

Bufferless multiplexer

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Bufferless Multiplexer

- Number of flows = N

$$C = 100 \text{ Mbps}$$

Greedy ON-OFF flows
($R = P$ or 0)



What is the Prob. That the rate from all flows exceeds capacity?

$$= \text{What is } P_{loss} = P\left(\sum_{j=1}^N R_j > C\right) \quad ?$$

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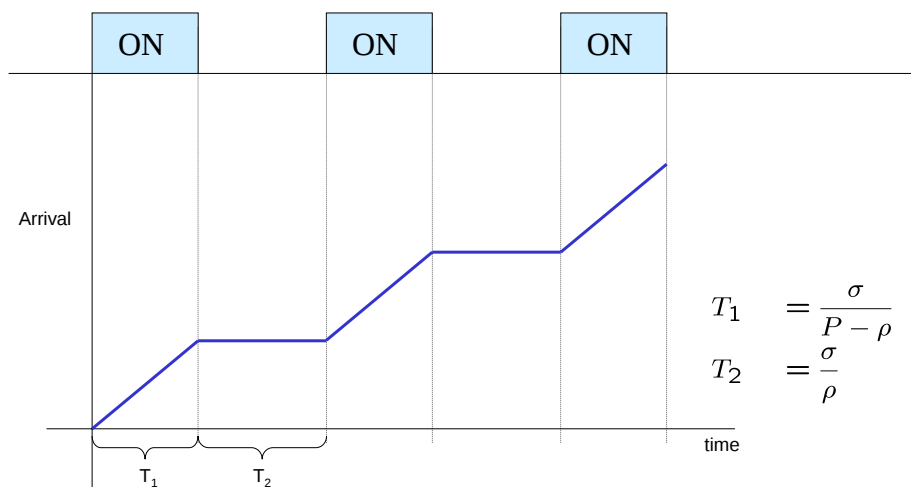
Greedy ON-OFF Source

- ON-OFF source that is constrained by a leaky bucket with parameters P , σ and ρ :
 - **ON Phase:**
 - Send at rate P (until leaky bucket is empty)
 - **OFF Phase:**
 - Do not send (until leaky bucket is full)



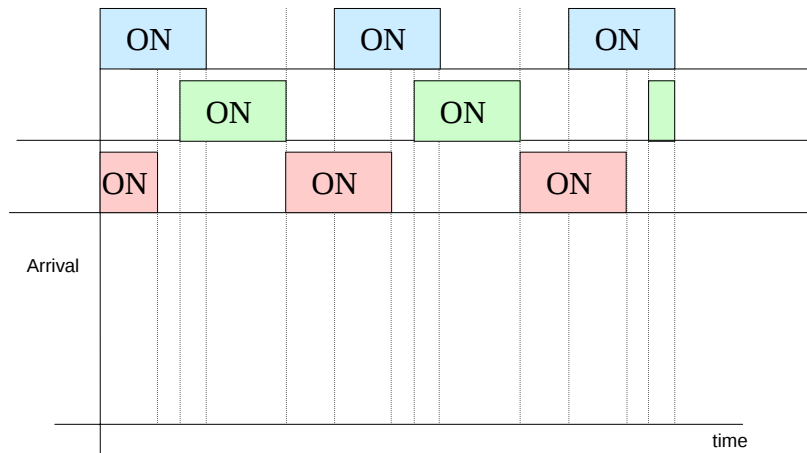
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Greedy ON-OFF Source



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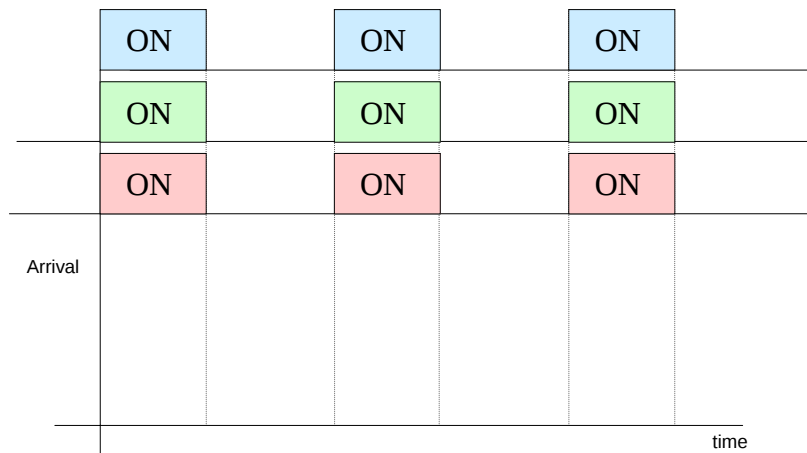
Multiple Greedy ON-OFF Sources



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Multiple Greedy ON-OFF Sources

- Worst-case



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CLT:
$$P_{loss} = Pr\left(\sum_{j=1}^N R_j > C\right) \approx 1 - \Phi\left(\frac{C - N\rho}{\sqrt{N(P\rho - \rho^2)}}\right)$$

Number of flows = N

$C = 100 \text{ Mbps}$

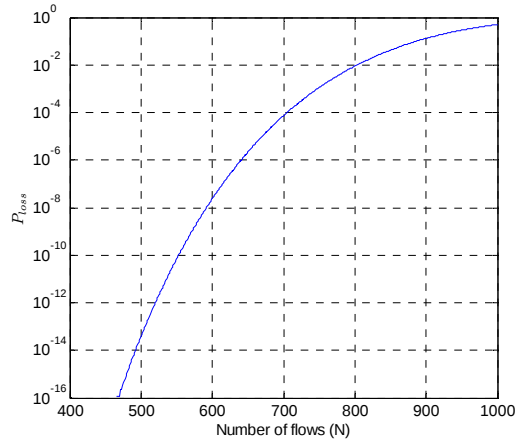
$P = 1 \text{ Mbps}$

$\rho = 0.1 \text{ Mbps}$

$\sigma = 100,000 \text{ bit}$

$$E[R] = \rho$$

$$Var[R] = P\rho - \rho^2$$



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Chernoff Bound:
$$P_{loss} = P\left(\sum_{j=1}^N R_j > C\right) < e^{-Nl(C/N)}$$

$$l(a) = a/P \cdot \ln\left(\frac{aP - \rho}{\rho P - a}\right) - \ln \frac{P - \rho}{P - a}$$

Number of flows = N

$C = 100 \text{ Mbps}$

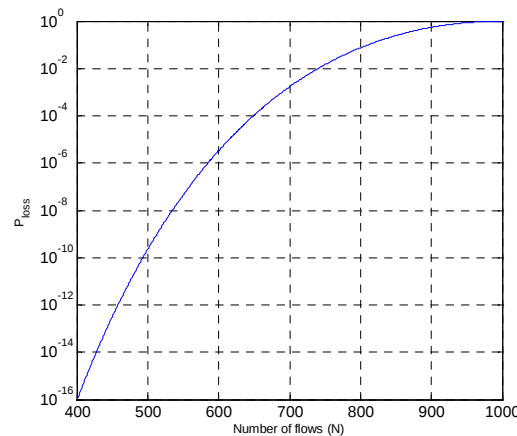
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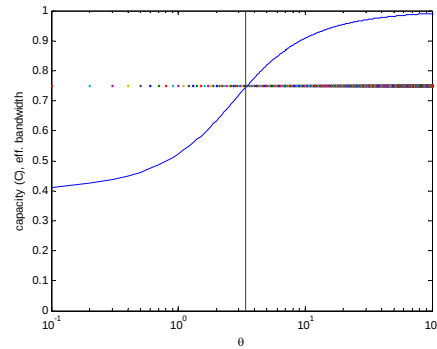
Effective bandwidth

- Slotted system
- Single source:
 - In each slot, source transmits with Prob. p
 - If source transmits, it sends at rate R
- $B_k = \{0, R\}$ transmission in slot k
- A_n are the transmissions in timer interval $[0, n]$

$$A_n = \sum_{k=1}^n B_k$$

$$E[e^{\theta B_k}] = 1 - p + pe^{R\theta}$$

$$\alpha_A(\theta) = \frac{1}{\theta} \ln(1 - p + pe^{R\theta})$$



$$R = 1 \text{ Mbps}$$

$$C = 0.75 \text{ Mbps}$$

$$p = 0.4$$