ECE 466 - Computer Networks II
Winter 2008

Problem Set #8

1. An exponentially distributed random variable X with parameter λ ($\lambda > 0$) is defined by the following probability density function:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & , \ x \geq 0 \\ 0 & , \ x < 0 \end{cases}$$

   (a) Derive the cumulative distribution function $F_X(x)$.
   (b) Derive $E[X]$ and the variance σ_X^2.
   (c) Derive the moment generating function $M_X(\theta)$ of X.
   (d) Derive the first and second moments using the moment generating function of X.
   (e) Derive the exact solution for $P(X > a)$.
   (f) Use the Markov Inequality to estimate $P(X > a)$.
   (g) Use the Chernoff Bound to estimate $P(X > a)$.
   (h) Compare the three results $P(X > a)$ for $\lambda = 1$.
   (i) Why would anyone use the Markov Inequality or Chernoff Bound to calculate $P(X > x)$ for the exponential distribution?

2. Consider a dual-leaky bucket constraint flow with peak rate parameter P, rate parameter ρ, and burst parameter σ. (In other workds, the source has an envelope $E(s) = \min\{P\ s, \sigma + \rho\ s\}$).

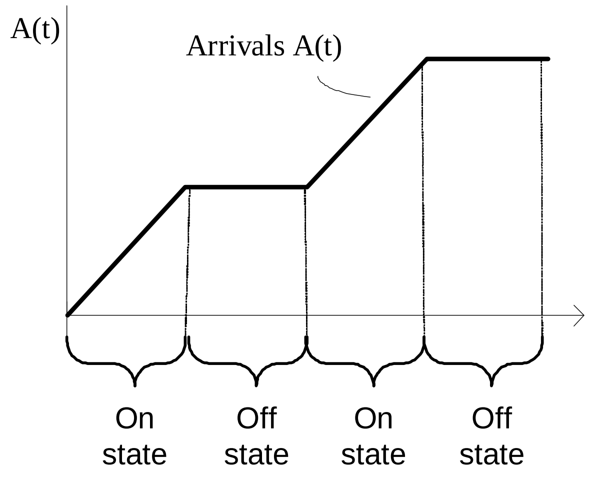
   Suppose the flow transmits in a *Greedy On-Off* fashion, which is described as follows: (1) The source transmits at rate P until the leaky-bucket is empty; (2) Then the source does not transmit until the leaky bucket is completely filled; (3) The source transmits at rate P until the leaky-bucket is empty; and so on. An arrival scenario is given in the Figure below:

   Define the discrete random variable $R \in \{0, P\}$ as follows:

$$P(R = P) = P(\text{source is in 'ON' state})$$
$$P(R = 0) = P(\text{source is in 'OFF' state})$$

   So, R describe the data rate of the source at a randomly selected time.

   (a) Determine $P(R = P)$ and $P(R = 0)$ in terms of P, ρ, and σ.
   (b) Determine $E[R]$, $E[R^2]$ and the variance σ_R^2.
   (c) Determine the moment generating function of R.



THE FOLLOWING PROBLEM ASSUMES THAT THE 'BUFFERLESS MULTIPLEXER' HAS BEEN COVERED IN THE LECTURE.

3. Consider a bufferless multiplexer with capacity $C = 100$ Mbps. We have N i.i.d dual-leaky bucket constrained On-Off flows (as described in the previous problem) with parameters:

$$\begin{aligned} P &= 1\ Mbps \\ \rho &= 0.1\ Mbps \\ \sigma &= 100,000\ bit \end{aligned}$$

Losses occur at the bufferless multiplexer, whenever the data rate from all flows exceeds the capacity C.

(a) Determine the maximum number of flows that can be supported so that no losses occur.

(b) Determine the maximum number of flows such that $P_{loss} \leq 10^{-6}$.

- Estimate the maximum number of flows using the Central Limit Theorem (CLT).
- Estimate the maximum number of flows using the Chernoff Bound.