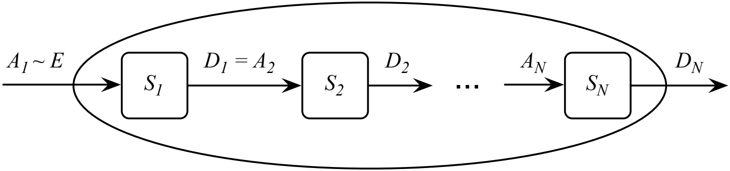
End-to-End Performance Bounds

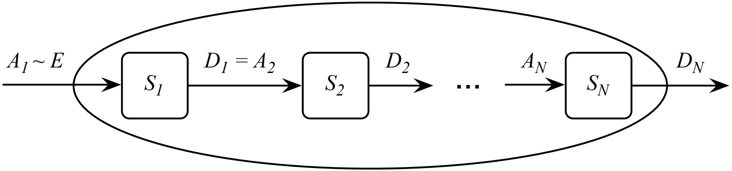
Network of Nodes



- Compute end-to-end delay bound
  - $A_1 \sim E$
  - $D_n \geq A_n \otimes S_n$

$$E(t) = \begin{cases} 0\ , & t \leq 0 \\ b + rt\ , & t > 0 \end{cases}\ , \qquad S_n(t) = \begin{cases} 0\ , & t \leq T \\ R_n(t - T_n)\ , & t > T \end{cases}$$

Two Analysis Approaches



- **Network Service Curve:** Obtain $S^{net} = S_1 \otimes S_2 \otimes \ldots \otimes S_N$, and compute delay bound using E and S^{net}.
- **Node-by-node iteration:** Compute arrival envelopes iteratively at each node.
  - 1. Node: Use envelope E and service curve S_1
  - 2. Node: Use $A_2 \sim E \oslash S_1$ and service curve S_2
  - n. Node: Use $A_n \sim (\ldots(E \oslash S_1) \oslash S_2) \oslash \ldots) \oslash S_{n-1}$ and S_n

Network Service Curve Method

- We have

$$S^{net} = S_1 \otimes S_2 \otimes \ldots \otimes S_N$$

- For latency-rate service curves

$$S^{net} = \begin{cases} 0 \, , & t \leq \sum_{n=1}^{N} T_n \\ \min_i \{R_i\} \left(t - \sum_i T_i\right) \, , & t > \sum_{n=1}^{N} T_n \end{cases} .$$

- Next: We need to compute $E \oslash S^{net}$!

Deconvolution

Deconvolution of token bucket envelope with latency-rate service curve:

$$E(t) = \begin{cases} 0\ , & t \le 0 \\ b + rt\ , & t > 0 \end{cases}\ , \qquad S(t) = \begin{cases} 0\ , & t \le T \\ R(t-T)\ , & t > T \end{cases}\ ,$$

$$E \oslash S(t) = \begin{cases} 0\ , & t \le -(T + \frac{b}{R}) \\ b + R(t+T)\ , & -(T + \frac{b}{R}) < t \le -T, \\ b + r(t+T)\ , & t > -T, \end{cases}\ .$$

Performance Bounds

- **Envelope for Departures**:

$$E \oslash S(t)$$

- **Backlog bound** (b^*):

$$b^* = E \oslash S(0)$$

- **Delay bound** (d^*):
  Smallest non-negative number d satisfying $E \oslash S(-d) \leq 0$.

Performance Bounds

- **Envelope for Departures**:

$$E \oslash S(t) = b + r(\tau + T)$$

- **Backlog bound**:

$$b^* = E \oslash S(0) = b + rT$$

- **Delay bound**:
  From

$$b + R(T - d) \leq 0 \ ,$$

  we get

$$d^* = \frac{b}{R} + T \ .$$

Bounds with Network Service Curve

Departure envelope : $E \oslash S^{net}(t) = b + r \sum_{n=1}^{N} T_n + r\,t$,

End-to-end backlog: $b^*_{net} = b + r \sum_{n=1}^{N} T_n$,

End-to-end delay : $d^*_{net} = \frac{b}{\min\limits_{n=1,\ldots,N} R_n} + \sum_{n=1}^{N} T_n$.

Node-by-Node Iteration:

Compute the output envelopes at each node:

$$
\begin{aligned}
D_1 &\sim E \oslash S_1 \\
D_2 &\sim (E \oslash S_1) \oslash S_2 \\
&\dots \\
D_N &\sim (\dots(E \oslash S_1) \oslash S_2) \oslash \dots) \oslash S_N
\end{aligned}
$$

Node-by-Node Iteration

Compute the output envelopes at each node:

$$D_1 \sim E \oslash S_1 = b + r\,T_1 + r\,t \ ,$$

$$D_2 \sim (E \oslash S_1) \oslash S_2 = b + r\,(T_1 + T_2) + r\,t \ ,$$

$$\ldots$$

$$D_N \sim (\ldots(E \oslash S_1) \oslash S_2) \oslash \ldots) \oslash S_N = b + r \sum_{i=1}^{N} T_i + r\,t \ ,$$

Node-by-Node Iteration

- E_n^{out}: departure envelope at n-th node,
- b_n^*: backlog envelope at n-th node,
- d_n^*: delay at n-th node. are given by

$$E_n^{out}(t) =$$
$$b_n^* =$$
$$d_n^* =$$

Node-by-Node Iteration

- E_n^{out}: departure envelope at n-th node,
- b_n^*: backlog envelope at n-th node,
- d_n^*: delay at n-th node. are given by

$$E_n^{out}(t) = b + r \sum_{i=1}^{n} T_i + r\,t \ ,$$

$$b_n^* = b + r \sum_{i=1}^{n} T_i \ ,$$

$$d_n^* = \frac{1}{R_n}(b + r \sum_{i=1}^{n-1} T_i) + T_n \ .$$

Node-by-Node Iteration: Envelope of Departures

The envelope of the departures from the network (E_{net}^{out})is E_n^{out} with $n = N$.

$$E_{net}^{out}(t) = E_N^{out}(t) = b + r \sum_{i=1}^{N} T_i + r\,t \ .$$

Node-by-Node Iteration: Backlog Bound

$$
\begin{aligned}
b^*_{net} &= \sum_{n=1}^{N} b^*_n \\
&= \sum_{n=1}^{N} \left(b + r \sum_{i=1}^{n} T_i \right) \\
&= N\ b + r \sum_{n=1}^{N} (N + 1 - n)\, T_n \ .
\end{aligned}
$$

Node-by-Node Iteration: Delay Bound

$$
\begin{aligned}
d^*_{net} &= \sum_{n=1}^{N} d^*_n \\
&= \sum_{n=1}^{N} \left(\frac{1}{R_n}(b + r \sum_{i=1}^{n-1} T_i) + T_n \right) \; .
\end{aligned}
$$

Comparison of the Two Methods

Simplify by assuming for all n:

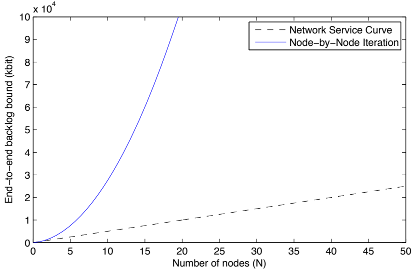
$$S_n(t) = \begin{cases} 0 \ , & t \leq T \\ R(t-T) \ , & t > T \end{cases}$$

	Network Service Curve	Node-by-Node Iteration
b^*_{net}	$b + NrT$	$Nb + (N^2 + N)\frac{rT}{2}$
d^*_{net}	$\frac{b}{R} + NT$	$\frac{Nb}{R} + NT + (N^2 - N)\frac{rT}{2R}$

End-to-End Bounds

- $b = 10\ kb,$
- $r = 100\ kbps,$
- $T = 5\ ms,$
- $R = 500\ kbps$

End-to-End Backlog Bound



End-to-End Delay Bound