Instructions: Read carefully before beginning.

- The time for this quiz is 50 minutes.
- You are permitted to use two handwritten aid sheets prepared by you and a calculator.
- You may not use your textbook, lecture notes, homework assignments, or any handouts from class (including solutions).
- Write your answers on the pages provided, using front and back if needed.
- Do not give aid or receive aid from other students.
- Show all steps of your solution.
- There are three problems. Each problem is worth 10 marks.

Name: ____________________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>
1. **EDF Scheduling. (10 marks)**

Consider an arrival scenario from three flows for an Earliest-Deadline-First (EDF) scheduler. The arrival times and packet sizes are as follows:

<table>
<thead>
<tr>
<th>Packet Identifier</th>
<th>Flow 1</th>
<th>Flow 2</th>
<th>Flow 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1a</td>
<td>1b</td>
<td>2b</td>
</tr>
<tr>
<td>Arrival time</td>
<td>0</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Packet Size</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2c</td>
<td>3</td>
<td>3a</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

Note: The arrival scenario is illustrated on the next page.

- Assume that the transmission rate of the link is \(C = 1\), i.e., it takes one time unit to transmit a packet of size 1, two time units to transmit a packet of size 2, etc.
- The delay bounds for arrivals from the flows are:
  - for Flow 1: \(d_1 = 3\),
  - for Flow 2: \(d_2 = 5\),
  - for Flow 3: \(d_3 = 6\).
- A packet transmission - once started - is not interrupted.
- For packets with equal deadlines the transmission order at an EDF scheduler is arbitrary.

(a) (4 marks) Provide the transmission schedule of the EDF scheduler (Provide an illustration or list the departure times of packets). Indicate which packet, if any, may experience a violation of its deadline.

(b) (4 marks) Increase the delay bound of Flow 2, \(d_2\), so that no packet has a deadline violation in the above scenario. The increase of \(d_2\) should be as small as possible. Provide the transmission schedule of EDF with the revised delay bounds.

(c) (2 marks) With the revised delay bounds from (b), does a Static Priority (SP) scheduler cause a violation of a delay bound? Support your answer.

**Solutions:**

see next page
Figure 1: Arrival Scenario for Problem 1. (Packet arrivals are indicated by arrows. Each arrival is labelled with the packet identifier.)
2. Fair Bandwidth Allocation. (10 marks)

Consider a link with capacity $C$ that sees arrivals from a set of flows. Each flow $i$ has an arrival rate of $r_i$. Each flow has a weight $\phi_i$.

(a) (5 marks) Given a link with capacity $C = 100$ Mbps, and arrival rates of five flows (given in Mbps):

$$r_1 = 5, r_2 = 15, r_3 = 20, r_4 = 35, r_5 = 45.$$ All flows have weight $\phi_i = 1$ for $i = 1, \ldots, 5$. Determine the fair share $f$ and the fair bandwidth allocation to each flow.

(b) (5 marks) Given a link with capacity $C = 100$ Mbps, and arrival rates of five flows (given in Mbps):

$$r_1 = 5, r_2 = 15, r_3 = 20, r_4 = 35, r_5 = 45.$$ The flows have weights:

$$\phi_1 = 1, \phi_2 = 2, \phi_3 = 4, \phi_4 = 4, \phi_5 = 6.$$ Determine the fair share $f$ and the fair bandwidth allocation to each flow.

Solutions:

(a) Note: The fair share must satisfy

$$\sum_i \min\{r_i, f\} = C.$$ We get $f = 60/2 = 30$

The allocation is:

$$a_1 = 5, a_2 = 15, a_3 = 20, a_4 = 30, a_5 = 30.$$ 

(b) Note: The fair share must satisfy

$$\sum_i \min\{r_i, \phi_i f\} = C.$$ Each flow $i$ receives an allocation of

$$a_i = \min\{r_i, \phi_i f\}$$ We get $f = 6.25$

The allocation is:

$$a_1 = 5, a_2 = 12.5, a_3 = 20, a_4 = 25, a_5 = 37.5.$$
3. WFQ Scheduling. \textit{(10 marks)}

Consider an arrival scenario of two flows, with arrival times and packet sizes given as follows:

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Packet Identifier & 1a & 1b & 2a & 2b & 2c \\
\hline
Arrival time & 0 & 5 & 1 & 2 & 3 \\
\hline
Packet Size & 2 & 1 & 1 & 2 & 1 \\
\hline
\end{tabular}
\end{center}

\textbf{Note:} The arrival scenario is illustrated on the next page.

- Assume that the transmission rate of the link is \((C = 1)\), i.e., it takes one time unit to transmit a packet of size 1, two time units to transmit a packet of size 2, etc..
- Assume the two flows have equal weights, i.e., \(\phi_1 = \phi_2 = 1\).

\textbf{(a)} \textit{(5 marks)} Devise a transmission schedule of a \underline{fluid-flow} WFQ scheduler. Provide the departure times of all packets.

\textbf{(b)} \textit{(5 marks)} Devise a transmission schedule of a \underline{packet-level} WFQ scheduler. Provide the departure times of all packets.

\textbf{Solutions:}

The problem as posed is quite simple and can be attempted without computing the system virtual time.
Figure 2: Arrival Scenario for Problem 3. (Packet arrivals are indicated by arrows. Each arrival is labelled with the packet identifier.)

Note: The order of packets $1b$ and $2c$ could be different.
This is a more detailed solution for Problem (3b), which follows the canonical solution method.

(a) Obtain system virtual time \( V(t) \).
(b) Compute virtual finishing times of packets (upon their arrival).
(c) Schedule available packets in order of virtual finishing times.

**Virtual Finishing Time**

\[
V(0) = 0
\]
\[
V(t_l - 1 + \tau) = V(t_l - 1) + \frac{C \tau}{\sum_{j \in B_l} \phi_j} \text{ for } \tau \leq t_l - t_l - 1
\]

where \( B_l \) is the set of backlogged flows in the time interval \([t_{l-1}, t_l)\).

![Figure 3: Virtual Finishing Times for Problem 3.](image)

**Virtual Finishing Time (of \(k\)-th packet from flow \(j\)):**

\[
F_j^k = \max\{F_j^{k-1}, V(a_j^k)\} + \frac{L_j^k}{\phi_j},
\]

where \(a_j^k\) is the actual arrival time, \(V(a_j^k)\) is the virtual system time for the arrival, and \(L_j^k\) is the packet size of the \(k\)-th packet from flow \(j\).
This gives the transmission order: $1a, 2a, 2b, 1b, 2c$. 

<table>
<thead>
<tr>
<th>Packet Identifier</th>
<th>Flow 1</th>
<th>Flow 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1a</td>
<td>1b</td>
</tr>
<tr>
<td>Arrival time</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Packet Size</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$V(a^k_j)$</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>$F^k_j$</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>