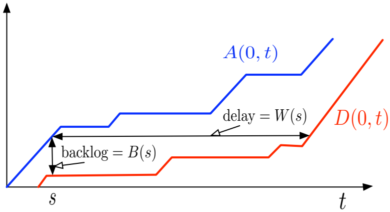
- **How we proceed:**
  1. Obtain service characterizations for the three channel models
  2. Apply network calculus to obtain end–to–end performance bounds
  3. Compare bounds

Network Calculus



- $(\min, +)$ dioid algebra
- Backlog: $B(s) = A(0,s) - D(0,s)$
- Delay: $W(s) = \inf\{u \geq 0 : A(0,s) \leq D(0,s+u)\}$
- Dynamic server [Chang 2000]

$$D(0,t) \geq \inf_{u \leq t}\{A(0,u) + S(u,t)\}$$
$$= A * S(0,t)$$



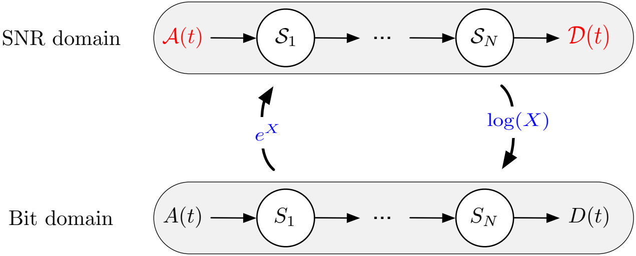
- Network service (multi node):

$$S_{\text{net}}(\tau,t) = S_1 * S_2 * \cdots * S_N(\tau,t)$$

Traffic and service measured in bits $\Longrightarrow$ 'Bit domain'

Network Calculus for Wireless Networks Analysis *(Infocom '13)*

- Characterization of wireless channels is simplified by performing analysis in an alternative domain (SNR domain)



- Equivalent transfer between the two domains using logarithmic functions

Service and Traffic Elements in the SNR Domain *(Infocom '13)*

- Service process for fading channel in the bit domain:

$$S(\tau, t) = \sum_{i=\tau}^{t-1} \log g(\gamma_i)$$

- Service process in the SNR domain

$$\mathcal{S}(\tau, t) = e^{S(\tau,t)} = \prod_{i=\tau}^{t-1} g(\gamma_i) \qquad \Longrightarrow \text{ simpler}$$

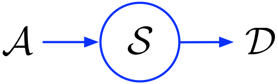
- SNR traffic processes:

$$\mathcal{A}(\tau, t) = e^{A(\tau,t)} \quad \text{and} \quad \mathcal{D}(\tau, t) = e^{D(\tau,t)}$$

$\Longrightarrow$ SNR domain is governed by $(\min, \times)$ dioid algebra

$(\min, \times)$ Network Calculus *(Infocom '13)*

- Service: $\mathcal{S}(\tau, t) = \prod_{i=\tau}^{t-1} g(\gamma_i)$
- Arrival: $\mathcal{A}(\tau, t) = \prod_{i=\tau}^{t-1} e^{a_i}$



$(\min, \times)$-convolution: $\mathcal{X} \otimes \mathcal{Y}(\tau, t) \stackrel{\triangle}{=} \inf_{\tau \le u \le t} \{\mathcal{X}(\tau, u) \cdot \mathcal{Y}(u, t)\}$

$(\min, \times)$-deconvolution: $\mathcal{X} \oslash \mathcal{Y}(\tau, t) \stackrel{\triangle}{=} \sup_{u \le \tau} \left\{ \frac{\mathcal{X}(u, t)}{\mathcal{Y}(u, \tau)} \right\}$

- Departures: $\mathcal{D}(0, t) \ge \mathcal{A} \otimes \mathcal{S}(0, t)$
- Backlog Bound: $\mathcal{B}(t) = \mathcal{A}(0, t)/\mathcal{D}(0, t) \le \mathcal{A} \oslash \mathcal{S}(t, t)$
- Delay Bound: $\mathcal{W}(t) \le \inf \left\{ d \ge 0 : \mathcal{A} \oslash \mathcal{S}(t + d, t) \le 1 \right\}$
- Network SNR server: $\mathcal{S}_{\text{net}}(\tau, t) = \mathcal{S}_1 \otimes \mathcal{S}_2 \otimes \cdots \otimes \mathcal{S}_N(\tau, t)$

Statistical Performance Bounds *(Infocom '13)*

- Mellin transform: $\mathcal{M}_X(s) = E[X^{s-1}]$
- $\mathsf{M}_{\text{net}}(s,\tau,t) \triangleq \sum_{u=0}^{\min(\tau,t)} \mathcal{M}_{\mathcal{A}}(1+s,u,t) \cdot \mathcal{M}_{\mathcal{S}_{\text{net}}}(1-s,u,\tau)$

- BACKLOG: $Pr\big(B(t) > b^{\varepsilon}_{\text{net}}\big) \leq \varepsilon$, where

$$b^{\varepsilon}_{\text{net}} = \inf_{s>0} \left\{ \frac{1}{s} \big(\log \mathsf{M}_{\text{net}}(s,t,t) - \log \varepsilon \big) \right\}$$

- DELAY: $Pr\big(W(t) > w^{\varepsilon}_{\text{net}}\big) \leq \varepsilon$, where

$$\inf_{s>0} \left\{ \mathsf{M}_{\text{net}}(s, t + w^{\varepsilon}{}_{\text{net}}, t) \right\} \leq \varepsilon$$

Service Characterization: Information–Theoretic Model

- Channel capacity is characterized by [Tse and Hanly 1998] as

$$C(\mathbf{h},\mathbf{p}) = \left\{\mathbf{r} : \sum_{i\in Q} r_i \leq W \log\left(1 + \frac{\sum_{i\in Q} |h_i|^2 p_i}{N_0 W}\right), \forall Q \subseteq \{1,\ldots,L\}\right\}$$

  - $\mathbf{r},\mathbf{p} \in \mathbb{R}^L$ are the rate and power allocation vectors
  - $|h_i|^2$ is the channel gain for user i
  - Capacity limit is achieved by coding, e.g., successive decoding

- Assume m active users at a node with power allocated equally
- $\mathcal{S}_j^{IT}$ is an SNR server for the through flow j at the node, where

$$\mathcal{S}_j^{IT}(\tau,t) = \prod_{u=\tau}^{t-1} g_j(\gamma_u) = \prod_{u=\tau}^{t-1}\left(1 + \frac{1}{m(u)} \sum_{i=1}^{m(u)} \frac{|h_i|^2 p_i}{N_0 W}\right)$$

  - This power scheme does not utilize user diversity

Service Characterization: Opportunistic Scheduler

- Maximizes throughput by scheduling user i at a node at time slot u if

$$\gamma_{i,u} = \max\{\gamma_{k,u} : k = 1, \ldots, m\} \triangleq \gamma_u^{\max}$$

- Users with i.i.d. channel gains can access the channel $1/m$ of the time, then $\mathcal{S}_j^{OS}$ is a dynamic SNR server for user j, where

$$\mathcal{S}_j^{OS}(\tau, t) = \prod_{u=\tau}^{t-1} [g(\gamma_u^{\max})]^{1/m} \triangleq \prod_{u=\tau}^{t-1} g_j(\gamma_u^{\max})$$

- The distribution of $g_j(\gamma_u^{\max})$ is given by

$$F_{g_j(\gamma_u^{\max})}(x) = \left[F_{|h_j|^2}\left(\frac{x^m - 1}{\bar{\gamma}}\right)\right]^m, \quad x \geq 1$$

Service Characterization: Random Access

- (Ciucu 2011) For the through flow j at a node, let $V_j(u)$ be the conditional virtual interference process during time slot u, then

$$V_j(u) = 1 - X_j(u) \cdot \prod_{i \neq j} (1 - X_i(u))$$

  - $X_i(u)$ are i.i.d. Bernoulli(p)
  - Then $V_j(u)$ is also Bernoulli($1 - q$)
  - $q = p(1-p)^{m-1}$ $\equiv$ probability of successful transmission

- User j can transmit successfully at the channel capacity rate when $V_j(u) = 0$, hence, an SNR server for user j is given by

$$\mathcal{S}_j^{RA}(\tau, t) = \frac{\prod_{u=\tau}^{t-1} g(\gamma_u)}{\prod_{u=\tau}^{t-1} [g(\gamma_u)]^{V_j(u)}} = \prod_{u=\tau}^{t-1} [g(\gamma_u)]^{1 - V_j(u)}$$

Mellin Transform for Service Processes

- Bounds are in terms of Mellin transform of service processes
- For i.i.d. fading channels: $\mathcal{M}_{\mathcal{S}_j^{\text{any}}}(s,\tau,t) = \left[\mathcal{M}_{g_j(\gamma)}(s)\right]^{t-\tau}$
- Assuming Rayleigh fading and average channel gain $\bar{\gamma}$

information–theoretic model	$\mathcal{M}_{g_j(\gamma)}(s) = \frac{1}{\Gamma(m)} \int_0^\infty x^{s-1} \left(\frac{m(x-1)}{\bar{\gamma}}\right)^{m-1} e^{-y}\, dx$
scheduling model	$\mathcal{M}_{g_j(\gamma_u^{\max})}(s) = \sum_{k=0}^{m-1} (-1)^k \binom{m-1}{k} \frac{m\bar{\gamma}^{\frac{s-1}{m}} e^{\frac{k+1}{\bar{\gamma}}}}{(k+1)^{\frac{s+m-1}{m}}} \Gamma\left(\frac{s+m-1}{m}, \frac{k+1}{\bar{\gamma}}\right)$
random access model	$\mathcal{M}_{\mathcal{S}_j^{RA}}(s,\tau,t) = \left[p(1-p)^{m-1} e^{\frac{1}{\bar{\gamma}}} \bar{\gamma}^{s-1} \Gamma(s, \bar{\gamma}^{-1})\right]^{t-\tau}$

Performance Bounds of N Multiaccess Channels

- **Arrivals**: $(\sigma(s), \rho(s))$ bounded arrivals [Chang 2000]

$$\mathcal{M}_{\mathcal{A}}(s,\tau,t) \leq e^{(s-1)\cdot(\rho(s-1)\cdot(t-\tau)+\sigma(s-1))}, \quad s > 1$$

- Backlog: $Pr\left(B(t) > b^{\varepsilon}_{\text{net}}\right) \leq \varepsilon$, where

$$b^{\varepsilon}_{\text{net}} = \inf_{s>0}\left\{\sigma(s) - \frac{1}{s}\left(N\log(1-U(s,m)) + \log\varepsilon\right)\right\}$$

- Delay: $Pr\left(W(t) > w^{\varepsilon}\right) \leq \varepsilon$, where

$$\inf_{s>0}\left\{\frac{e^{s(-\rho(s)w^{\varepsilon}+\sigma(s))}}{(1-U(s,m))^{N}} \cdot \min\left\{1, (U(s,m))^{w^{\varepsilon}}(w^{\varepsilon})^{N-1}\right\}\right\} \leq \varepsilon$$

Values of $U(s,m)$

- Information–theoretic model ($m=2$):

$$U^{IT}(s,2) = e^{s\rho(s)}\frac{1}{\Gamma(2)}e^{2/\bar\gamma}(\frac{\bar\gamma}{2})^{1-s} \quad \cdot \Big(\Gamma(2-s,\frac{2}{\bar\gamma}) - \frac{2}{\bar\gamma}\Gamma(1-s,\frac{2}{\bar\gamma})\Big)$$

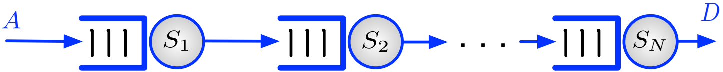
- Random access model:

$$U^{RA}(s,m) = e^{s\rho(s)}p(1-p)^{m-1}e^{\frac{1}{\bar\gamma}}\bar\gamma^{-s}\Gamma(1-s,\bar\gamma^{-1})$$

- Opportunistic scheduler:

$$U(s,m) = \sum_{k=0}^{m-1}(-1)^k\binom{m-1}{k}\frac{m\bar\gamma^{\frac{-s}{m}}e^{s\rho(s)+\frac{k+1}{\bar\gamma}}}{(k+1)^{1-\frac{s}{m}}}\,\Gamma\Big(\frac{m-s}{m},\frac{k+1}{\bar\gamma}\Big)$$

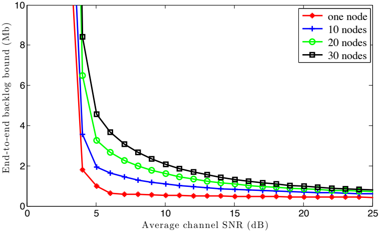
Numerical Results for N Rayleigh Channels



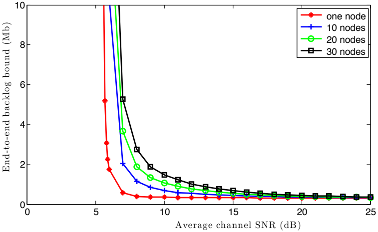
Model parameters

- Sampling interval $\Delta t = 1$ ms
- Transmission bandwidth $W = 20$ kHz
- Arrival (σ, ρ) bounded traffic
- Traffic burst $\sigma = 50$ kb
- Average arrival rate $\rho = 30$ kbps
- Average SNR $\bar{\gamma} = 0$ to 40 dB
- Number of nodes $N = 1$ to 30
- Number of active transmitters $m = 2$
- Bound violation probability $\varepsilon = 10^{-4}$

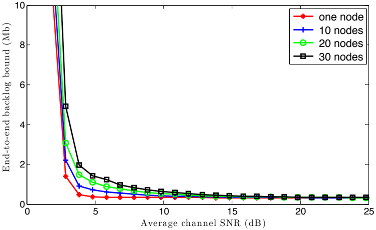
Backlog Bounds for N multiaccess Rayleigh Channels



Information–theoretic



Random access



Opportunistic scheduler

Conclusions

- A common service characterization for three different multiaccess approaches
  - Physical layer (multiaccess channel capacity)
  - MAC layer (random access)
  - Link/Network layer (scheduling)
- **Key:** Analysis in SNR domain using $(\min, \times)$ dioid algebra
- We enabled a qualitative comparison of end–to–end performance bounds